

# Computer algebra independent integration tests

3-Logarithms/3.1.2-d-x<sup>m</sup>-a+b-log-c-x<sup>n</sup>-p

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3.117	$\int \sqrt{\log(ax^n)} dx$	386
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3.131	$\int \frac{1}{\sqrt{\log(ax^n)}} dx$	428
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3.133	$\int \frac{1}{x^2 \sqrt{\log(ax^n)}} dx$	434
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3.135	$\int \frac{x^3}{\log^{\frac{3}{2}}(ax^n)} dx$	440
3.136	$\int \frac{x^2}{\log^{\frac{3}{2}}(ax^n)} dx$	443
3.137	$\int \frac{x}{\log^{\frac{3}{2}}(ax^n)} dx$	446

3.138	$\int \frac{1}{\log^{\frac{3}{2}}(ax^n)} dx$	449
3.139	$\int \frac{1}{x \log^{\frac{3}{2}}(ax^n)} dx$	452
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3.141	$\int \frac{1}{x^3 \log^{\frac{3}{2}}(ax^n)} dx$	458
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3.143	$\int \frac{x^2}{\log^{\frac{5}{2}}(ax^n)} dx$	464
3.144	$\int \frac{x}{\log^{\frac{5}{2}}(ax^n)} dx$	467
3.145	$\int \frac{1}{\log^{\frac{5}{2}}(ax^n)} dx$	470
3.146	$\int \frac{1}{x \log^{\frac{5}{2}}(ax^n)} dx$	473
3.147	$\int \frac{1}{x^2 \log^{\frac{5}{2}}(ax^n)} dx$	476
3.148	$\int \frac{1}{x^3 \log^{\frac{5}{2}}(ax^n)} dx$	479
3.149	$\int (dx)^m \left( a + \frac{a(1+m) \log(cx^n)}{n} \right) dx$	482
3.150	$\int (dx)^m (a + b \log(cx^n))^3 dx$	485
3.151	$\int (dx)^m (a + b \log(cx^n))^2 dx$	488
3.152	$\int (dx)^m (a + b \log(cx^n)) dx$	492
3.153	$\int \frac{(dx)^m}{a + b \log(cx^n)} dx$	495
3.154	$\int \frac{(dx)^m}{(a + b \log(cx^n))^2} dx$	498
3.155	$\int \frac{(dx)^m}{(a + b \log(cx^n))^3} dx$	501
3.156	$\int (dx)^{-1+n} \log^3(cx^n) dx$	504
3.157	$\int (dx)^{-1+n} \log^2(cx^n) dx$	507
3.158	$\int (dx)^{-1+n} \log(cx^n) dx$	510
3.159	$\int \frac{(dx)^{-1+n}}{\log(cx^n)} dx$	513
3.160	$\int \frac{(dx)^{-1+n}}{\log^2(cx^n)} dx$	516
3.161	$\int \frac{(dx)^{-1+n}}{\log^3(cx^n)} dx$	519
3.162	$\int x^m \log^{\frac{2}{3}}(ax^n) dx$	522
3.163	$\int x^m \sqrt{\log(ax^n)} dx$	525
3.164	$\int \frac{x^m}{\sqrt{\log(ax^n)}} dx$	528
3.165	$\int \frac{x^m}{\log^{\frac{3}{2}}(ax^n)} dx$	531
3.166	$\int \frac{x^m}{\log^{\frac{5}{2}}(ax^n)} dx$	534
3.167	$\int (dx)^m (a + b \log(cx^n))^p dx$	537
3.168	$\int x^2 (a + b \log(cx^n))^p dx$	540
3.169	$\int x (a + b \log(cx^n))^p dx$	543
3.170	$\int (a + b \log(cx^n))^p dx$	546
3.171	$\int \frac{(a + b \log(cx^n))^p}{x} dx$	549
3.172	$\int \frac{(a + b \log(cx^n))^p}{x^2} dx$	552
3.173	$\int \frac{(a + b \log(cx^n))^p}{x^3} dx$	555
3.174	$\int \frac{(a + b \log(cx^n))^p}{x^4} dx$	558
3.175	$\int (dx)^m (a + b \log(cx))^p dx$	561
3.176	$\int x^2 (a + b \log(cx))^p dx$	564

3.177	$\int x(a + b \log(cx))^p dx$	567
3.178	$\int (a + b \log(cx))^p dx$	570
3.179	$\int \frac{(a+b \log(cx))^p}{x} dx$	573
3.180	$\int \frac{(a+b \log(cx))^p}{x^2} dx$	576
3.181	$\int \frac{(a+b \log(cx))^p}{x^3} dx$	579
3.182	$\int \frac{(a+b \log(cx))^p}{x^4} dx$	582
3.183	$\int (dx)^m (a + b \log(c\sqrt{x}))^p dx$	585
3.184	$\int x^2 (a + b \log(c\sqrt{x}))^p dx$	588
3.185	$\int x (a + b \log(c\sqrt{x}))^p dx$	591
3.186	$\int (a + b \log(c\sqrt{x}))^p dx$	594
3.187	$\int \frac{(a+b \log(c\sqrt{x}))^p}{x} dx$	597
3.188	$\int \frac{(a+b \log(c\sqrt{x}))^p}{x^2} dx$	600
3.189	$\int \frac{(a+b \log(c\sqrt{x}))^p}{x^3} dx$	603
3.190	$\int \frac{(a+b \log(c\sqrt{x}))^p}{x^4} dx$	606
3.191	$\int x^{-1+n} (a + b \log(cx^n))^p dx$	609
3.192	$\int (dx^q)^m (a + b \log(cx^n))^p dx$	612
3.193	$\int (d1x^{q1})^{m1} (d2x^{q2})^{m2} (a + b \log(cx^n))^p dx$	615

#### 4 Listing of Grading functions

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# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 193 ]. This is test number [ 56 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

### 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. ( 193 )	% 0. ( 0 )
Mathematica	% 100. ( 193 )	% 0. ( 0 )
Maple	% 50.78 ( 98 )	% 49.22 ( 95 )
Maxima	% 49.74 ( 96 )	% 50.26 ( 97 )
Fricas	% 63.73 ( 123 )	% 36.27 ( 70 )
Sympy	% 36.79 ( 71 )	% 63.21 ( 122 )
Giac	% 52.33 ( 101 )	% 47.67 ( 92 )

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

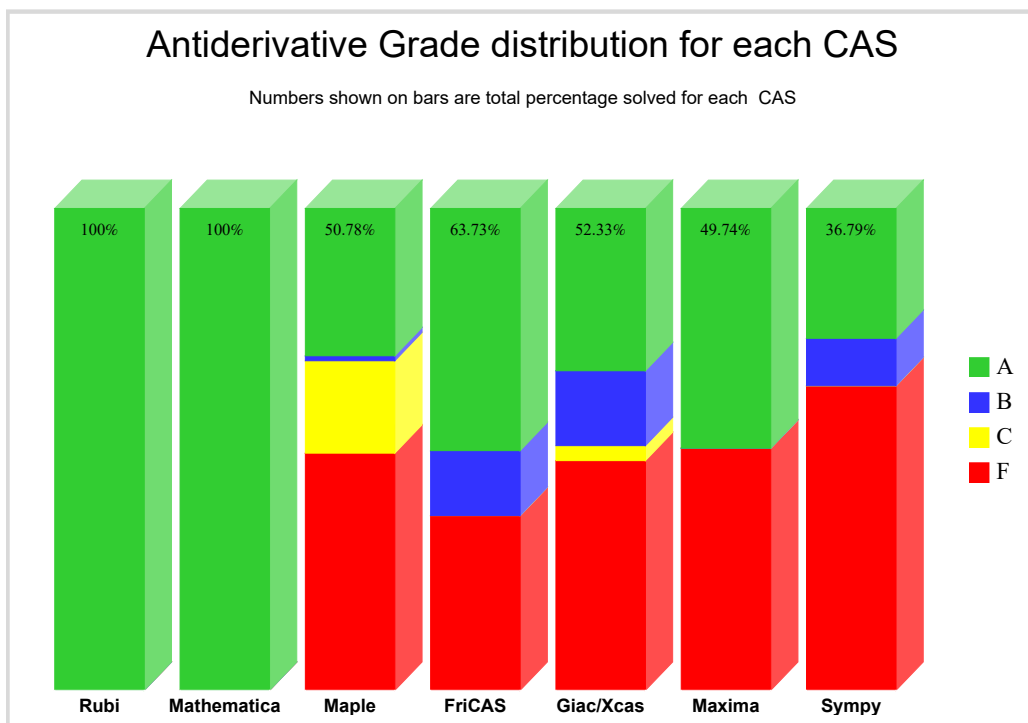
grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

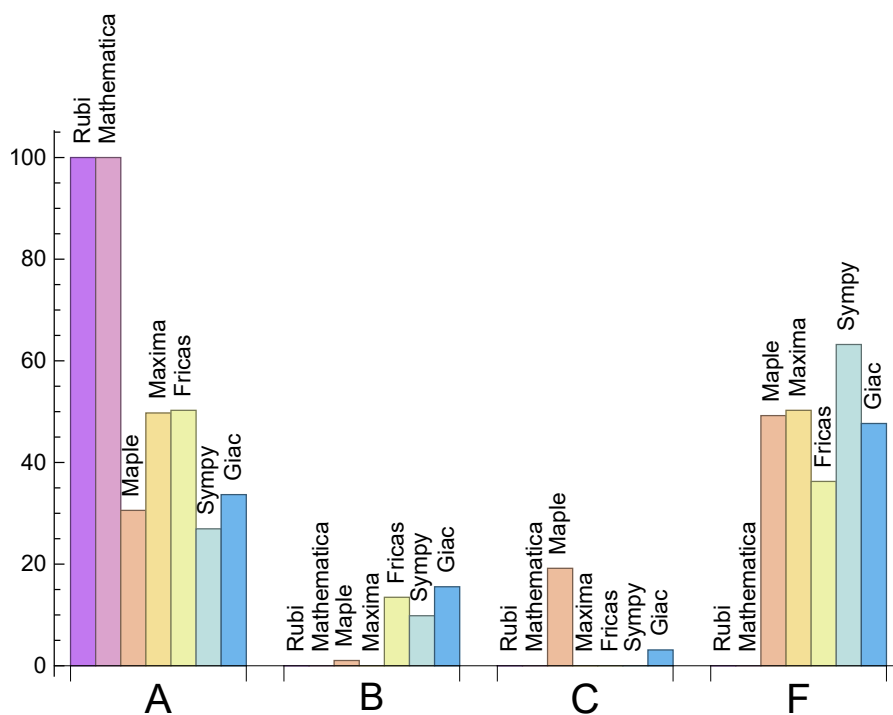
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	100.	0.	0.	0.
Maple	30.57	1.04	19.17	49.22
Maxima	49.74	0.	0.	50.26
Fricas	50.26	13.47	0.	36.27
Sympy	26.94	9.84	0.	63.21
Giac	33.68	15.54	3.11	47.67



The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



### 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.05	56.56	1.	58.	1.
Mathematica	0.04	53.03	0.95	54.	1.
Maple	0.11	461.81	6.82	37.	1.17
Maxima	1.13	52.04	1.27	30.	1.2
Fricas	0.87	172.19	3.46	100.	3.11
Sympy	11.56	83.24	1.9	41.	1.64
Giac	1.26	179.65	2.96	50.	1.44

## 1.4 list of integrals that has no closed form antiderivative

{}

## 1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

## 1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```

from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')

```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in->

```

def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

```

```

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1

```

For Sympy, called directly from Python, the following code is used

```

try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

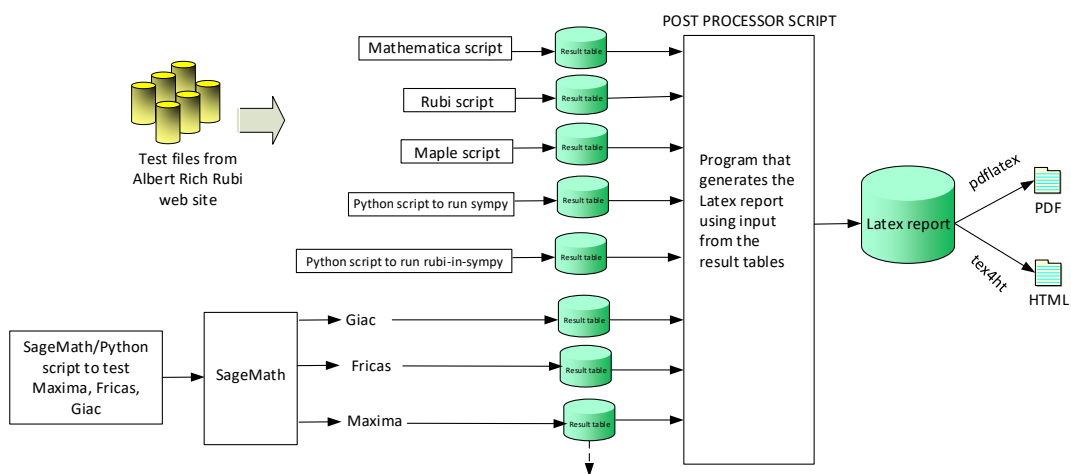
except Exception as ee:
    leafCount =1

```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

### High level overview of the CAS independent integration test build system

Nasser M. Abbasi  
June 22, 2018



# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193 }

B grade: { }

C grade: { }

F grade: { }

#### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193 }

B grade: { }

C grade: { }

F grade: { }

#### 2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 45, 46, 47, 53, 69, 77, 85, 92, 98, 118, 125, 132, 139, 146, 171, 179, 187 }

B grade: { 54, 61 }

C grade: { 43, 44, 48, 49, 50, 51, 52, 55, 56, 57, 58, 59, 60, 62, 63, 64, 68, 76, 84, 89, 90, 91, 93, 94, 95, 96, 97, 99, 100, 110, 149, 150, 151, 152, 156, 157, 158 }

F grade: { 65, 66, 67, 70, 71, 72, 73, 74, 75, 78, 79, 80, 81, 82, 83, 86, 87, 88, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 147, 148, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193 }

## 2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 69, 77, 85, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 118, 125, 132, 139, 146, 176, 177, 178, 180, 181, 182, 184, 185, 186, 188, 189, 190 }

B grade: { }

C grade: { }

F grade: { 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 179, 183, 187, 191, 192, 193 }

## 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 53, 55, 56, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 118, 132, 139, 149, 152, 153, 154, 156, 157, 158, 159, 160, 161, 170, 171, 178, 179, 187 }

B grade: { 50, 51, 52, 54, 57, 58, 59, 60, 61, 62, 63, 64, 81, 82, 83, 84, 85, 86, 87, 88, 95, 125, 146, 150, 151, 155 }

C grade: { }

F grade: { 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 147, 148, 162, 163, 164, 165, 166, 167, 168, 169, 172, 173, 174, 175, 176, 177, 180, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193 }

## 2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 25, 26, 32, 33, 39, 40, 43, 44, 45, 46, 47, 48, 49, 54, 69, 77, 90, 91, 92, 93, 94, 118, 125, 132, 139, 149, 157, 158, 171, 179, 187 }

B grade: { 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 96, 97, 98, 99, 100 }

C grade: { }

F grade: { 22, 23, 24, 27, 28, 29, 30, 31, 34, 35, 36, 37, 38, 41, 42, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 95, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193 }



## 2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 43, 44, 45, 46, 47, 48, 49, 55, 56, 65, 66, 67, 68, 77, 85, 92, 93, 94, 98, 105, 118, 125, 132, 139, 146, 157, 158, 171, 179, 187 }

B grade: { 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 69, 73, 74, 75, 76, 81, 82, 83, 84, 99, 100, 111, 149, 150, 151, 152, 156 }

C grade: { 89, 90, 91, 95, 96, 97 }

F grade: { 26, 27, 28, 34, 35, 41, 42, 70, 71, 72, 78, 79, 80, 86, 87, 88, 101, 102, 103, 104, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 147, 148, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	39	14	20
normalized size	1	1.	1.	0.84	1.05	2.05	0.74	1.05
time (sec)	N/A	0.008	0.001	0.037	0.955	0.788	0.092	1.073

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	38	14	20
normalized size	1	1.	1.	0.84	1.05	2.	0.74	1.05
time (sec)	N/A	0.007	0.001	0.036	0.959	0.839	0.097	1.081

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	38	14	20
normalized size	1	1.	1.	0.84	1.05	2.	0.74	1.05
time (sec)	N/A	0.004	0.001	0.037	0.96	0.815	0.087	1.105

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	22	22	7	22
normalized size	1	1.	1.	1.1	2.2	2.2	0.7	2.2
time (sec)	N/A	0.001	0.001	0.036	0.99	0.845	0.089	1.07

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	22	7	11
normalized size	1	1.	1.	0.9	1.1	2.2	0.7	1.1
time (sec)	N/A	0.006	0.001	0.035	0.969	0.8	0.085	1.233

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	20	26	10	20
normalized size	1	1.	1.	1.07	1.33	1.73	0.67	1.33
time (sec)	N/A	0.007	0.001	0.034	0.99	0.86	0.098	1.617

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	36	17	20
normalized size	1	1.	1.	0.84	1.05	1.89	0.89	1.05
time (sec)	N/A	0.007	0.001	0.035	0.972	0.826	0.132	1.098

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	28	68	26	35
normalized size	1	1.	1.	0.84	0.88	2.12	0.81	1.09
time (sec)	N/A	0.02	0.001	0.036	0.975	0.829	0.11	1.119

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	28	68	29	35
normalized size	1	1.	1.	0.84	0.88	2.12	0.91	1.09
time (sec)	N/A	0.02	0.001	0.036	0.992	0.756	0.109	1.199

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	28	66	26	35
normalized size	1	1.	1.	0.84	0.88	2.06	0.81	1.09
time (sec)	N/A	0.011	0.001	0.036	0.978	0.745	0.113	1.109

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	22	47	19	26
normalized size	1	1.	1.	1.05	1.16	2.47	1.	1.37
time (sec)	N/A	0.005	0.001	0.036	0.996	0.816	0.096	1.097

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	22	7	11
normalized size	1	1.	1.	0.9	1.1	2.2	0.7	1.1
time (sec)	N/A	0.012	0.001	0.035	0.968	0.79	0.087	1.11

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	26	46	20	35
normalized size	1	1.	1.	1.04	1.	1.77	0.77	1.35
time (sec)	N/A	0.02	0.001	0.036	0.967	0.795	0.111	1.12

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	28	57	29	35
normalized size	1	1.	1.	0.84	0.88	1.78	0.91	1.09
time (sec)	N/A	0.019	0.001	0.037	0.985	0.805	0.129	1.103

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	38	39	100	42	50
normalized size	1	1.	1.	0.84	0.87	2.22	0.93	1.11
time (sec)	N/A	0.036	0.001	0.036	0.991	0.767	0.123	1.107

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	38	39	96	41	50
normalized size	1	1.	1.	0.84	0.87	2.13	0.91	1.11
time (sec)	N/A	0.031	0.001	0.036	1.074	0.781	0.122	1.102

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	38	39	95	42	50
normalized size	1	1.	1.	0.84	0.87	2.11	0.93	1.11
time (sec)	N/A	0.018	0.001	0.033	1.005	0.823	0.122	1.099

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	32	70	29	38
normalized size	1	1.	1.	1.04	1.14	2.5	1.04	1.36
time (sec)	N/A	0.008	0.001	0.035	1.022	0.748	0.11	1.13

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	22	7	11
normalized size	1	1.	1.	0.9	1.1	2.2	0.7	1.1
time (sec)	N/A	0.011	0.001	0.033	1.06	0.873	0.091	1.079

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	38	36	66	31	50
normalized size	1	1.	1.	1.03	0.97	1.78	0.84	1.35
time (sec)	N/A	0.034	0.001	0.036	1.076	0.836	0.139	1.471

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	38	39	77	44	50
normalized size	1	1.	1.	0.84	0.87	1.71	0.98	1.11
time (sec)	N/A	0.033	0.001	0.035	1.064	0.885	0.15	1.135

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	15	36	0	15
normalized size	1	1.	1.	1.27	1.36	3.27	0.	1.36
time (sec)	N/A	0.024	0.014	0.04	1.121	0.731	0.	1.132

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	15	36	0	15
normalized size	1	1.	1.	1.27	1.36	3.27	0.	1.36
time (sec)	N/A	0.023	0.014	0.039	1.14	0.699	0.	1.12

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	15	36	0	15
normalized size	1	1.	1.	1.27	1.36	3.27	0.	1.36
time (sec)	N/A	0.017	0.013	0.043	1.132	0.836	0.	1.108

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	14	12	28	5	12
normalized size	1	1.	1.	1.75	1.5	3.5	0.62	1.5
time (sec)	N/A	0.003	0.005	0.037	1.182	0.844	0.51	1.105

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	7	20	5	0
normalized size	1	1.	1.	1.2	1.4	4.	1.	0.
time (sec)	N/A	0.012	0.005	0.033	1.031	0.853	0.101	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	12	34	0	0
normalized size	1	1.	1.	1.11	1.33	3.78	0.	0.
time (sec)	N/A	0.024	0.014	0.038	1.219	0.841	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	15	42	0	0
normalized size	1	1.	1.	1.27	1.36	3.82	0.	0.
time (sec)	N/A	0.024	0.014	0.038	1.169	0.724	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	26	18	84	0	32
normalized size	1	1.	1.	1.08	0.75	3.5	0.	1.33
time (sec)	N/A	0.038	0.015	0.033	1.184	0.707	0.	1.122

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	26	18	84	0	32
normalized size	1	1.	1.	1.08	0.75	3.5	0.	1.33
time (sec)	N/A	0.037	0.015	0.033	1.229	0.756	0.	1.104

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	26	18	84	0	32
normalized size	1	1.	1.	1.08	0.75	3.5	0.	1.33
time (sec)	N/A	0.026	0.013	0.036	1.254	0.887	0.	1.141

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	24	16	68	12	26
normalized size	1	1.	1.	1.33	0.89	3.78	0.67	1.44
time (sec)	N/A	0.005	0.004	0.036	1.235	0.792	0.488	1.12

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	18	7	11
normalized size	1	1.	1.	1.12	1.38	2.25	0.88	1.38
time (sec)	N/A	0.012	0.001	0.033	1.094	0.768	0.083	1.105

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	12	76	0	0
normalized size	1	1.	1.	0.95	0.55	3.45	0.	0.
time (sec)	N/A	0.035	0.015	0.036	1.247	0.578	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	26	18	92	0	0
normalized size	1	1.	1.	1.08	0.75	3.83	0.	0.
time (sec)	N/A	0.037	0.015	0.035	1.261	0.687	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	37	18	124	0	47
normalized size	1	1.	1.	1.	0.49	3.35	0.	1.27
time (sec)	N/A	0.052	0.016	0.046	1.255	0.857	0.	1.125

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	37	18	123	0	47
normalized size	1	1.	1.	0.9	0.44	3.	0.	1.15
time (sec)	N/A	0.053	0.015	0.034	1.125	0.756	0.	1.121

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	37	18	123	0	47
normalized size	1	1.	1.	1.	0.49	3.32	0.	1.27
time (sec)	N/A	0.031	0.005	0.045	1.153	0.853	0.	1.134

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	18	99	26	39
normalized size	1	1.	1.	0.97	0.53	2.91	0.76	1.15
time (sec)	N/A	0.009	0.005	0.035	1.165	0.664	0.519	1.118

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	23	10	11
normalized size	1	1.	1.	0.9	1.1	2.3	1.	1.1
time (sec)	N/A	0.012	0.001	0.033	1.109	0.861	0.09	1.11

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	33	12	100	0	0
normalized size	1	1.	1.	0.85	0.31	2.56	0.	0.
time (sec)	N/A	0.05	0.015	0.034	1.166	0.846	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	36	18	119	0	0
normalized size	1	1.	1.	1.	0.5	3.31	0.	0.
time (sec)	N/A	0.049	0.016	0.033	1.241	0.717	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	32	112	35	84	36	42
normalized size	1	1.	1.19	4.15	1.3	3.11	1.33	1.56
time (sec)	N/A	0.013	0.002	0.21	1.109	0.787	1.397	1.112

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	32	112	35	82	36	42
normalized size	1	1.	1.19	4.15	1.3	3.04	1.33	1.56
time (sec)	N/A	0.012	0.001	0.167	1.026	0.862	0.819	1.097

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	32	29	35	82	36	42
normalized size	1	1.	1.19	1.07	1.3	3.04	1.33	1.56
time (sec)	N/A	0.007	0.001	0.052	1.169	0.803	0.46	1.175

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	55	19	27
normalized size	1	1.	1.	1.06	1.33	3.06	1.06	1.5
time (sec)	N/A	0.006	0.001	0.038	1.103	0.794	0.239	1.178

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	21	27	27	57	36	26
normalized size	1	1.	0.95	1.23	1.23	2.59	1.64	1.18
time (sec)	N/A	0.012	0.001	0.035	1.117	0.878	6.95	1.139

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	26	112	35	51	24	32
normalized size	1	1.	1.13	4.87	1.52	2.22	1.04	1.39
time (sec)	N/A	0.013	0.001	0.082	1.189	0.961	0.474	1.175

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	32	111	35	68	37	36
normalized size	1	1.	1.19	4.11	1.3	2.52	1.37	1.33
time (sec)	N/A	0.013	0.001	0.073	1.138	0.835	1.24	1.15

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	43	691	96	244	131	150
normalized size	1	1.	0.83	13.29	1.85	4.69	2.52	2.88
time (sec)	N/A	0.036	0.016	0.191	1.128	0.891	2.911	1.157

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	46	692	96	247	143	150
normalized size	1	1.	0.88	13.31	1.85	4.75	2.75	2.88
time (sec)	N/A	0.037	0.019	0.204	1.086	0.848	1.735	1.186

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	41	692	95	243	126	146
normalized size	1	1.	0.79	13.31	1.83	4.67	2.42	2.81
time (sec)	N/A	0.023	0.013	0.213	1.119	0.879	1.02	1.15



Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	33	63	77	197	109	119
normalized size	1	1.	0.77	1.47	1.79	4.58	2.53	2.77
time (sec)	N/A	0.013	0.008	0.05	1.089	0.792	0.607	1.197

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	56	27	136	60	76
normalized size	1	1.	1.	2.55	1.23	6.18	2.73	3.45
time (sec)	N/A	0.024	0.003	0.037	1.188	0.838	22.342	1.242

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	35	704	95	182	110	116
normalized size	1	1.	0.76	15.3	2.07	3.96	2.39	2.52
time (sec)	N/A	0.035	0.011	0.122	1.118	0.86	1.075	1.156

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	41	703	96	204	128	122
normalized size	1	1.	0.79	13.52	1.85	3.92	2.46	2.35
time (sec)	N/A	0.035	0.013	0.122	1.159	0.864	1.742	1.23

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	66	2649	182	517	338	354
normalized size	1	1.	0.86	34.4	2.36	6.71	4.39	4.6
time (sec)	N/A	0.063	0.029	0.318	1.135	0.866	7.593	1.263

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	67	2650	181	512	311	346
normalized size	1	1.	0.87	34.42	2.35	6.65	4.04	4.49
time (sec)	N/A	0.06	0.014	0.332	1.184	0.954	4.036	1.291

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	60	2650	182	505	337	354
normalized size	1	1.	0.78	34.42	2.36	6.56	4.38	4.6
time (sec)	N/A	0.039	0.03	0.334	1.124	0.814	2.492	1.185

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	50	2641	153	435	270	296
normalized size	1	1.	0.76	40.02	2.32	6.59	4.09	4.48
time (sec)	N/A	0.024	0.009	0.299	1.22	0.899	1.394	1.23

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	75	27	255	92	154
normalized size	1	1.	1.	3.41	1.23	11.59	4.18	7.
time (sec)	N/A	0.022	0.003	0.036	1.102	0.81	37.442	1.224

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	52	2674	180	406	272	266
normalized size	1	1.	0.75	38.75	2.61	5.88	3.94	3.86
time (sec)	N/A	0.06	0.018	0.24	1.159	0.882	1.601	1.219

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	60	2673	182	436	338	274
normalized size	1	1.	0.78	34.71	2.36	5.66	4.39	3.56
time (sec)	N/A	0.059	0.021	0.237	1.176	0.938	1.69	1.222

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	60	2674	184	443	313	275
normalized size	1	1.	0.78	34.73	2.39	5.75	4.06	3.57
time (sec)	N/A	0.061	0.024	0.249	1.1	0.788	3.781	1.172

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	108	0	65
normalized size	1	1.	1.	0.	0.	2.12	0.	1.27
time (sec)	N/A	0.056	0.058	0.167	0.	0.904	0.	1.195

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	108	0	65
normalized size	1	1.	1.	0.	0.	2.12	0.	1.27
time (sec)	N/A	0.055	0.055	0.168	0.	0.856	0.	1.183

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	108	0	65
normalized size	1	1.	1.	0.	0.	2.12	0.	1.27
time (sec)	N/A	0.045	0.051	0.155	0.	0.819	0.	1.2

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	241	0	100	0	57
normalized size	1	1.	1.	5.02	0.	2.08	0.	1.19
time (sec)	N/A	0.036	0.041	0.267	0.	0.837	0.	1.214

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	51	32	61
normalized size	1	1.	1.	1.06	1.33	2.83	1.78	3.39
time (sec)	N/A	0.025	0.016	0.038	1.058	0.828	1.989	1.235

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	100	0	0
normalized size	1	1.	1.	0.	0.	2.08	0.	0.
time (sec)	N/A	0.051	0.049	0.151	0.	0.737	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	108	0	0
normalized size	1	1.	1.	0.	0.	2.12	0.	0.
time (sec)	N/A	0.051	0.05	0.165	0.	0.785	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	108	0	0
normalized size	1	1.	1.	0.	0.	2.12	0.	0.
time (sec)	N/A	0.052	0.052	0.184	0.	0.8	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	70	0	0	257	0	352
normalized size	1	1.	0.92	0.	0.	3.38	0.	4.63
time (sec)	N/A	0.079	0.118	0.747	0.	0.834	0.	1.282

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	70	0	0	257	0	352
normalized size	1	1.	0.92	0.	0.	3.38	0.	4.63
time (sec)	N/A	0.079	0.121	0.691	0.	0.888	0.	1.339

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	70	0	0	257	0	352
normalized size	1	1.	0.92	0.	0.	3.38	0.	4.63
time (sec)	N/A	0.059	0.118	0.65	0.	0.899	0.	1.326

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	66	351	0	240	0	321
normalized size	1	1.	0.94	5.01	0.	3.43	0.	4.59
time (sec)	N/A	0.04	0.098	0.265	0.	0.952	0.	1.222

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	27	59	70	28
normalized size	1	1.	1.	1.05	1.35	2.95	3.5	1.4
time (sec)	N/A	0.024	0.005	0.034	1.132	0.805	49.5	1.188

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	76	0	0	220	0	0
normalized size	1	1.	1.04	0.	0.	3.01	0.	0.
time (sec)	N/A	0.075	0.089	0.659	0.	0.773	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	80	0	0	247	0	0
normalized size	1	1.	1.05	0.	0.	3.25	0.	0.
time (sec)	N/A	0.078	0.091	0.725	0.	0.952	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	80	0	0	247	0	0
normalized size	1	1.	1.05	0.	0.	3.25	0.	0.
time (sec)	N/A	0.075	0.094	0.756	0.	0.857	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	89	0	0	520	0	1389
normalized size	1	1.	0.88	0.	0.	5.15	0.	13.75
time (sec)	N/A	0.108	0.135	0.705	0.	0.827	0.	1.441

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	89	0	0	518	0	1389
normalized size	1	1.	0.85	0.	0.	4.93	0.	13.23
time (sec)	N/A	0.108	0.131	0.662	0.	0.95	0.	1.6

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	89	0	0	518	0	1389
normalized size	1	1.	0.88	0.	0.	5.13	0.	13.75
time (sec)	N/A	0.08	0.13	0.652	0.	0.783	0.	1.804

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	82	460	0	489	0	1326
normalized size	1	1.	0.84	4.69	0.	4.99	0.	13.53
time (sec)	N/A	0.052	0.114	0.268	0.	0.762	0.	1.329

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	27	150	0	28
normalized size	1	1.	1.	0.95	1.23	6.82	0.	1.27
time (sec)	N/A	0.024	0.004	0.035	1.096	0.792	0.	1.288

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	94	0	0	473	0	0
normalized size	1	1.	0.92	0.	0.	4.64	0.	0.
time (sec)	N/A	0.106	0.094	0.719	0.	0.904	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	89	0	0	524	0	0
normalized size	1	1.	0.89	0.	0.	5.24	0.	0.
time (sec)	N/A	0.108	0.119	0.709	0.	0.739	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	89	0	0	524	0	0
normalized size	1	1.	0.85	0.	0.	4.99	0.	0.
time (sec)	N/A	0.107	0.115	0.741	0.	0.739	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	29	128	55	119	0	158
normalized size	1	1.	0.71	3.12	1.34	2.9	0.	3.85
time (sec)	N/A	0.016	0.014	0.108	1.193	0.913	0.	1.437

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	29	128	55	108	70	146
normalized size	1	1.	0.71	3.12	1.34	2.63	1.71	3.56
time (sec)	N/A	0.016	0.01	0.082	1.011	0.934	46.05	1.525

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	29	124	55	88	70	142
normalized size	1	1.	0.71	3.02	1.34	2.15	1.71	3.46
time (sec)	N/A	0.014	0.007	0.08	1.135	0.916	2.538	1.553

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	24	42	55	69	63	55
normalized size	1	1.	0.65	1.14	1.49	1.86	1.7	1.49
time (sec)	N/A	0.015	0.006	0.043	1.172	0.87	1.68	1.374

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	24	122	55	78	65	58
normalized size	1	1.	0.65	3.3	1.49	2.11	1.76	1.57
time (sec)	N/A	0.016	0.007	0.083	1.178	0.865	5.865	1.362

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	29	128	55	92	71	90
normalized size	1	1.	0.71	3.12	1.34	2.24	1.73	2.2
time (sec)	N/A	0.015	0.009	0.088	1.05	0.867	39.234	1.408

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	61	716	138	321	0	574
normalized size	1	1.	0.84	9.81	1.89	4.4	0.	7.86
time (sec)	N/A	0.045	0.02	0.139	1.155	0.796	0.	1.998

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	61	716	138	294	216	521
normalized size	1	1.	0.84	9.81	1.89	4.03	2.96	7.14
time (sec)	N/A	0.046	0.018	0.134	1.101	0.899	124.624	1.944

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	61	710	138	243	216	517
normalized size	1	1.	0.84	9.73	1.89	3.33	2.96	7.08
time (sec)	N/A	0.041	0.016	0.126	1.102	0.896	7.888	1.971

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	54	107	138	203	199	159
normalized size	1	1.	0.81	1.6	2.06	3.03	2.97	2.37
time (sec)	N/A	0.041	0.013	0.054	1.181	0.956	4.274	1.303

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	54	707	136	212	201	201
normalized size	1	1.	0.81	10.55	2.03	3.16	3.	3.
time (sec)	N/A	0.047	0.013	0.131	1.084	0.939	5.55	1.27

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	61	716	138	236	218	288
normalized size	1	1.	0.84	9.81	1.89	3.23	2.99	3.95
time (sec)	N/A	0.046	0.015	0.137	1.131	0.849	42.533	1.303

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.081	0.118	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.071	0.116	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.069	0.115	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.063	0.108	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	66
normalized size	1	1.	0.97	0.	0.	0.	0.	1.03
time (sec)	N/A	0.061	0.071	0.109	0.	0.	0.	1.286

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.073	0.108	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	84	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.154	4.648	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	84	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	0.144	4.5	0.	0.	0.	0.



Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	84	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	0.133	4.658	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	83	427	0	0	0	0
normalized size	1	1.	0.85	4.36	0.	0.	0.	0.
time (sec)	N/A	0.088	0.115	1.596	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	93	0	0	0	0	379
normalized size	1	1.	0.95	0.	0.	0.	0.	3.87
time (sec)	N/A	0.093	0.12	4.557	0.	0.	0.	1.313

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	94	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	0.123	4.664	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	85	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	0.038	0.485	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	61	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.022	0.293	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	67	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.033	0.174	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	67	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.032	0.177	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.016	0.181	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	45	29	19
normalized size	1	1.	1.	0.82	1.06	2.65	1.71	1.12
time (sec)	N/A	0.014	0.002	0.038	1.019	1.029	1.924	1.232

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	65	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.047	0.178	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	73	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.048	0.184	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.043	0.166	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	76	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	0.051	0.171	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	79	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.057	0.173	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	72	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.038	0.17	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	104	75	19
normalized size	1	1.	1.	0.82	1.06	6.12	4.41	1.12
time (sec)	N/A	0.014	0.002	0.039	1.098	0.994	37.164	1.334

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	79	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.056	0.169	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	88	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	0.064	0.174	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.005	0.178	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.009	0.169	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.009	0.181	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.004	0.178	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	39	24	19
normalized size	1	1.	1.	0.93	1.2	2.6	1.6	1.27
time (sec)	N/A	0.013	0.001	0.039	1.101	0.943	3.27	1.284

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	52	0	0	0	0	0
normalized size	1	1.	1.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.034	0.171	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	60	0	0	0	0	0
normalized size	1	1.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.035	0.174	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	73	0	0	0	0	0
normalized size	1	1.	1.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.045	0.189	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	78	0	0	0	0	0
normalized size	1	1.	1.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.049	0.175	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	78	0	0	0	0	0
normalized size	1	1.	1.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.046	0.171	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	69	0	0	0	0	0
normalized size	1	1.	1.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.034	0.175	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	70	48	19
normalized size	1	1.	1.	0.93	1.2	4.67	3.2	1.27
time (sec)	N/A	0.014	0.002	0.039	1.139	0.944	165.01	1.198

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	58	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.049	0.173	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	66	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.047	0.168	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	0.059	0.17	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	92	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	0.064	0.168	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	92	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.064	0.168	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	83	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.048	0.174	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	108	0	19
normalized size	1	1.	1.	0.82	1.06	6.35	0.	1.12
time (sec)	N/A	0.014	0.002	0.039	0.98	0.984	0.	1.27

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	70	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.054	0.182	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	78	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.053	0.178	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	17	260	0	74	27	289
normalized size	1	1.	0.81	12.38	0.	3.52	1.29	13.76
time (sec)	N/A	0.02	0.011	0.117	0.	1.006	1.356	1.351

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	76	9684	0	1242	0	1530
normalized size	1	1.	0.66	83.48	0.	10.71	0.	13.19
time (sec)	N/A	0.091	0.045	0.698	0.	1.058	0.	1.93

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	76	2126	0	478	0	543
normalized size	1	1.	0.94	26.25	0.	5.9	0.	6.7
time (sec)	N/A	0.046	0.032	0.197	0.	1.046	0.	1.31

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	32	371	0	142	0	128
normalized size	1	1.	0.7	8.07	0.	3.09	0.	2.78
time (sec)	N/A	0.015	0.012	0.103	0.	1.035	0.	1.285

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	67	0	0	163	0	0
normalized size	1	1.	1.02	0.	0.	2.47	0.	0.
time (sec)	N/A	0.074	0.106	0.245	0.	0.989	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	89	0	0	332	0	0
normalized size	1	1.	0.89	0.	0.	3.32	0.	0.
time (sec)	N/A	0.093	0.233	1.303	0.	1.003	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	113	0	0	765	0	0
normalized size	1	1.	0.8	0.	0.	5.39	0.	0.
time (sec)	N/A	0.139	0.357	1.312	0.	1.033	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	40	2008	0	197	0	219
normalized size	1	1.	0.54	27.14	0.	2.66	0.	2.96
time (sec)	N/A	0.053	0.008	0.257	0.	0.995	0.	1.268

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	30	750	0	113	163	123
normalized size	1	1.	0.57	14.15	0.	2.13	3.08	2.32
time (sec)	N/A	0.032	0.006	0.121	0.	1.027	113.492	1.254

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	20	263	0	55	68	57
normalized size	1	1.	0.62	8.22	0.	1.72	2.12	1.78
time (sec)	N/A	0.011	0.004	0.088	0.	0.993	39.605	1.386

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	0	0	53	0	0
normalized size	1	1.	1.	0.	0.	1.96	0.	0.
time (sec)	N/A	0.037	0.008	0.254	0.	1.014	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	132	0	0
normalized size	1	1.	1.	0.	0.	2.69	0.	0.
time (sec)	N/A	0.053	0.017	1.202	0.	1.006	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	61	0	0	240	0	0
normalized size	1	1.	0.79	0.	0.	3.12	0.	0.
time (sec)	N/A	0.076	0.024	1.195	0.	1.01	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	101	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	0.194	0.206	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	86	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.051	0.178	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.011	0.201	0.	0.	0.	0.



Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	86	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.194	0.185	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	103	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	0.413	0.188	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	107	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.135	1.076	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.087	0.479	0.	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.081	0.411	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	132	0	0
normalized size	1	1.	1.	0.	0.	1.65	0.	0.
time (sec)	N/A	0.038	0.068	0.344	0.	1.308	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	0	99	56	36
normalized size	1	1.	1.	1.04	0.	3.81	2.15	1.38
time (sec)	N/A	0.032	0.008	0.04	0.	1.035	2.609	1.318

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	78	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.073	0.36	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.078	0.174	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.08	0.185	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	82	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	0.109	0.198	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	59	0	0	0
normalized size	1	1.	1.	0.	0.94	0.	0.	0.
time (sec)	N/A	0.059	0.036	0.058	1.195	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	59	0	0	0
normalized size	1	1.	1.	0.	0.94	0.	0.	0.
time (sec)	N/A	0.047	0.031	0.051	1.261	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	59	86	0	0
normalized size	1	1.	1.	0.	1.05	1.54	0.	0.
time (sec)	N/A	0.033	0.026	0.048	1.248	1.053	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	0	63	39	28
normalized size	1	1.	1.	1.05	0.	3.	1.86	1.33
time (sec)	N/A	0.028	0.006	0.037	0.	1.03	1.333	1.286

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	48	0	54	0	0	0
normalized size	1	1.	0.92	0.	1.04	0.	0.	0.
time (sec)	N/A	0.049	0.033	0.054	1.259	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	59	0	0	0
normalized size	1	1.	1.	0.	0.94	0.	0.	0.
time (sec)	N/A	0.055	0.035	0.047	1.335	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	59	0	0	0
normalized size	1	1.	1.	0.	0.94	0.	0.	0.
time (sec)	N/A	0.054	0.035	0.048	1.262	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	103	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	0.188	0.086	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	65	0	0	0
normalized size	1	1.	1.	0.	0.81	0.	0.	0.
time (sec)	N/A	0.062	0.052	0.043	1.288	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	65	0	0	0
normalized size	1	1.	1.	0.	0.87	0.	0.	0.
time (sec)	N/A	0.055	0.045	0.043	1.335	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	65	0	0	0
normalized size	1	1.	1.	0.	0.89	0.	0.	0.
time (sec)	N/A	0.035	0.026	0.044	1.247	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	25	0	82	48	34
normalized size	1	1.	1.	0.96	0.	3.15	1.85	1.31
time (sec)	N/A	0.028	0.007	0.04	0.	1.043	16.297	1.263

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	65	0	0	0
normalized size	1	1.	1.	0.	0.89	0.	0.	0.
time (sec)	N/A	0.052	0.048	0.046	1.351	0.	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	65	0	0	0
normalized size	1	1.	1.	0.	0.87	0.	0.	0.
time (sec)	N/A	0.058	0.045	0.041	1.247	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	65	0	0	0
normalized size	1	1.	1.	0.	0.81	0.	0.	0.
time (sec)	N/A	0.058	0.046	0.04	1.341	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	0.024	1.768	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	118	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	0.151	1.508	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	142	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.168	0.227	19.344	0.	0.	0.	0.

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [117] had the largest ratio of [ 0.4 ]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.	8	0.125
2	A	1	1	1.	8	0.125
3	A	1	1	1.	6	0.167
4	A	1	1	1.	4	0.25
5	A	1	1	1.	8	0.125
6	A	1	1	1.	8	0.125
7	A	1	1	1.	8	0.125
8	A	2	2	1.	10	0.2
9	A	2	2	1.	10	0.2
10	A	2	2	1.	8	0.25
11	A	2	2	1.	6	0.333
12	A	2	2	1.	10	0.2
13	A	2	2	1.	10	0.2
14	A	2	2	1.	10	0.2
15	A	3	2	1.	10	0.2
16	A	3	2	1.	10	0.2
17	A	3	2	1.	8	0.25
18	A	3	2	1.	6	0.333
19	A	2	2	1.	10	0.2
20	A	3	2	1.	10	0.2
21	A	3	2	1.	10	0.2
22	A	2	2	1.	10	0.2
23	A	2	2	1.	10	0.2
24	A	2	2	1.	8	0.25
25	A	1	1	1.	6	0.167
26	A	2	2	1.	10	0.2
27	A	2	2	1.	10	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
28	A	2	2	1.	10	0.2
29	A	3	3	1.	10	0.3
30	A	3	3	1.	10	0.3
31	A	3	3	1.	8	0.375
32	A	2	2	1.	6	0.333
33	A	2	2	1.	10	0.2
34	A	3	3	1.	10	0.3
35	A	3	3	1.	10	0.3
36	A	4	3	1.	10	0.3
37	A	4	3	1.	10	0.3
38	A	4	3	1.	8	0.375
39	A	3	2	1.	6	0.333
40	A	2	2	1.	10	0.2
41	A	4	3	1.	10	0.3
42	A	4	3	1.	10	0.3
43	A	1	1	1.	14	0.071
44	A	1	1	1.	14	0.071
45	A	1	1	1.	12	0.083
46	A	2	1	1.	10	0.1
47	A	1	1	1.	14	0.071
48	A	1	1	1.	14	0.071
49	A	1	1	1.	14	0.071
50	A	2	2	1.	16	0.125
51	A	2	2	1.	16	0.125
52	A	2	2	1.	14	0.143
53	A	3	2	1.	12	0.167
54	A	2	2	1.	16	0.125
55	A	2	2	1.	16	0.125
56	A	2	2	1.	16	0.125
57	A	3	2	1.	16	0.125
58	A	3	2	1.	16	0.125
59	A	3	2	1.	14	0.143
60	A	4	2	1.	12	0.167
61	A	2	2	1.	16	0.125
62	A	3	2	1.	16	0.125
63	A	3	2	1.	16	0.125
64	A	3	2	1.	16	0.125
65	A	2	2	1.	16	0.125
66	A	2	2	1.	16	0.125
67	A	2	2	1.	14	0.143
68	A	2	2	1.	12	0.167
69	A	2	2	1.	16	0.125
70	A	2	2	1.	16	0.125
71	A	2	2	1.	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
72	A	2	2	1.	16	0.125
73	A	3	3	1.	16	0.188
74	A	3	3	1.	16	0.188
75	A	3	3	1.	14	0.214
76	A	3	3	1.	12	0.25
77	A	2	2	1.	16	0.125
78	A	3	3	1.	16	0.188
79	A	3	3	1.	16	0.188
80	A	3	3	1.	16	0.188
81	A	4	3	1.	16	0.188
82	A	4	3	1.	16	0.188
83	A	4	3	1.	14	0.214
84	A	4	3	1.	12	0.25
85	A	2	2	1.	16	0.125
86	A	4	3	1.	16	0.188
87	A	4	3	1.	16	0.188
88	A	4	3	1.	16	0.188
89	A	1	1	1.	18	0.056
90	A	1	1	1.	18	0.056
91	A	1	1	1.	18	0.056
92	A	1	1	1.	18	0.056
93	A	1	1	1.	18	0.056
94	A	1	1	1.	18	0.056
95	A	2	2	1.	20	0.1
96	A	2	2	1.	20	0.1
97	A	2	2	1.	20	0.1
98	A	2	2	1.	20	0.1
99	A	2	2	1.	20	0.1
100	A	2	2	1.	20	0.1
101	A	2	2	1.	20	0.1
102	A	2	2	1.	20	0.1
103	A	2	2	1.	20	0.1
104	A	2	2	1.	20	0.1
105	A	2	2	1.	20	0.1
106	A	2	2	1.	20	0.1
107	A	3	3	1.	20	0.15
108	A	3	3	1.	20	0.15
109	A	3	3	1.	20	0.15
110	A	3	3	1.	20	0.15
111	A	3	3	1.	20	0.15
112	A	3	3	1.	20	0.15
113	A	4	4	1.	14	0.286
114	A	4	4	1.	14	0.286
115	A	4	4	1.	14	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
116	A	4	4	1.	12	0.333
117	A	4	4	1.	10	0.4
118	A	2	2	1.	14	0.143
119	A	4	4	1.	14	0.286
120	A	4	4	1.	14	0.286
121	A	5	4	1.	14	0.286
122	A	5	4	1.	14	0.286
123	A	5	4	1.	12	0.333
124	A	5	4	1.	10	0.4
125	A	2	2	1.	14	0.143
126	A	5	4	1.	14	0.286
127	A	5	4	1.	14	0.286
128	A	3	3	1.	14	0.214
129	A	3	3	1.	14	0.214
130	A	3	3	1.	12	0.25
131	A	3	3	1.	10	0.3
132	A	2	2	1.	14	0.143
133	A	3	3	1.	14	0.214
134	A	3	3	1.	14	0.214
135	A	4	4	1.	14	0.286
136	A	4	4	1.	14	0.286
137	A	4	4	1.	12	0.333
138	A	4	4	1.	10	0.4
139	A	2	2	1.	14	0.143
140	A	4	4	1.	14	0.286
141	A	4	4	1.	14	0.286
142	A	5	4	1.	14	0.286
143	A	5	4	1.	14	0.286
144	A	5	4	1.	12	0.333
145	A	5	4	1.	10	0.4
146	A	2	2	1.	14	0.143
147	A	5	4	1.	14	0.286
148	A	5	4	1.	14	0.286
149	A	1	1	1.	22	0.045
150	A	3	2	1.	18	0.111
151	A	2	2	1.	18	0.111
152	A	1	1	1.	16	0.062
153	A	2	2	1.	18	0.111
154	A	3	3	1.	18	0.167
155	A	4	3	1.	18	0.167
156	A	3	2	1.	16	0.125
157	A	2	2	1.	16	0.125
158	A	1	1	1.	14	0.071
159	A	3	3	1.	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
160	A	4	4	1.	16	0.25
161	A	5	4	1.	16	0.25
162	A	5	4	1.	14	0.286
163	A	4	4	1.	14	0.286
164	A	3	3	1.	14	0.214
165	A	4	4	1.	14	0.286
166	A	5	4	1.	14	0.286
167	A	2	2	1.	18	0.111
168	A	2	2	1.	16	0.125
169	A	2	2	1.	14	0.143
170	A	2	2	1.	12	0.167
171	A	2	2	1.	16	0.125
172	A	2	2	1.	16	0.125
173	A	2	2	1.	16	0.125
174	A	2	2	1.	16	0.125
175	A	2	2	1.	16	0.125
176	A	2	2	1.	14	0.143
177	A	2	2	1.	12	0.167
178	A	2	2	1.	10	0.2
179	A	2	2	1.	14	0.143
180	A	2	2	1.	14	0.143
181	A	2	2	1.	14	0.143
182	A	2	2	1.	14	0.143
183	A	2	2	1.	20	0.1
184	A	2	2	1.	18	0.111
185	A	2	2	1.	16	0.125
186	A	2	2	1.	14	0.143
187	A	2	2	1.	18	0.111
188	A	2	2	1.	18	0.111
189	A	2	2	1.	18	0.111
190	A	2	2	1.	18	0.111
191	A	2	2	1.	18	0.111
192	A	3	3	1.	20	0.15
193	A	4	3	1.	27	0.111



# Chapter 3

## Listing of integrals

### 3.1 $\int x^3 \log(cx) dx$

Optimal. Leaf size=19

$$\frac{1}{4}x^4 \log(cx) - \frac{x^4}{16}$$

[Out]  $-x^4/16 + (x^4*\text{Log}[c*x])/4$

---

**Rubi [A]** time = 0.0075869, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2304}

$$\frac{1}{4}x^4 \log(cx) - \frac{x^4}{16}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3*\text{Log}[c*x], x]$

[Out]  $-x^4/16 + (x^4*\text{Log}[c*x])/4$

#### Rule 2304

$\text{Int}[(a + \text{Log}[(c_*)*(x_*)^{(n_*)}]*(b_*))*((d_*)*(x_*))^{(m_*)}, x\_Symbol] :>$   
 $\text{Simp}[(d*x)^{(m+1)}*(a + b*\text{Log}[c*x^n])/(d*(m+1)), x] - \text{Simp}[(b*n*(d*x)^{(m+1)})/(d*(m+1)^2), x] /;$  FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\int x^3 \log(cx) dx = -\frac{x^4}{16} + \frac{1}{4}x^4 \log(cx)$$

**Mathematica [A]** time = 0.001066, size = 19, normalized size = 1.

$$\frac{1}{4}x^4 \log(cx) - \frac{x^4}{16}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Log[c\*x],x]

[Out]  $-x^4/16 + (x^4*\text{Log}[c*x])/4$

**Maple [A]** time = 0.037, size = 16, normalized size = 0.8

$$-\frac{x^4}{16} + \frac{x^4 \ln(cx)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*ln(c\*x),x)

[Out]  $-1/16*x^4+1/4*x^4*\ln(c*x)$

**Maxima [A]** time = 0.954678, size = 20, normalized size = 1.05

$$\frac{1}{4}x^4 \log(cx) - \frac{1}{16}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*log(c\*x),x, algorithm="maxima")

[Out]  $1/4*x^4*\log(c*x) - 1/16*x^4$

**Fricas [A]** time = 0.787972, size = 39, normalized size = 2.05

$$\frac{1}{4}x^4 \log(cx) - \frac{1}{16}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*log(c\*x),x, algorithm="fricas")

[Out]  $1/4*x^4*\log(c*x) - 1/16*x^4$

**Sympy [A]** time = 0.091888, size = 14, normalized size = 0.74

$$\frac{x^4 \log(cx)}{4} - \frac{x^4}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*ln(c\*x),x)

[Out]  $x**4*\log(c*x)/4 - x**4/16$

**Giac [A]** time = 1.07294, size = 20, normalized size = 1.05

$$\frac{1}{4}x^4 \log(cx) - \frac{1}{16}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*x),x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(c*x) - 1/16*x^4
```

## 3.2 $\int x^2 \log(cx) dx$

**Optimal.** Leaf size=19

$$\frac{1}{3}x^3 \log(cx) - \frac{x^3}{9}$$

[Out]  $-x^3/9 + (x^3 \text{Log}[c*x])/3$

**Rubi [A]** time = 0.0066558, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2304}

$$\frac{1}{3}x^3 \log(cx) - \frac{x^3}{9}$$

Antiderivative was successfully verified.

[In] Int[x^2\*Log[c\*x],x]

[Out]  $-x^3/9 + (x^3 \text{Log}[c*x])/3$

### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :>  
Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

### Rubi steps

$$\int x^2 \log(cx) dx = -\frac{x^3}{9} + \frac{1}{3}x^3 \log(cx)$$

**Mathematica [A]** time = 0.0009417, size = 19, normalized size = 1.

$$\frac{1}{3}x^3 \log(cx) - \frac{x^3}{9}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Log[c\*x],x]

[Out]  $-x^3/9 + (x^3 \text{Log}[c*x])/3$

**Maple [A]** time = 0.036, size = 16, normalized size = 0.8

$$-\frac{x^3}{9} + \frac{x^3 \ln(cx)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*ln(c\*x),x)

[Out]  $-1/9*x^3+1/3*x^3*\ln(c*x)$

---

**Maxima [A]** time = 0.959366, size = 20, normalized size = 1.05

$$\frac{1}{3}x^3 \log(cx) - \frac{1}{9}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*log(c*x),x, algorithm="maxima")`

[Out]  $1/3*x^3*\log(c*x) - 1/9*x^3$

---

**Fricas [A]** time = 0.838674, size = 38, normalized size = 2.

$$\frac{1}{3}x^3 \log(cx) - \frac{1}{9}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*log(c*x),x, algorithm="fricas")`

[Out]  $1/3*x^3*\log(c*x) - 1/9*x^3$

---

**Sympy [A]** time = 0.097191, size = 14, normalized size = 0.74

$$\frac{x^3 \log(cx)}{3} - \frac{x^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*ln(c*x),x)`

[Out]  $x**3*\log(c*x)/3 - x**3/9$

---

**Giac [A]** time = 1.08101, size = 20, normalized size = 1.05

$$\frac{1}{3}x^3 \log(cx) - \frac{1}{9}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*log(c*x),x, algorithm="giac")`

[Out]  $1/3*x^3*\log(c*x) - 1/9*x^3$

### 3.3 $\int x \log(cx) dx$

**Optimal.** Leaf size=19

$$\frac{1}{2}x^2 \log(cx) - \frac{x^2}{4}$$

[Out]  $-x^2/4 + (x^2 \cdot \text{Log}[c \cdot x])/2$

**Rubi [A]** time = 0.0039277, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2304}

$$\frac{1}{2}x^2 \log(cx) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

[In] Int[x\*Log[c\*x],x]

[Out]  $-x^2/4 + (x^2 \cdot \text{Log}[c \cdot x])/2$

**Rule 2304**

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :>  
Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

**Rubi steps**

$$\int x \log(cx) dx = -\frac{x^2}{4} + \frac{1}{2}x^2 \log(cx)$$

**Mathematica [A]** time = 0.0006873, size = 19, normalized size = 1.

$$\frac{1}{2}x^2 \log(cx) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Log[c\*x],x]

[Out]  $-x^2/4 + (x^2 \cdot \text{Log}[c \cdot x])/2$

**Maple [A]** time = 0.037, size = 16, normalized size = 0.8

$$-\frac{x^2}{4} + \frac{x^2 \ln(cx)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*ln(c\*x),x)



[Out]  $-1/4*x^2+1/2*x^2*\ln(c*x)$

---

**Maxima [A]** time = 0.960376, size = 20, normalized size = 1.05

$$\frac{1}{2}x^2 \log(cx) - \frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(c*x),x, algorithm="maxima")`

[Out]  $1/2*x^2*\log(c*x) - 1/4*x^2$

---

**Fricas [A]** time = 0.815068, size = 38, normalized size = 2.

$$\frac{1}{2}x^2 \log(cx) - \frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(c*x),x, algorithm="fricas")`

[Out]  $1/2*x^2*\log(c*x) - 1/4*x^2$

---

**Sympy [A]** time = 0.087329, size = 14, normalized size = 0.74

$$\frac{x^2 \log(cx)}{2} - \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*ln(c*x),x)`

[Out]  $x**2*\log(c*x)/2 - x**2/4$

---

**Giac [A]** time = 1.10545, size = 20, normalized size = 1.05

$$\frac{1}{2}x^2 \log(cx) - \frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(c*x),x, algorithm="giac")`

[Out]  $1/2*x^2*\log(c*x) - 1/4*x^2$

### 3.4 $\int \log(cx) dx$

**Optimal.** Leaf size=10

$$x \log(cx) - x$$

[Out] -x + x\*Log[c\*x]

**Rubi [A]** time = 0.0013316, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {2295}

$$x \log(cx) - x$$

Antiderivative was successfully verified.

[In] Int[Log[c\*x],x]

[Out] -x + x\*Log[c\*x]

Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

Rubi steps

$$\int \log(cx) dx = -x + x \log(cx)$$

**Mathematica [A]** time = 0.0006168, size = 10, normalized size = 1.

$$x \log(cx) - x$$

Antiderivative was successfully verified.

[In] Integrate[Log[c\*x],x]

[Out] -x + x\*Log[c\*x]

**Maple [A]** time = 0.036, size = 11, normalized size = 1.1

$$-x + x \ln(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c\*x),x)

[Out] -x+x\*ln(c\*x)

---

**Maxima [A]** time = 0.989558, size = 22, normalized size = 2.2

$$\frac{cx \log(cx) - cx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x),x, algorithm="maxima")

[Out] (c\*x\*log(c\*x) - c\*x)/c

---

**Fricas [A]** time = 0.845204, size = 22, normalized size = 2.2

$$x \log(cx) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x),x, algorithm="fricas")

[Out] x\*log(c\*x) - x

---

**Sympy [A]** time = 0.089146, size = 7, normalized size = 0.7

$$x \log(cx) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(c\*x),x)

[Out] x\*log(c\*x) - x

---

**Giac [A]** time = 1.07016, size = 22, normalized size = 2.2

$$\frac{cx \log(cx) - cx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x),x, algorithm="giac")

[Out] (c\*x\*log(c\*x) - c\*x)/c

$$3.5 \quad \int \frac{\log(cx)}{x} dx$$

**Optimal.** Leaf size=10

$$\frac{1}{2} \log^2(cx)$$

[Out] Log[c\*x]^2/2

**Rubi [A]** time = 0.0059887, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2301}

$$\frac{1}{2} \log^2(cx)$$

Antiderivative was successfully verified.

[In] Int[Log[c\*x]/x,x]

[Out] Log[c\*x]^2/2

Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] :> Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

Rubi steps

$$\int \frac{\log(cx)}{x} dx = \frac{1}{2} \log^2(cx)$$

**Mathematica [A]** time = 0.0007479, size = 10, normalized size = 1.

$$\frac{1}{2} \log^2(cx)$$

Antiderivative was successfully verified.

[In] Integrate[Log[c\*x]/x,x]

[Out] Log[c\*x]^2/2

**Maple [A]** time = 0.035, size = 9, normalized size = 0.9

$$\frac{(\ln(cx))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c\*x)/x,x)

[Out]  $\frac{1}{2} \ln(c*x)^2$

---

**Maxima [A]** time = 0.969151, size = 11, normalized size = 1.1

$$\frac{1}{2} \log(cx)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)/x,x, algorithm="maxima")

[Out]  $\frac{1}{2} \log(c*x)^2$

---

**Fricas [A]** time = 0.800344, size = 22, normalized size = 2.2

$$\frac{1}{2} \log(cx)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)/x,x, algorithm="fricas")

[Out]  $\frac{1}{2} \log(c*x)^2$

---

**Sympy [A]** time = 0.084852, size = 7, normalized size = 0.7

$$\frac{\log(cx)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(c\*x)/x,x)

[Out]  $\log(c*x)**2/2$

---

**Giac [A]** time = 1.23289, size = 11, normalized size = 1.1

$$\frac{1}{2} \log(cx)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)/x,x, algorithm="giac")

[Out]  $\frac{1}{2} \log(c*x)^2$

### 3.6 $\int \frac{\log(cx)}{x^2} dx$

**Optimal.** Leaf size=15

$$-\frac{\log(cx)}{x} - \frac{1}{x}$$

[Out]  $-x^{(-1)} - \text{Log}[c*x]/x$

**Rubi [A]** time = 0.0071283, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2304}

$$-\frac{\log(cx)}{x} - \frac{1}{x}$$

Antiderivative was successfully verified.

[In] Int[Log[c\*x]/x^2,x]

[Out]  $-x^{(-1)} - \text{Log}[c*x]/x$

#### Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_)^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rubi steps

$$\int \frac{\log(cx)}{x^2} dx = -\frac{1}{x} - \frac{\log(cx)}{x}$$

**Mathematica [A]** time = 0.0008004, size = 15, normalized size = 1.

$$-\frac{\log(cx)}{x} - \frac{1}{x}$$

Antiderivative was successfully verified.

[In] Integrate[Log[c\*x]/x^2,x]

[Out]  $-x^{(-1)} - \text{Log}[c*x]/x$

**Maple [A]** time = 0.034, size = 16, normalized size = 1.1

$$-x^{-1} - \frac{\ln(cx)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c\*x)/x^2,x)

[Out]  $-1/x - \ln(c*x)/x$

**Maxima [A]** time = 0.990233, size = 20, normalized size = 1.33

$$-\frac{\log(cx)}{x} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x)/x^2,x, algorithm="maxima")`

[Out]  $-\log(c*x)/x - 1/x$

**Fricas [A]** time = 0.860201, size = 26, normalized size = 1.73

$$-\frac{\log(cx) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x)/x^2,x, algorithm="fricas")`

[Out]  $-(\log(c*x) + 1)/x$

**Sympy [A]** time = 0.097856, size = 10, normalized size = 0.67

$$-\frac{\log(cx)}{x} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(c*x)/x**2,x)`

[Out]  $-\log(c*x)/x - 1/x$

**Giac [A]** time = 1.61667, size = 20, normalized size = 1.33

$$-\frac{\log(cx)}{x} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x)/x^2,x, algorithm="giac")`

[Out]  $-\log(c*x)/x - 1/x$

### 3.7 $\int \frac{\log(cx)}{x^3} dx$

**Optimal.** Leaf size=19

$$-\frac{\log(cx)}{2x^2} - \frac{1}{4x^2}$$

[Out]  $-1/(4*x^2) - \text{Log}[c*x]/(2*x^2)$

**Rubi [A]** time = 0.0066814, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2304}

$$-\frac{\log(cx)}{2x^2} - \frac{1}{4x^2}$$

Antiderivative was successfully verified.

[In] Int[Log[c\*x]/x^3,x]

[Out]  $-1/(4*x^2) - \text{Log}[c*x]/(2*x^2)$

#### Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_)^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rubi steps

$$\int \frac{\log(cx)}{x^3} dx = -\frac{1}{4x^2} - \frac{\log(cx)}{2x^2}$$

**Mathematica [A]** time = 0.0008228, size = 19, normalized size = 1.

$$-\frac{\log(cx)}{2x^2} - \frac{1}{4x^2}$$

Antiderivative was successfully verified.

[In] Integrate[Log[c\*x]/x^3,x]

[Out]  $-1/(4*x^2) - \text{Log}[c*x]/(2*x^2)$

**Maple [A]** time = 0.035, size = 16, normalized size = 0.8

$$-\frac{1}{4x^2} - \frac{\ln(cx)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c\*x)/x^3,x)



[Out]  $-1/4/x^2 - 1/2 \cdot \ln(cx)/x^2$

**Maxima [A]** time = 0.97228, size = 20, normalized size = 1.05

$$-\frac{\log(cx)}{2x^2} - \frac{1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x)/x^3,x, algorithm="maxima")`

[Out]  $-1/2 \cdot \log(cx)/x^2 - 1/4/x^2$

**Fricas [A]** time = 0.826076, size = 36, normalized size = 1.89

$$-\frac{2 \log(cx) + 1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x)/x^3,x, algorithm="fricas")`

[Out]  $-1/4 \cdot (2 \cdot \log(cx) + 1)/x^2$

**Sympy [A]** time = 0.131911, size = 17, normalized size = 0.89

$$-\frac{\log(cx)}{2x^2} - \frac{1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(c*x)/x**3,x)`

[Out]  $-\log(cx)/(2 \cdot x^{**2}) - 1/(4 \cdot x^{**2})$

**Giac [A]** time = 1.09793, size = 20, normalized size = 1.05

$$-\frac{\log(cx)}{2x^2} - \frac{1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x)/x^3,x, algorithm="giac")`

[Out]  $-1/2 \cdot \log(cx)/x^2 - 1/4/x^2$

### 3.8 $\int x^3 \log^2(cx) dx$

**Optimal.** Leaf size=32

$$\frac{1}{4}x^4 \log^2(cx) - \frac{1}{8}x^4 \log(cx) + \frac{x^4}{32}$$

[Out]  $x^4/32 - (x^4 \cdot \text{Log}[c \cdot x])/8 + (x^4 \cdot \text{Log}[c \cdot x]^2)/4$

**Rubi [A]** time = 0.0199876, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2305, 2304}

$$\frac{1}{4}x^4 \log^2(cx) - \frac{1}{8}x^4 \log(cx) + \frac{x^4}{32}$$

Antiderivative was successfully verified.

[In] Int[x^3\*Log[c\*x]^2,x]

[Out]  $x^4/32 - (x^4 \cdot \text{Log}[c \cdot x])/8 + (x^4 \cdot \text{Log}[c \cdot x]^2)/4$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int x^3 \log^2(cx) dx &= \frac{1}{4}x^4 \log^2(cx) - \frac{1}{2} \int x^3 \log(cx) dx \\ &= \frac{x^4}{32} - \frac{1}{8}x^4 \log(cx) + \frac{1}{4}x^4 \log^2(cx) \end{aligned}$$

**Mathematica [A]** time = 0.0012158, size = 32, normalized size = 1.

$$\frac{1}{4}x^4 \log^2(cx) - \frac{1}{8}x^4 \log(cx) + \frac{x^4}{32}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Log[c\*x]^2,x]

[Out]  $x^4/32 - (x^4 \cdot \text{Log}[c \cdot x])/8 + (x^4 \cdot \text{Log}[c \cdot x]^2)/4$

**Maple [A]** time = 0.036, size = 27, normalized size = 0.8

$$\frac{x^4}{32} - \frac{x^4 \ln(cx)}{8} + \frac{x^4 (\ln(cx))^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*ln(c\*x)^2,x)

[Out] 1/32\*x^4-1/8\*x^4\*ln(c\*x)+1/4\*x^4\*ln(c\*x)^2

---

**Maxima [A]** time = 0.975037, size = 28, normalized size = 0.88

$$\frac{1}{32} (8 \log(cx)^2 - 4 \log(cx) + 1)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*log(c\*x)^2,x, algorithm="maxima")

[Out] 1/32\*(8\*log(c\*x)^2 - 4\*log(c\*x) + 1)\*x^4

---

**Fricas [A]** time = 0.828518, size = 68, normalized size = 2.12

$$\frac{1}{4} x^4 \log(cx)^2 - \frac{1}{8} x^4 \log(cx) + \frac{1}{32} x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*log(c\*x)^2,x, algorithm="fricas")

[Out] 1/4\*x^4\*log(c\*x)^2 - 1/8\*x^4\*log(c\*x) + 1/32\*x^4

---

**Sympy [A]** time = 0.110012, size = 26, normalized size = 0.81

$$\frac{x^4 \log(cx)^2}{4} - \frac{x^4 \log(cx)}{8} + \frac{x^4}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*ln(c\*x)\*\*2,x)

[Out] x\*\*4\*log(c\*x)\*\*2/4 - x\*\*4\*log(c\*x)/8 + x\*\*4/32

---

**Giac [A]** time = 1.1193, size = 35, normalized size = 1.09

$$\frac{1}{4} x^4 \log(cx)^2 - \frac{1}{8} x^4 \log(cx) + \frac{1}{32} x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(c*x)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(c*x)^2 - 1/8*x^4*log(c*x) + 1/32*x^4
```

### 3.9 $\int x^2 \log^2(cx) dx$

**Optimal.** Leaf size=32

$$\frac{1}{3}x^3 \log^2(cx) - \frac{2}{9}x^3 \log(cx) + \frac{2x^3}{27}$$

[Out]  $(2*x^3)/27 - (2*x^3*\text{Log}[c*x])/9 + (x^3*\text{Log}[c*x]^2)/3$

**Rubi [A]** time = 0.0197903, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2305, 2304}

$$\frac{1}{3}x^3 \log^2(cx) - \frac{2}{9}x^3 \log(cx) + \frac{2x^3}{27}$$

Antiderivative was successfully verified.

[In] Int[x^2\*Log[c\*x]^2,x]

[Out]  $(2*x^3)/27 - (2*x^3*\text{Log}[c*x])/9 + (x^3*\text{Log}[c*x]^2)/3$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int x^2 \log^2(cx) dx &= \frac{1}{3}x^3 \log^2(cx) - \frac{2}{3} \int x^2 \log(cx) dx \\ &= \frac{2x^3}{27} - \frac{2}{9}x^3 \log(cx) + \frac{1}{3}x^3 \log^2(cx) \end{aligned}$$

**Mathematica [A]** time = 0.0012867, size = 32, normalized size = 1.

$$\frac{1}{3}x^3 \log^2(cx) - \frac{2}{9}x^3 \log(cx) + \frac{2x^3}{27}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Log[c\*x]^2,x]

[Out]  $(2*x^3)/27 - (2*x^3*\text{Log}[c*x])/9 + (x^3*\text{Log}[c*x]^2)/3$

**Maple [A]** time = 0.036, size = 27, normalized size = 0.8

$$\frac{2x^3}{27} - \frac{2x^3 \ln(cx)}{9} + \frac{x^3 (\ln(cx))^2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*ln(c\*x)^2,x)

[Out] 2/27\*x^3-2/9\*x^3\*ln(c\*x)+1/3\*x^3\*ln(c\*x)^2

---

**Maxima [A]** time = 0.991852, size = 28, normalized size = 0.88

$$\frac{1}{27} (9 \log(cx)^2 - 6 \log(cx) + 2)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*log(c\*x)^2,x, algorithm="maxima")

[Out] 1/27\*(9\*log(c\*x)^2 - 6\*log(c\*x) + 2)\*x^3

---

**Fricas [A]** time = 0.755554, size = 68, normalized size = 2.12

$$\frac{1}{3} x^3 \log(cx)^2 - \frac{2}{9} x^3 \log(cx) + \frac{2}{27} x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*log(c\*x)^2,x, algorithm="fricas")

[Out] 1/3\*x^3\*log(c\*x)^2 - 2/9\*x^3\*log(c\*x) + 2/27\*x^3

---

**Sympy [A]** time = 0.108755, size = 29, normalized size = 0.91

$$\frac{x^3 \log(cx)^2}{3} - \frac{2x^3 \log(cx)}{9} + \frac{2x^3}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*ln(c\*x)\*\*2,x)

[Out] x\*\*3\*log(c\*x)\*\*2/3 - 2\*x\*\*3\*log(c\*x)/9 + 2\*x\*\*3/27

---

**Giac [A]** time = 1.19884, size = 35, normalized size = 1.09

$$\frac{1}{3} x^3 \log(cx)^2 - \frac{2}{9} x^3 \log(cx) + \frac{2}{27} x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*x)^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(c*x)^2 - 2/9*x^3*log(c*x) + 2/27*x^3
```

### 3.10 $\int x \log^2(cx) dx$

**Optimal.** Leaf size=32

$$\frac{1}{2}x^2 \log^2(cx) - \frac{1}{2}x^2 \log(cx) + \frac{x^2}{4}$$

[Out]  $x^2/4 - (x^2 \cdot \text{Log}[c \cdot x])/2 + (x^2 \cdot \text{Log}[c \cdot x]^2)/2$

**Rubi [A]** time = 0.0108939, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {2305, 2304}

$$\frac{1}{2}x^2 \log^2(cx) - \frac{1}{2}x^2 \log(cx) + \frac{x^2}{4}$$

Antiderivative was successfully verified.

[In] Int[x\*Log[c\*x]^2,x]

[Out]  $x^2/4 - (x^2 \cdot \text{Log}[c \cdot x])/2 + (x^2 \cdot \text{Log}[c \cdot x]^2)/2$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int x \log^2(cx) dx &= \frac{1}{2}x^2 \log^2(cx) - \int x \log(cx) dx \\ &= \frac{x^2}{4} - \frac{1}{2}x^2 \log(cx) + \frac{1}{2}x^2 \log^2(cx) \end{aligned}$$

**Mathematica [A]** time = 0.0009494, size = 32, normalized size = 1.

$$\frac{1}{2}x^2 \log^2(cx) - \frac{1}{2}x^2 \log(cx) + \frac{x^2}{4}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Log[c\*x]^2,x]

[Out]  $x^2/4 - (x^2 \cdot \text{Log}[c \cdot x])/2 + (x^2 \cdot \text{Log}[c \cdot x]^2)/2$



**Maple [A]** time = 0.036, size = 27, normalized size = 0.8

$$\frac{x^2}{4} - \frac{x^2 \ln(cx)}{2} + \frac{x^2 (\ln(cx))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*ln(c\*x)^2,x)

[Out] 1/4\*x^2-1/2\*x^2\*ln(c\*x)+1/2\*x^2\*ln(c\*x)^2

---

**Maxima [A]** time = 0.977959, size = 28, normalized size = 0.88

$$\frac{1}{4} (2 \log(cx)^2 - 2 \log(cx) + 1) x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*log(c\*x)^2,x, algorithm="maxima")

[Out] 1/4\*(2\*log(c\*x)^2 - 2\*log(c\*x) + 1)\*x^2

---

**Fricas [A]** time = 0.744731, size = 66, normalized size = 2.06

$$\frac{1}{2} x^2 \log(cx)^2 - \frac{1}{2} x^2 \log(cx) + \frac{1}{4} x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*log(c\*x)^2,x, algorithm="fricas")

[Out] 1/2\*x^2\*log(c\*x)^2 - 1/2\*x^2\*log(c\*x) + 1/4\*x^2

---

**Sympy [A]** time = 0.112876, size = 26, normalized size = 0.81

$$\frac{x^2 \log(cx)^2}{2} - \frac{x^2 \log(cx)}{2} + \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*ln(c\*x)\*\*2,x)

[Out] x\*\*2\*log(c\*x)\*\*2/2 - x\*\*2\*log(c\*x)/2 + x\*\*2/4

---

**Giac [A]** time = 1.10867, size = 35, normalized size = 1.09

$$\frac{1}{2} x^2 \log(cx)^2 - \frac{1}{2} x^2 \log(cx) + \frac{1}{4} x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(c*x)^2 - 1/2*x^2*log(c*x) + 1/4*x^2
```

### 3.11 $\int \log^2(cx) dx$

**Optimal.** Leaf size=19

$$x \log^2(cx) - 2x \log(cx) + 2x$$

[Out]  $2*x - 2*x*\text{Log}[c*x] + x*\text{Log}[c*x]^2$

**Rubi [A]** time = 0.0045393, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2296, 2295}

$$x \log^2(cx) - 2x \log(cx) + 2x$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Log}[c*x]^2, x]$

[Out]  $2*x - 2*x*\text{Log}[c*x] + x*\text{Log}[c*x]^2$

#### Rule 2296

$\text{Int}[(a + \text{Log}[(c_*)*(x_)^{(n_*)}])*(b_*)^{(p_*)}, x\_Symbol] \rightarrow \text{Simp}[x*(a + b*\text{Log}[c*x^n])^p, x] - \text{Dist}[b*n*p, \text{Int}[(a + b*\text{Log}[c*x^n])^{(p-1)}, x], x] /;$   
 $\text{FreeQ}[\{a, b, c, n\}, x] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{IntegerQ}[2*p]$

#### Rule 2295

$\text{Int}[\text{Log}[(c_*)*(x_)^{(n_*)}], x\_Symbol] \rightarrow \text{Simp}[x*\text{Log}[c*x^n], x] - \text{Simp}[n*x, x] /;$   
 $\text{FreeQ}[\{c, n\}, x]$

#### Rubi steps

$$\begin{aligned} \int \log^2(cx) dx &= x \log^2(cx) - 2 \int \log(cx) dx \\ &= 2x - 2x \log(cx) + x \log^2(cx) \end{aligned}$$

**Mathematica [A]** time = 0.000852, size = 19, normalized size = 1.

$$x \log^2(cx) - 2x \log(cx) + 2x$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[\text{Log}[c*x]^2, x]$

[Out]  $2*x - 2*x*\text{Log}[c*x] + x*\text{Log}[c*x]^2$

**Maple [A]** time = 0.036, size = 20, normalized size = 1.1

$$2x - 2x \ln(cx) + x (\ln(cx))^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(c*x)^2,x)`

[Out]  $2*x-2*x*\ln(c*x)+x*\ln(c*x)^2$

---

**Maxima [A]** time = 0.995963, size = 22, normalized size = 1.16

$$(\log(cx)^2 - 2 \log(cx) + 2)x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x)^2,x, algorithm="maxima")`

[Out]  $(\log(c*x)^2 - 2*\log(c*x) + 2)*x$

---

**Fricas [A]** time = 0.815711, size = 47, normalized size = 2.47

$$x \log(cx)^2 - 2x \log(cx) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x)^2,x, algorithm="fricas")`

[Out]  $x*\log(c*x)^2 - 2*x*\log(c*x) + 2*x$

---

**Sympy [A]** time = 0.095907, size = 19, normalized size = 1.

$$x \log(cx)^2 - 2x \log(cx) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(c*x)**2,x)`

[Out]  $x*\log(c*x)**2 - 2*x*\log(c*x) + 2*x$

---

**Giac [A]** time = 1.09678, size = 26, normalized size = 1.37

$$x \log(cx)^2 - 2x \log(cx) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x)^2,x, algorithm="giac")`

[Out]  $x*\log(c*x)^2 - 2*x*\log(c*x) + 2*x$

$$3.12 \quad \int \frac{\log^2(cx)}{x} dx$$

**Optimal.** Leaf size=10

$$\frac{1}{3} \log^3(cx)$$

[Out] Log[c\*x]^3/3

**Rubi [A]** time = 0.0121408, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2302, 30}

$$\frac{1}{3} \log^3(cx)$$

Antiderivative was successfully verified.

[In] Int[Log[c\*x]^2/x,x]

[Out] Log[c\*x]^3/3

**Rule 2302**

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] := Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

**Rule 30**

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

**Rubi steps**

$$\begin{aligned} \int \frac{\log^2(cx)}{x} dx &= \text{Subst} \left( \int x^2 dx, x, \log(cx) \right) \\ &= \frac{1}{3} \log^3(cx) \end{aligned}$$

**Mathematica [A]** time = 0.0009469, size = 10, normalized size = 1.

$$\frac{1}{3} \log^3(cx)$$

Antiderivative was successfully verified.

[In] Integrate[Log[c\*x]^2/x,x]

[Out] Log[c\*x]^3/3

**Maple [A]** time = 0.035, size = 9, normalized size = 0.9

$$\frac{(\ln(cx))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c\*x)^2/x,x)

[Out] 1/3\*ln(c\*x)^3

---

**Maxima [A]** time = 0.968349, size = 11, normalized size = 1.1

$$\frac{1}{3} \log(cx)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^2/x,x, algorithm="maxima")

[Out] 1/3\*log(c\*x)^3

---

**Fricas [A]** time = 0.790002, size = 22, normalized size = 2.2

$$\frac{1}{3} \log(cx)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^2/x,x, algorithm="fricas")

[Out] 1/3\*log(c\*x)^3

---

**Sympy [A]** time = 0.086911, size = 7, normalized size = 0.7

$$\frac{\log(cx)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(c\*x)\*\*2/x,x)

[Out] log(c\*x)\*\*3/3

---

**Giac [A]** time = 1.10985, size = 11, normalized size = 1.1

$$\frac{1}{3} \log(cx)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x)^2/x,x, algorithm="giac")
```

```
[Out] 1/3*log(c*x)^3
```

### 3.13 $\int \frac{\log^2(cx)}{x^2} dx$

**Optimal.** Leaf size=26

$$-\frac{\log^2(cx)}{x} - \frac{2 \log(cx)}{x} - \frac{2}{x}$$

[Out]  $-2/x - (2*\text{Log}[c*x])/x - \text{Log}[c*x]^2/x$

**Rubi [A]** time = 0.0197428, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2305, 2304}

$$-\frac{\log^2(cx)}{x} - \frac{2 \log(cx)}{x} - \frac{2}{x}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Log}[c*x]^2/x^2, x]$

[Out]  $-2/x - (2*\text{Log}[c*x])/x - \text{Log}[c*x]^2/x$

#### Rule 2305

$\text{Int}[(a_. + \text{Log}[c_.*(x_.)^{n_.}]*b_.)^{p_.}*((d_.)*(x_.))^{m_.}, x\_Symbol] \rightarrow \text{Simp}[(d*x)^{m+1}*(a + b*\text{Log}[c*x^n])^p/(d*(m+1)), x] - \text{Dist}[(b*n*p)/(m+1), \text{Int}[(d*x)^m*(a + b*\text{Log}[c*x^n])^{p-1}, x], x] /;$   $\text{FreeQ}\{a, b, c, d, m, n\}, x \} \&\& \text{NeQ}[m, -1] \&\& \text{GtQ}[p, 0]$

#### Rule 2304

$\text{Int}[(a_. + \text{Log}[c_.*(x_.)^{n_.}]*b_.)*((d_.)*(x_.))^{m_.}, x\_Symbol] \rightarrow \text{Simp}[(d*x)^{m+1}*(a + b*\text{Log}[c*x^n])]/(d*(m+1)), x] - \text{Simp}[(b*n*(d*x)^{m+1})/(d*(m+1)^2), x] /;$   $\text{FreeQ}\{a, b, c, d, m, n\}, x \} \&\& \text{NeQ}[m, -1]$

#### Rubi steps

$$\begin{aligned} \int \frac{\log^2(cx)}{x^2} dx &= -\frac{\log^2(cx)}{x} + 2 \int \frac{\log(cx)}{x^2} dx \\ &= -\frac{2}{x} - \frac{2 \log(cx)}{x} - \frac{\log^2(cx)}{x} \end{aligned}$$

**Mathematica [A]** time = 0.0010911, size = 26, normalized size = 1.

$$-\frac{\log^2(cx)}{x} - \frac{2 \log(cx)}{x} - \frac{2}{x}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[\text{Log}[c*x]^2/x^2, x]$

[Out]  $-2/x - (2*\text{Log}[c*x])/x - \text{Log}[c*x]^2/x$



---

**Maple [A]** time = 0.036, size = 27, normalized size = 1.

$$-2x^{-1} - 2\frac{\ln(cx)}{x} - \frac{(\ln(cx))^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c\*x)^2/x^2,x)

[Out] -2/x-2\*ln(c\*x)/x-ln(c\*x)^2/x

---

**Maxima [A]** time = 0.966634, size = 26, normalized size = 1.

$$-\frac{\log(cx)^2 + 2\log(cx) + 2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^2/x^2,x, algorithm="maxima")

[Out] -(log(c\*x)^2 + 2\*log(c\*x) + 2)/x

---

**Fricas [A]** time = 0.794552, size = 46, normalized size = 1.77

$$-\frac{\log(cx)^2 + 2\log(cx) + 2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^2/x^2,x, algorithm="fricas")

[Out] -(log(c\*x)^2 + 2\*log(c\*x) + 2)/x

---

**Sympy [A]** time = 0.111153, size = 20, normalized size = 0.77

$$-\frac{\log(cx)^2}{x} - \frac{2\log(cx)}{x} - \frac{2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(c\*x)\*\*2/x\*\*2,x)

[Out] -log(c\*x)\*\*2/x - 2\*log(c\*x)/x - 2/x

---

**Giac [A]** time = 1.11971, size = 35, normalized size = 1.35

$$-\frac{\log(cx)^2}{x} - \frac{2\log(cx)}{x} - \frac{2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x)^2/x^2,x, algorithm="giac")
```

```
[Out] -log(c*x)^2/x - 2*log(c*x)/x - 2/x
```

$$3.14 \quad \int \frac{\log^2(cx)}{x^3} dx$$

**Optimal.** Leaf size=32

$$-\frac{\log^2(cx)}{2x^2} - \frac{\log(cx)}{2x^2} - \frac{1}{4x^2}$$

[Out]  $-1/(4*x^2) - \text{Log}[c*x]/(2*x^2) - \text{Log}[c*x]^2/(2*x^2)$

**Rubi [A]** time = 0.0189641, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2305, 2304}

$$-\frac{\log^2(cx)}{2x^2} - \frac{\log(cx)}{2x^2} - \frac{1}{4x^2}$$

Antiderivative was successfully verified.

[In] Int[Log[c\*x]^2/x^3,x]

[Out]  $-1/(4*x^2) - \text{Log}[c*x]/(2*x^2) - \text{Log}[c*x]^2/(2*x^2)$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*(d\*x)^(m + 1))/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{\log^2(cx)}{x^3} dx &= -\frac{\log^2(cx)}{2x^2} + \int \frac{\log(cx)}{x^3} dx \\ &= -\frac{1}{4x^2} - \frac{\log(cx)}{2x^2} - \frac{\log^2(cx)}{2x^2} \end{aligned}$$

**Mathematica [A]** time = 0.0012215, size = 32, normalized size = 1.

$$-\frac{\log^2(cx)}{2x^2} - \frac{\log(cx)}{2x^2} - \frac{1}{4x^2}$$

Antiderivative was successfully verified.

[In] Integrate[Log[c\*x]^2/x^3,x]

[Out]  $-1/(4*x^2) - \text{Log}[c*x]/(2*x^2) - \text{Log}[c*x]^2/(2*x^2)$

---

**Maple [A]** time = 0.037, size = 27, normalized size = 0.8

$$-\frac{1}{4x^2} - \frac{\ln(cx)}{2x^2} - \frac{(\ln(cx))^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c\*x)^2/x^3,x)

[Out] -1/4/x^2-1/2\*ln(c\*x)/x^2-1/2\*ln(c\*x)^2/x^2

---

**Maxima [A]** time = 0.985426, size = 28, normalized size = 0.88

$$-\frac{2 \log(cx)^2 + 2 \log(cx) + 1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^2/x^3,x, algorithm="maxima")

[Out] -1/4\*(2\*log(c\*x)^2 + 2\*log(c\*x) + 1)/x^2

---

**Fricas [A]** time = 0.805439, size = 57, normalized size = 1.78

$$-\frac{2 \log(cx)^2 + 2 \log(cx) + 1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^2/x^3,x, algorithm="fricas")

[Out] -1/4\*(2\*log(c\*x)^2 + 2\*log(c\*x) + 1)/x^2

---

**Sympy [A]** time = 0.128722, size = 29, normalized size = 0.91

$$-\frac{\log(cx)^2}{2x^2} - \frac{\log(cx)}{2x^2} - \frac{1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(c\*x)\*\*2/x\*\*3,x)

[Out] -log(c\*x)\*\*2/(2\*x\*\*2) - log(c\*x)/(2\*x\*\*2) - 1/(4\*x\*\*2)

---

**Giac [A]** time = 1.1034, size = 35, normalized size = 1.09

$$-\frac{\log(cx)^2}{2x^2} - \frac{\log(cx)}{2x^2} - \frac{1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x)^2/x^3,x, algorithm="giac")
```

```
[Out] -1/2*log(c*x)^2/x^2 - 1/2*log(c*x)/x^2 - 1/4/x^2
```

### 3.15 $\int x^3 \log^3(cx) dx$

**Optimal.** Leaf size=45

$$\frac{1}{4}x^4 \log^3(cx) - \frac{3}{16}x^4 \log^2(cx) + \frac{3}{32}x^4 \log(cx) - \frac{3x^4}{128}$$

[Out]  $(-3*x^4)/128 + (3*x^4*\text{Log}[c*x])/32 - (3*x^4*\text{Log}[c*x]^2)/16 + (x^4*\text{Log}[c*x]^3)/4$

**Rubi [A]** time = 0.03594, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2305, 2304}

$$\frac{1}{4}x^4 \log^3(cx) - \frac{3}{16}x^4 \log^2(cx) + \frac{3}{32}x^4 \log(cx) - \frac{3x^4}{128}$$

Antiderivative was successfully verified.

[In] Int[x^3\*Log[c\*x]^3,x]

[Out]  $(-3*x^4)/128 + (3*x^4*\text{Log}[c*x])/32 - (3*x^4*\text{Log}[c*x]^2)/16 + (x^4*\text{Log}[c*x]^3)/4$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int x^3 \log^3(cx) dx &= \frac{1}{4}x^4 \log^3(cx) - \frac{3}{4} \int x^3 \log^2(cx) dx \\ &= -\frac{3}{16}x^4 \log^2(cx) + \frac{1}{4}x^4 \log^3(cx) + \frac{3}{8} \int x^3 \log(cx) dx \\ &= -\frac{3x^4}{128} + \frac{3}{32}x^4 \log(cx) - \frac{3}{16}x^4 \log^2(cx) + \frac{1}{4}x^4 \log^3(cx) \end{aligned}$$

**Mathematica [A]** time = 0.0014539, size = 45, normalized size = 1.

$$\frac{1}{4}x^4 \log^3(cx) - \frac{3}{16}x^4 \log^2(cx) + \frac{3}{32}x^4 \log(cx) - \frac{3x^4}{128}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Log[c\*x]^3,x]

[Out]  $(-3x^4)/128 + (3x^4 \text{Log}[cx])/32 - (3x^4 \text{Log}[cx]^2)/16 + (x^4 \text{Log}[cx]^3)/4$

**Maple [A]** time = 0.036, size = 38, normalized size = 0.8

$$-\frac{3x^4}{128} + \frac{3x^4 \ln(cx)}{32} - \frac{3x^4 (\ln(cx))^2}{16} + \frac{x^4 (\ln(cx))^3}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*ln(c*x)^3,x)`

[Out]  $-3/128*x^4+3/32*x^4*\ln(c*x)-3/16*x^4*\ln(c*x)^2+1/4*x^4*\ln(c*x)^3$

**Maxima [A]** time = 0.99127, size = 39, normalized size = 0.87

$$\frac{1}{128} (32 \log(cx)^3 - 24 \log(cx)^2 + 12 \log(cx) - 3)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*log(c*x)^3,x, algorithm="maxima")`

[Out]  $1/128*(32*\log(c*x)^3 - 24*\log(c*x)^2 + 12*\log(c*x) - 3)*x^4$

**Fricas [A]** time = 0.766697, size = 100, normalized size = 2.22

$$\frac{1}{4} x^4 \log(cx)^3 - \frac{3}{16} x^4 \log(cx)^2 + \frac{3}{32} x^4 \log(cx) - \frac{3}{128} x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*log(c*x)^3,x, algorithm="fricas")`

[Out]  $1/4*x^4*\log(c*x)^3 - 3/16*x^4*\log(c*x)^2 + 3/32*x^4*\log(c*x) - 3/128*x^4$

**Sympy [A]** time = 0.123327, size = 42, normalized size = 0.93

$$\frac{x^4 \log(cx)^3}{4} - \frac{3x^4 \log(cx)^2}{16} + \frac{3x^4 \log(cx)}{32} - \frac{3x^4}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*ln(c*x)**3,x)`

[Out]  $x**4*\log(c*x)**3/4 - 3*x**4*\log(c*x)**2/16 + 3*x**4*\log(c*x)/32 - 3*x**4/128$

**Giac [A]** time = 1.10661, size = 50, normalized size = 1.11

$$\frac{1}{4}x^4 \log(cx)^3 - \frac{3}{16}x^4 \log(cx)^2 + \frac{3}{32}x^4 \log(cx) - \frac{3}{128}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*log(c\*x)^3,x, algorithm="giac")

[Out] 1/4\*x^4\*log(c\*x)^3 - 3/16\*x^4\*log(c\*x)^2 + 3/32\*x^4\*log(c\*x) - 3/128\*x^4



### 3.16 $\int x^2 \log^3(cx) dx$

**Optimal.** Leaf size=45

$$\frac{1}{3}x^3 \log^3(cx) - \frac{1}{3}x^3 \log^2(cx) + \frac{2}{9}x^3 \log(cx) - \frac{2x^3}{27}$$

[Out]  $(-2x^3)/27 + (2x^3 \text{Log}[c*x])/9 - (x^3 \text{Log}[c*x]^2)/3 + (x^3 \text{Log}[c*x]^3)/3$

**Rubi [A]** time = 0.0314186, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2305, 2304}

$$\frac{1}{3}x^3 \log^3(cx) - \frac{1}{3}x^3 \log^2(cx) + \frac{2}{9}x^3 \log(cx) - \frac{2x^3}{27}$$

Antiderivative was successfully verified.

[In] Int[x^2\*Log[c\*x]^3,x]

[Out]  $(-2x^3)/27 + (2x^3 \text{Log}[c*x])/9 - (x^3 \text{Log}[c*x]^2)/3 + (x^3 \text{Log}[c*x]^3)/3$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int x^2 \log^3(cx) dx &= \frac{1}{3}x^3 \log^3(cx) - \int x^2 \log^2(cx) dx \\ &= -\frac{1}{3}x^3 \log^2(cx) + \frac{1}{3}x^3 \log^3(cx) + \frac{2}{3} \int x^2 \log(cx) dx \\ &= -\frac{2x^3}{27} + \frac{2}{9}x^3 \log(cx) - \frac{1}{3}x^3 \log^2(cx) + \frac{1}{3}x^3 \log^3(cx) \end{aligned}$$

**Mathematica [A]** time = 0.0014576, size = 45, normalized size = 1.

$$\frac{1}{3}x^3 \log^3(cx) - \frac{1}{3}x^3 \log^2(cx) + \frac{2}{9}x^3 \log(cx) - \frac{2x^3}{27}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Log[c\*x]^3,x]

[Out]  $(-2x^3)/27 + (2x^3 \text{Log}[c*x])/9 - (x^3 \text{Log}[c*x]^2)/3 + (x^3 \text{Log}[c*x]^3)/3$

---

**Maple [A]** time = 0.036, size = 38, normalized size = 0.8

$$-\frac{2x^3}{27} + \frac{2x^3 \ln(cx)}{9} - \frac{x^3 (\ln(cx))^2}{3} + \frac{x^3 (\ln(cx))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*ln(c\*x)^3,x)

[Out] -2/27\*x^3+2/9\*x^3\*ln(c\*x)-1/3\*x^3\*ln(c\*x)^2+1/3\*x^3\*ln(c\*x)^3

---

**Maxima [A]** time = 1.07407, size = 39, normalized size = 0.87

$$\frac{1}{27} (9 \log(cx)^3 - 9 \log(cx)^2 + 6 \log(cx) - 2)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*log(c\*x)^3,x, algorithm="maxima")

[Out] 1/27\*(9\*log(c\*x)^3 - 9\*log(c\*x)^2 + 6\*log(c\*x) - 2)\*x^3

---

**Fricas [A]** time = 0.781229, size = 96, normalized size = 2.13

$$\frac{1}{3} x^3 \log(cx)^3 - \frac{1}{3} x^3 \log(cx)^2 + \frac{2}{9} x^3 \log(cx) - \frac{2}{27} x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*log(c\*x)^3,x, algorithm="fricas")

[Out] 1/3\*x^3\*log(c\*x)^3 - 1/3\*x^3\*log(c\*x)^2 + 2/9\*x^3\*log(c\*x) - 2/27\*x^3

---

**Sympy [A]** time = 0.121807, size = 41, normalized size = 0.91

$$\frac{x^3 \log(cx)^3}{3} - \frac{x^3 \log(cx)^2}{3} + \frac{2x^3 \log(cx)}{9} - \frac{2x^3}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*ln(c\*x)\*\*3,x)

[Out] x\*\*3\*log(c\*x)\*\*3/3 - x\*\*3\*log(c\*x)\*\*2/3 + 2\*x\*\*3\*log(c\*x)/9 - 2\*x\*\*3/27

---

**Giac [A]** time = 1.10201, size = 50, normalized size = 1.11

$$\frac{1}{3} x^3 \log(cx)^3 - \frac{1}{3} x^3 \log(cx)^2 + \frac{2}{9} x^3 \log(cx) - \frac{2}{27} x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(c*x)^3,x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(c*x)^3 - 1/3*x^3*log(c*x)^2 + 2/9*x^3*log(c*x) - 2/27*x^3
```

### 3.17 $\int x \log^3(cx) dx$

**Optimal.** Leaf size=45

$$\frac{1}{2}x^2 \log^3(cx) - \frac{3}{4}x^2 \log^2(cx) + \frac{3}{4}x^2 \log(cx) - \frac{3x^2}{8}$$

[Out]  $(-3*x^2)/8 + (3*x^2*\text{Log}[c*x])/4 - (3*x^2*\text{Log}[c*x]^2)/4 + (x^2*\text{Log}[c*x]^3)/2$

**Rubi [A]** time = 0.0181902, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {2305, 2304}

$$\frac{1}{2}x^2 \log^3(cx) - \frac{3}{4}x^2 \log^2(cx) + \frac{3}{4}x^2 \log(cx) - \frac{3x^2}{8}$$

Antiderivative was successfully verified.

[In] Int[x\*Log[c\*x]^3,x]

[Out]  $(-3*x^2)/8 + (3*x^2*\text{Log}[c*x])/4 - (3*x^2*\text{Log}[c*x]^2)/4 + (x^2*\text{Log}[c*x]^3)/2$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int x \log^3(cx) dx &= \frac{1}{2}x^2 \log^3(cx) - \frac{3}{2} \int x \log^2(cx) dx \\ &= -\frac{3}{4}x^2 \log^2(cx) + \frac{1}{2}x^2 \log^3(cx) + \frac{3}{2} \int x \log(cx) dx \\ &= -\frac{3x^2}{8} + \frac{3}{4}x^2 \log(cx) - \frac{3}{4}x^2 \log^2(cx) + \frac{1}{2}x^2 \log^3(cx) \end{aligned}$$

**Mathematica [A]** time = 0.0011034, size = 45, normalized size = 1.

$$\frac{1}{2}x^2 \log^3(cx) - \frac{3}{4}x^2 \log^2(cx) + \frac{3}{4}x^2 \log(cx) - \frac{3x^2}{8}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Log[c\*x]^3,x]

[Out]  $(-3*x^2)/8 + (3*x^2*\text{Log}[c*x])/4 - (3*x^2*\text{Log}[c*x]^2)/4 + (x^2*\text{Log}[c*x]^3)/2$

---

**Maple [A]** time = 0.033, size = 38, normalized size = 0.8

$$-\frac{3x^2}{8} + \frac{3x^2 \ln(cx)}{4} - \frac{3x^2 (\ln(cx))^2}{4} + \frac{x^2 (\ln(cx))^3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*ln(c\*x)^3,x)

[Out] -3/8\*x^2+3/4\*x^2\*ln(c\*x)-3/4\*x^2\*ln(c\*x)^2+1/2\*x^2\*ln(c\*x)^3

---

**Maxima [A]** time = 1.0054, size = 39, normalized size = 0.87

$$\frac{1}{8} (4 \log(cx)^3 - 6 \log(cx)^2 + 6 \log(cx) - 3)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*log(c\*x)^3,x, algorithm="maxima")

[Out] 1/8\*(4\*log(c\*x)^3 - 6\*log(c\*x)^2 + 6\*log(c\*x) - 3)\*x^2

---

**Fricas [A]** time = 0.823187, size = 95, normalized size = 2.11

$$\frac{1}{2} x^2 \log(cx)^3 - \frac{3}{4} x^2 \log(cx)^2 + \frac{3}{4} x^2 \log(cx) - \frac{3}{8} x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*log(c\*x)^3,x, algorithm="fricas")

[Out] 1/2\*x^2\*log(c\*x)^3 - 3/4\*x^2\*log(c\*x)^2 + 3/4\*x^2\*log(c\*x) - 3/8\*x^2

---

**Sympy [A]** time = 0.122258, size = 42, normalized size = 0.93

$$\frac{x^2 \log(cx)^3}{2} - \frac{3x^2 \log(cx)^2}{4} + \frac{3x^2 \log(cx)}{4} - \frac{3x^2}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*ln(c\*x)\*\*3,x)

[Out] x\*\*2\*log(c\*x)\*\*3/2 - 3\*x\*\*2\*log(c\*x)\*\*2/4 + 3\*x\*\*2\*log(c\*x)/4 - 3\*x\*\*2/8

---

**Giac [A]** time = 1.09851, size = 50, normalized size = 1.11

$$\frac{1}{2} x^2 \log(cx)^3 - \frac{3}{4} x^2 \log(cx)^2 + \frac{3}{4} x^2 \log(cx) - \frac{3}{8} x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(c*x)^3,x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(c*x)^3 - 3/4*x^2*log(c*x)^2 + 3/4*x^2*log(c*x) - 3/8*x^2
```

### 3.18 $\int \log^3(cx) dx$

**Optimal.** Leaf size=28

$$x \log^3(cx) - 3x \log^2(cx) + 6x \log(cx) - 6x$$

[Out]  $-6*x + 6*x*\text{Log}[c*x] - 3*x*\text{Log}[c*x]^2 + x*\text{Log}[c*x]^3$

**Rubi [A]** time = 0.0079641, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2296, 2295}

$$x \log^3(cx) - 3x \log^2(cx) + 6x \log(cx) - 6x$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Log}[c*x]^3, x]$

[Out]  $-6*x + 6*x*\text{Log}[c*x] - 3*x*\text{Log}[c*x]^2 + x*\text{Log}[c*x]^3$

#### Rule 2296

$\text{Int}[(a + \text{Log}[c*x^n])^p, x] - \text{Dist}[b*n*p, \text{Int}[(a + b*\text{Log}[c*x^n])^{p-1}, x], x] /;$   
 $\text{FreeQ}\{a, b, c, n, x\} \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{IntegerQ}[2*p]$

#### Rule 2295

$\text{Int}[\text{Log}[c*x^n], x] - \text{Simp}[n*x, x] /;$   
 $\text{FreeQ}\{c, n, x\}$

#### Rubi steps

$$\begin{aligned} \int \log^3(cx) dx &= x \log^3(cx) - 3 \int \log^2(cx) dx \\ &= -3x \log^2(cx) + x \log^3(cx) + 6 \int \log(cx) dx \\ &= -6x + 6x \log(cx) - 3x \log^2(cx) + x \log^3(cx) \end{aligned}$$

**Mathematica [A]** time = 0.0009562, size = 28, normalized size = 1.

$$x \log^3(cx) - 3x \log^2(cx) + 6x \log(cx) - 6x$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[\text{Log}[c*x]^3, x]$

[Out]  $-6*x + 6*x*\text{Log}[c*x] - 3*x*\text{Log}[c*x]^2 + x*\text{Log}[c*x]^3$

**Maple [A]** time = 0.035, size = 29, normalized size = 1.

$$-6x + 6x \ln(cx) - 3x (\ln(cx))^2 + x (\ln(cx))^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(c*x)^3,x)`

[Out] `-6*x+6*x*ln(c*x)-3*x*ln(c*x)^2+x*ln(c*x)^3`

**Maxima [A]** time = 1.02186, size = 32, normalized size = 1.14

$$(\log(cx)^3 - 3 \log(cx)^2 + 6 \log(cx) - 6)x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x)^3,x, algorithm="maxima")`

[Out] `(log(c*x)^3 - 3*log(c*x)^2 + 6*log(c*x) - 6)*x`

**Fricas [A]** time = 0.747944, size = 70, normalized size = 2.5

$$x \log(cx)^3 - 3x \log(cx)^2 + 6x \log(cx) - 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x)^3,x, algorithm="fricas")`

[Out] `x*log(c*x)^3 - 3*x*log(c*x)^2 + 6*x*log(c*x) - 6*x`

**Sympy [A]** time = 0.110192, size = 29, normalized size = 1.04

$$x \log(cx)^3 - 3x \log(cx)^2 + 6x \log(cx) - 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(c*x)**3,x)`

[Out] `x*log(c*x)**3 - 3*x*log(c*x)**2 + 6*x*log(c*x) - 6*x`

**Giac [A]** time = 1.13001, size = 38, normalized size = 1.36

$$x \log(cx)^3 - 3x \log(cx)^2 + 6x \log(cx) - 6x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x)^3,x, algorithm="giac")`

[Out] `x*log(c*x)^3 - 3*x*log(c*x)^2 + 6*x*log(c*x) - 6*x`



$$3.19 \quad \int \frac{\log^3(cx)}{x} dx$$

**Optimal.** Leaf size=10

$$\frac{1}{4} \log^4(cx)$$

[Out] Log[c\*x]^4/4

**Rubi [A]** time = 0.0112918, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2302, 30}

$$\frac{1}{4} \log^4(cx)$$

Antiderivative was successfully verified.

[In] Int[Log[c\*x]^3/x, x]

[Out] Log[c\*x]^4/4

**Rule 2302**

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] := Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

**Rule 30**

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

**Rubi steps**

$$\begin{aligned} \int \frac{\log^3(cx)}{x} dx &= \text{Subst} \left( \int x^3 dx, x, \log(cx) \right) \\ &= \frac{1}{4} \log^4(cx) \end{aligned}$$

**Mathematica [A]** time = 0.0009738, size = 10, normalized size = 1.

$$\frac{1}{4} \log^4(cx)$$

Antiderivative was successfully verified.

[In] Integrate[Log[c\*x]^3/x, x]

[Out] Log[c\*x]^4/4

**Maple [A]** time = 0.033, size = 9, normalized size = 0.9

$$\frac{(\ln(cx))^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c\*x)^3/x,x)

[Out] 1/4\*ln(c\*x)^4

---

**Maxima [A]** time = 1.05993, size = 11, normalized size = 1.1

$$\frac{1}{4} \log(cx)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^3/x,x, algorithm="maxima")

[Out] 1/4\*log(c\*x)^4

---

**Fricas [A]** time = 0.872809, size = 22, normalized size = 2.2

$$\frac{1}{4} \log(cx)^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^3/x,x, algorithm="fricas")

[Out] 1/4\*log(c\*x)^4

---

**Sympy [A]** time = 0.091152, size = 7, normalized size = 0.7

$$\frac{\log(cx)^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(c\*x)\*\*3/x,x)

[Out] log(c\*x)\*\*4/4

---

**Giac [A]** time = 1.0786, size = 11, normalized size = 1.1

$$\frac{1}{4} \log(cx)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x)^3/x,x, algorithm="giac")
```

```
[Out] 1/4*log(c*x)^4
```

### 3.20 $\int \frac{\log^3(cx)}{x^2} dx$

**Optimal.** Leaf size=37

$$-\frac{\log^3(cx)}{x} - \frac{3\log^2(cx)}{x} - \frac{6\log(cx)}{x} - \frac{6}{x}$$

[Out]  $-6/x - (6*\text{Log}[c*x])/x - (3*\text{Log}[c*x]^2)/x - \text{Log}[c*x]^3/x$

**Rubi [A]** time = 0.0339765, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2305, 2304}

$$-\frac{\log^3(cx)}{x} - \frac{3\log^2(cx)}{x} - \frac{6\log(cx)}{x} - \frac{6}{x}$$

Antiderivative was successfully verified.

[In] Int[Log[c\*x]^3/x^2,x]

[Out]  $-6/x - (6*\text{Log}[c*x])/x - (3*\text{Log}[c*x]^2)/x - \text{Log}[c*x]^3/x$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{\log^3(cx)}{x^2} dx &= -\frac{\log^3(cx)}{x} + 3 \int \frac{\log^2(cx)}{x^2} dx \\ &= -\frac{\log^3(cx)}{x} - \frac{3\log^2(cx)}{x} + 6 \int \frac{\log(cx)}{x^2} dx \\ &= -\frac{6}{x} - \frac{6\log(cx)}{x} - \frac{3\log^2(cx)}{x} - \frac{\log^3(cx)}{x} \end{aligned}$$

**Mathematica [A]** time = 0.0014595, size = 37, normalized size = 1.

$$-\frac{\log^3(cx)}{x} - \frac{3\log^2(cx)}{x} - \frac{6\log(cx)}{x} - \frac{6}{x}$$

Antiderivative was successfully verified.

[In] Integrate[Log[c\*x]^3/x^2,x]

[Out]  $-6/x - (6*\text{Log}[c*x])/x - (3*\text{Log}[c*x]^2)/x - \text{Log}[c*x]^3/x$

---

**Maple [A]** time = 0.036, size = 38, normalized size = 1.

$$-6x^{-1} - 6 \frac{\ln(cx)}{x} - 3 \frac{(\ln(cx))^2}{x} - \frac{(\ln(cx))^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(c*x)^3/x^2,x)`

[Out]  $-6/x - 6*\ln(c*x)/x - 3*\ln(c*x)^2/x - \ln(c*x)^3/x$

---

**Maxima [A]** time = 1.0759, size = 36, normalized size = 0.97

$$\frac{\log(cx)^3 + 3 \log(cx)^2 + 6 \log(cx) + 6}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x)^3/x^2,x, algorithm="maxima")`

[Out]  $-(\log(c*x)^3 + 3*\log(c*x)^2 + 6*\log(c*x) + 6)/x$

---

**Fricas [A]** time = 0.836164, size = 66, normalized size = 1.78

$$\frac{\log(cx)^3 + 3 \log(cx)^2 + 6 \log(cx) + 6}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(c*x)^3/x^2,x, algorithm="fricas")`

[Out]  $-(\log(c*x)^3 + 3*\log(c*x)^2 + 6*\log(c*x) + 6)/x$

---

**Sympy [A]** time = 0.138687, size = 31, normalized size = 0.84

$$-\frac{\log(cx)^3}{x} - \frac{3 \log(cx)^2}{x} - \frac{6 \log(cx)}{x} - \frac{6}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(c*x)**3/x**2,x)`

[Out]  $-\log(c*x)**3/x - 3*\log(c*x)**2/x - 6*\log(c*x)/x - 6/x$

---

**Giac [A]** time = 1.47079, size = 50, normalized size = 1.35

$$-\frac{\log(cx)^3}{x} - \frac{3 \log(cx)^2}{x} - \frac{6 \log(cx)}{x} - \frac{6}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(c*x)^3/x^2,x, algorithm="giac")
```

```
[Out] -log(c*x)^3/x - 3*log(c*x)^2/x - 6*log(c*x)/x - 6/x
```

$$3.21 \quad \int \frac{\log^3(cx)}{x^3} dx$$

**Optimal.** Leaf size=45

$$-\frac{\log^3(cx)}{2x^2} - \frac{3\log^2(cx)}{4x^2} - \frac{3\log(cx)}{4x^2} - \frac{3}{8x^2}$$

[Out]  $-3/(8*x^2) - (3*\text{Log}[c*x])/(4*x^2) - (3*\text{Log}[c*x]^2)/(4*x^2) - \text{Log}[c*x]^3/(2*x^2)$

**Rubi [A]** time = 0.0331861, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2305, 2304}

$$-\frac{\log^3(cx)}{2x^2} - \frac{3\log^2(cx)}{4x^2} - \frac{3\log(cx)}{4x^2} - \frac{3}{8x^2}$$

Antiderivative was successfully verified.

[In] Int[Log[c\*x]^3/x^3, x]

[Out]  $-3/(8*x^2) - (3*\text{Log}[c*x])/(4*x^2) - (3*\text{Log}[c*x]^2)/(4*x^2) - \text{Log}[c*x]^3/(2*x^2)$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m+1)), x] - Dist[(b\*n\*p)/(m+1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p-1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a + b\*Log[c\*x^n]))/(d\*(m+1)), x] - Simp[(b\*n\*(d\*x)^(m+1))/(d\*(m+1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{\log^3(cx)}{x^3} dx &= -\frac{\log^3(cx)}{2x^2} + \frac{3}{2} \int \frac{\log^2(cx)}{x^3} dx \\ &= -\frac{3\log^2(cx)}{4x^2} - \frac{\log^3(cx)}{2x^2} + \frac{3}{2} \int \frac{\log(cx)}{x^3} dx \\ &= -\frac{3}{8x^2} - \frac{3\log(cx)}{4x^2} - \frac{3\log^2(cx)}{4x^2} - \frac{\log^3(cx)}{2x^2} \end{aligned}$$

**Mathematica [A]** time = 0.0014731, size = 45, normalized size = 1.

$$-\frac{\log^3(cx)}{2x^2} - \frac{3\log^2(cx)}{4x^2} - \frac{3\log(cx)}{4x^2} - \frac{3}{8x^2}$$

Antiderivative was successfully verified.

[In] Integrate[Log[c\*x]^3/x^3,x]

[Out]  $-3/(8*x^2) - (3*\text{Log}[c*x])/(4*x^2) - (3*\text{Log}[c*x]^2)/(4*x^2) - \text{Log}[c*x]^3/(2*x^2)$

**Maple [A]** time = 0.035, size = 38, normalized size = 0.8

$$-\frac{3}{8x^2} - \frac{3 \ln(cx)}{4x^2} - \frac{3 (\ln(cx))^2}{4x^2} - \frac{(\ln(cx))^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(c\*x)^3/x^3,x)

[Out]  $-3/8/x^2-3/4*\ln(c*x)/x^2-3/4*\ln(c*x)^2/x^2-1/2*\ln(c*x)^3/x^2$

**Maxima [A]** time = 1.06371, size = 39, normalized size = 0.87

$$-\frac{4 \log(cx)^3 + 6 \log(cx)^2 + 6 \log(cx) + 3}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^3/x^3,x, algorithm="maxima")

[Out]  $-1/8*(4*\log(c*x)^3 + 6*\log(c*x)^2 + 6*\log(c*x) + 3)/x^2$

**Fricas [A]** time = 0.885273, size = 77, normalized size = 1.71

$$-\frac{4 \log(cx)^3 + 6 \log(cx)^2 + 6 \log(cx) + 3}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^3/x^3,x, algorithm="fricas")

[Out]  $-1/8*(4*\log(c*x)^3 + 6*\log(c*x)^2 + 6*\log(c*x) + 3)/x^2$

**Sympy [A]** time = 0.150497, size = 44, normalized size = 0.98

$$-\frac{\log(cx)^3}{2x^2} - \frac{3 \log(cx)^2}{4x^2} - \frac{3 \log(cx)}{4x^2} - \frac{3}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(c\*x)\*\*3/x\*\*3,x)

[Out]  $-\log(c*x)**3/(2*x**2) - 3*\log(c*x)**2/(4*x**2) - 3*\log(c*x)/(4*x**2) - 3/(8*x**2)$



---

**Giac [A]** time = 1.13499, size = 50, normalized size = 1.11

$$-\frac{\log(cx)^3}{2x^2} - \frac{3\log(cx)^2}{4x^2} - \frac{3\log(cx)}{4x^2} - \frac{3}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(c\*x)^3/x^3,x, algorithm="giac")

[Out] -1/2\*log(c\*x)^3/x^2 - 3/4\*log(c\*x)^2/x^2 - 3/4\*log(c\*x)/x^2 - 3/8/x^2

$$3.22 \quad \int \frac{x^3}{\log(cx)} dx$$

**Optimal.** Leaf size=11

$$\frac{\text{Ei}(4 \log(cx))}{c^4}$$

[Out] ExpIntegralEi[4\*Log[c\*x]]/c^4

**Rubi [A]** time = 0.0238933, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2309, 2178}

$$\frac{\text{Ei}(4 \log(cx))}{c^4}$$

Antiderivative was successfully verified.

[In] Int[x^3/Log[c\*x],x]

[Out] ExpIntegralEi[4\*Log[c\*x]]/c^4

#### Rule 2309

Int[((a\_.) + Log[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[E^((m + 1)\*x)\*(a + b\*x)^p, x], x, Log[c\*x]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{x^3}{\log(cx)} dx &= \frac{\text{Subst}\left(\int \frac{e^{4x}}{x} dx, x, \log(cx)\right)}{c^4} \\ &= \frac{\text{Ei}(4 \log(cx))}{c^4} \end{aligned}$$

**Mathematica [A]** time = 0.0143969, size = 11, normalized size = 1.

$$\frac{\text{Ei}(4 \log(cx))}{c^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Log[c\*x],x]

[Out] ExpIntegralEi[4\*Log[c\*x]]/c^4

---

**Maple [A]** time = 0.04, size = 14, normalized size = 1.3

$$-\frac{\text{Ei}(1, -4 \ln(cx))}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/ln(c\*x), x)

[Out] -1/c^4\*Ei(1, -4\*ln(c\*x))

---

**Maxima [A]** time = 1.1214, size = 15, normalized size = 1.36

$$\frac{\text{Ei}(4 \log(cx))}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(c\*x), x, algorithm="maxima")

[Out] Ei(4\*log(c\*x))/c^4

---

**Fricas [A]** time = 0.731077, size = 36, normalized size = 3.27

$$\frac{\log\_integral(c^4x^4)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(c\*x), x, algorithm="fricas")

[Out] log\_integral(c^4\*x^4)/c^4

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\log(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/ln(c\*x), x)

[Out] Integral(x\*\*3/log(c\*x), x)

---

**Giac [A]** time = 1.13243, size = 15, normalized size = 1.36

$$\frac{\text{Ei}(4 \log(cx))}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(c*x),x, algorithm="giac")
```

```
[Out] Ei(4*log(c*x))/c^4
```

$$3.23 \quad \int \frac{x^2}{\log(cx)} dx$$

**Optimal.** Leaf size=11

$$\frac{\text{Ei}(3 \log(cx))}{c^3}$$

[Out] ExpIntegralEi[3\*Log[c\*x]]/c^3

**Rubi [A]** time = 0.0234093, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2309, 2178}

$$\frac{\text{Ei}(3 \log(cx))}{c^3}$$

Antiderivative was successfully verified.

[In] Int[x^2/Log[c\*x],x]

[Out] ExpIntegralEi[3\*Log[c\*x]]/c^3

#### Rule 2309

Int[((a\_.) + Log[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[E^((m + 1)\*x)\*(a + b\*x)^p, x], x, Log[c\*x]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{x^2}{\log(cx)} dx &= \frac{\text{Subst}\left(\int \frac{e^{3x}}{x} dx, x, \log(cx)\right)}{c^3} \\ &= \frac{\text{Ei}(3 \log(cx))}{c^3} \end{aligned}$$

**Mathematica [A]** time = 0.0143752, size = 11, normalized size = 1.

$$\frac{\text{Ei}(3 \log(cx))}{c^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Log[c\*x],x]

[Out] ExpIntegralEi[3\*Log[c\*x]]/c^3

---

**Maple [A]** time = 0.039, size = 14, normalized size = 1.3

$$-\frac{\text{Ei}(1, -3 \ln(cx))}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/ln(c\*x), x)

[Out] -1/c^3\*Ei(1, -3\*ln(c\*x))

---

**Maxima [A]** time = 1.13978, size = 15, normalized size = 1.36

$$\frac{\text{Ei}(3 \log(cx))}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/log(c\*x), x, algorithm="maxima")

[Out] Ei(3\*log(c\*x))/c^3

---

**Fricas [A]** time = 0.699408, size = 36, normalized size = 3.27

$$\frac{\log\_integral(c^3x^3)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/log(c\*x), x, algorithm="fricas")

[Out] log\_integral(c^3\*x^3)/c^3

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\log(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/ln(c\*x), x)

[Out] Integral(x\*\*2/log(c\*x), x)

---

**Giac [A]** time = 1.12029, size = 15, normalized size = 1.36

$$\frac{\text{Ei}(3 \log(cx))}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(c*x),x, algorithm="giac")
```

```
[Out] Ei(3*log(c*x))/c^3
```

### 3.24 $\int \frac{x}{\log(cx)} dx$

**Optimal.** Leaf size=11

$$\frac{\text{Ei}(2 \log(cx))}{c^2}$$

[Out] ExpIntegralEi[2\*Log[c\*x]]/c^2

**Rubi [A]** time = 0.0174104, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {2309, 2178}

$$\frac{\text{Ei}(2 \log(cx))}{c^2}$$

Antiderivative was successfully verified.

[In] Int[x/Log[c\*x],x]

[Out] ExpIntegralEi[2\*Log[c\*x]]/c^2

#### Rule 2309

Int[((a\_.) + Log[(c\_.)\*(x\_)])\*(b\_.))^(p\_)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[E^((m + 1)\*x)\*(a + b\*x)^p, x], x, Log[c\*x]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{x}{\log(cx)} dx &= \frac{\text{Subst}\left(\int \frac{e^{2x}}{x} dx, x, \log(cx)\right)}{c^2} \\ &= \frac{\text{Ei}(2 \log(cx))}{c^2} \end{aligned}$$

**Mathematica [A]** time = 0.013095, size = 11, normalized size = 1.

$$\frac{\text{Ei}(2 \log(cx))}{c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/Log[c\*x],x]

[Out] ExpIntegralEi[2\*Log[c\*x]]/c^2



**Maple [A]** time = 0.043, size = 14, normalized size = 1.3

$$-\frac{\text{Ei}(1, -2 \ln(cx))}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/ln(c\*x), x)

[Out] -1/c^2\*Ei(1, -2\*ln(c\*x))

**Maxima [A]** time = 1.13168, size = 15, normalized size = 1.36

$$\frac{\text{Ei}(2 \log(cx))}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(c\*x), x, algorithm="maxima")

[Out] Ei(2\*log(c\*x))/c^2

**Fricas [A]** time = 0.836101, size = 36, normalized size = 3.27

$$\frac{\log\_integral(c^2x^2)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(c\*x), x, algorithm="fricas")

[Out] log\_integral(c^2\*x^2)/c^2

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\log(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/ln(c\*x), x)

[Out] Integral(x/log(c\*x), x)

**Giac [A]** time = 1.10774, size = 15, normalized size = 1.36

$$\frac{\text{Ei}(2 \log(cx))}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/log(c*x),x, algorithm="giac")
```

```
[Out] Ei(2*log(c*x))/c^2
```

$$3.25 \quad \int \frac{1}{\log(cx)} dx$$

**Optimal.** Leaf size=8

$$\frac{\text{li}(cx)}{c}$$

[Out] LogIntegral[c\*x]/c

**Rubi [A]** time = 0.0028761, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2298}

$$\frac{\text{li}(cx)}{c}$$

Antiderivative was successfully verified.

[In] Int[Log[c\*x]^(-1), x]

[Out] LogIntegral[c\*x]/c

**Rule 2298**

Int[Log[(c\_.)\*(x\_)]^(-1), x\_Symbol] :> Simp[LogIntegral[c\*x]/c, x] /; FreeQ[c, x]

**Rubi steps**

$$\int \frac{1}{\log(cx)} dx = \frac{\text{li}(cx)}{c}$$

**Mathematica [A]** time = 0.0045917, size = 8, normalized size = 1.

$$\frac{\text{li}(cx)}{c}$$

Antiderivative was successfully verified.

[In] Integrate[Log[c\*x]^(-1), x]

[Out] LogIntegral[c\*x]/c

**Maple [A]** time = 0.037, size = 14, normalized size = 1.8

$$-\frac{\text{Ei}(1, -\ln(cx))}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/ln(c\*x), x)

[Out]  $-1/c \cdot \text{Ei}(1, -\ln(cx))$

---

**Maxima [A]** time = 1.18215, size = 12, normalized size = 1.5

$$\frac{\text{Ei}(\log(cx))}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/log(c*x),x, algorithm="maxima")`

[Out]  $\text{Ei}(\log(cx))/c$

---

**Fricas [A]** time = 0.844042, size = 28, normalized size = 3.5

$$\frac{\log\_integral(cx)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/log(c*x),x, algorithm="fricas")`

[Out]  $\log\_integral(cx)/c$

---

**Sympy [A]** time = 0.510081, size = 5, normalized size = 0.62

$$\frac{\text{li}(cx)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/ln(c*x),x)`

[Out]  $\text{li}(cx)/c$

---

**Giac [A]** time = 1.10455, size = 12, normalized size = 1.5

$$\frac{\text{Ei}(\log(cx))}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/log(c*x),x, algorithm="giac")`

[Out]  $\text{Ei}(\log(cx))/c$

$$3.26 \quad \int \frac{1}{x \log(cx)} dx$$

**Optimal.** Leaf size=5

$$\log(\log(cx))$$

[Out] Log[Log[c\*x]]

**Rubi [A]** time = 0.0116483, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2302, 29}

$$\log(\log(cx))$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Log[c\*x]),x]

[Out] Log[Log[c\*x]]

#### Rule 2302

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] := Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

#### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

#### Rubi steps

$$\int \frac{1}{x \log(cx)} dx = \text{Subst} \left( \int \frac{1}{x} dx, x, \log(cx) \right) \\ = \log(\log(cx))$$

**Mathematica [A]** time = 0.0046747, size = 5, normalized size = 1.

$$\log(\log(cx))$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Log[c\*x]),x]

[Out] Log[Log[c\*x]]

**Maple [A]** time = 0.033, size = 6, normalized size = 1.2

$$\ln(\ln(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(c*x),x)
```

```
[Out] ln(ln(c*x))
```

---

**Maxima [A]** time = 1.03103, size = 7, normalized size = 1.4

$$\log(\log(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*x),x, algorithm="maxima")
```

```
[Out] log(log(c*x))
```

---

**Fricas [A]** time = 0.852844, size = 20, normalized size = 4.

$$\log(\log(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*x),x, algorithm="fricas")
```

```
[Out] log(log(c*x))
```

---

**Sympy [A]** time = 0.100574, size = 5, normalized size = 1.

$$\log(\log(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(c*x),x)
```

```
[Out] log(log(c*x))
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*x),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.27 \quad \int \frac{1}{x^2 \log(cx)} dx$$

**Optimal.** Leaf size=9

$$c\text{Ei}(-\log(cx))$$

[Out] c\*ExpIntegralEi[-Log[c\*x]]

**Rubi [A]** time = 0.0236958, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2309, 2178}

$$c\text{Ei}(-\log(cx))$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*Log[c\*x]),x]

[Out] c\*ExpIntegralEi[-Log[c\*x]]

Rule 2309

Int[((a\_.) + Log[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[E^((m + 1)\*x)\*(a + b\*x)^p, x], x, Log[c\*x]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]

Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rubi steps

$$\int \frac{1}{x^2 \log(cx)} dx = c \text{Subst} \left( \int \frac{e^{-x}}{x} dx, x, \log(cx) \right) = c\text{Ei}(-\log(cx))$$

**Mathematica [A]** time = 0.0137685, size = 9, normalized size = 1.

$$c\text{Ei}(-\log(cx))$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*Log[c\*x]),x]

[Out] c\*ExpIntegralEi[-Log[c\*x]]

**Maple [A]** time = 0.038, size = 10, normalized size = 1.1

$$-c\text{Ei}(1, \ln(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/ln(c*x),x)`

[Out] `-c*Ei(1,ln(c*x))`

---

**Maxima [A]** time = 1.21867, size = 12, normalized size = 1.33

$$c\operatorname{Ei}(-\log(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/log(c*x),x, algorithm="maxima")`

[Out] `c*Ei(-log(c*x))`

---

**Fricas [A]** time = 0.840993, size = 34, normalized size = 3.78

$$c\log\_integral\left(\frac{1}{cx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/log(c*x),x, algorithm="fricas")`

[Out] `c*log_integral(1/(c*x))`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \log(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/ln(c*x),x)`

[Out] `Integral(1/(x**2*log(c*x)), x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \log(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/log(c*x),x, algorithm="giac")`

[Out] `integrate(1/(x^2*log(c*x)), x)`



$$3.28 \quad \int \frac{1}{x^3 \log(cx)} dx$$

**Optimal.** Leaf size=11

$$c^2 \text{Ei}(-2 \log(cx))$$

[Out] c^2\*ExpIntegralEi[-2\*Log[c\*x]]

**Rubi [A]** time = 0.0241797, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2309, 2178}

$$c^2 \text{Ei}(-2 \log(cx))$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*Log[c\*x]),x]

[Out] c^2\*ExpIntegralEi[-2\*Log[c\*x]]

**Rule 2309**

Int[((a\_.) + Log[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/c^(m + 1), Subst[Int[E^((m + 1)\*x)\*(a + b\*x)^p, x], x, Log[c\*x]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]

**Rule 2178**

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

**Rubi steps**

$$\begin{aligned} \int \frac{1}{x^3 \log(cx)} dx &= c^2 \text{Subst} \left( \int \frac{e^{-2x}}{x} dx, x, \log(cx) \right) \\ &= c^2 \text{Ei}(-2 \log(cx)) \end{aligned}$$

**Mathematica [A]** time = 0.013946, size = 11, normalized size = 1.

$$c^2 \text{Ei}(-2 \log(cx))$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*Log[c\*x]),x]

[Out] c^2\*ExpIntegralEi[-2\*Log[c\*x]]

**Maple [A]** time = 0.038, size = 14, normalized size = 1.3

$$-c^2 \text{Ei}(1, 2 \ln(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/ln(c*x),x)`

[Out] `-c^2*Ei(1,2*ln(c*x))`

---

**Maxima [A]** time = 1.16887, size = 15, normalized size = 1.36

$$c^2 \operatorname{Ei}(-2 \log(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/log(c*x),x, algorithm="maxima")`

[Out] `c^2*Ei(-2*log(c*x))`

---

**Fricas [A]** time = 0.724369, size = 42, normalized size = 3.82

$$c^2 \log\_integral\left(\frac{1}{c^2 x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/log(c*x),x, algorithm="fricas")`

[Out] `c^2*log_integral(1/(c^2*x^2))`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \log(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/ln(c*x),x)`

[Out] `Integral(1/(x**3*log(c*x)), x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \log(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/log(c*x),x, algorithm="giac")`

[Out] `integrate(1/(x^3*log(c*x)), x)`

$$3.29 \quad \int \frac{x^3}{\log^2(cx)} dx$$

**Optimal.** Leaf size=24

$$\frac{4\text{Ei}(4\log(cx))}{c^4} - \frac{x^4}{\log(cx)}$$

[Out] (4\*ExpIntegralEi[4\*Log[c\*x]])/c^4 - x^4/Log[c\*x]

**Rubi [A]** time = 0.0375445, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {2306, 2309, 2178}

$$\frac{4\text{Ei}(4\log(cx))}{c^4} - \frac{x^4}{\log(cx)}$$

Antiderivative was successfully verified.

[In] Int[x^3/Log[c\*x]^2,x]

[Out] (4\*ExpIntegralEi[4\*Log[c\*x]])/c^4 - x^4/Log[c\*x]

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2309

Int[((a\_.) + Log[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/c^(m + 1), Subst[Int[E^((m + 1)\*x)\*(a + b\*x)^p, x], x, Log[c\*x]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{x^3}{\log^2(cx)} dx &= -\frac{x^4}{\log(cx)} + 4 \int \frac{x^3}{\log(cx)} dx \\ &= -\frac{x^4}{\log(cx)} + \frac{4 \text{Subst}\left(\int \frac{e^{4x}}{x} dx, x, \log(cx)\right)}{c^4} \\ &= \frac{4\text{Ei}(4\log(cx))}{c^4} - \frac{x^4}{\log(cx)} \end{aligned}$$

**Mathematica [A]** time = 0.0146853, size = 24, normalized size = 1.

$$\frac{4\text{Ei}(4\log(cx))}{c^4} - \frac{x^4}{\log(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Log[c\*x]^2,x]

[Out] (4\*ExpIntegralEi[4\*Log[c\*x]])/c^4 - x^4/Log[c\*x]

**Maple [A]** time = 0.033, size = 26, normalized size = 1.1

$$-\frac{x^4}{\ln(cx)} - 4 \frac{\text{Ei}(1, -4 \ln(cx))}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/ln(c\*x)^2,x)

[Out] -x^4/ln(c\*x)-4/c^4\*Ei(1,-4\*ln(c\*x))

**Maxima [A]** time = 1.18444, size = 18, normalized size = 0.75

$$\frac{4\Gamma(-1, -4 \log(cx))}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(c\*x)^2,x, algorithm="maxima")

[Out] 4\*gamma(-1, -4\*log(c\*x))/c^4

**Fricas [A]** time = 0.707463, size = 84, normalized size = 3.5

$$\frac{c^4 x^4 - 4 \log(cx) \log\_integral(c^4 x^4)}{c^4 \log(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(c\*x)^2,x, algorithm="fricas")

[Out] -(c^4\*x^4 - 4\*log(c\*x)\*log\_integral(c^4\*x^4))/(c^4\*log(c\*x))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{x^4}{\log(cx)} + 4 \int \frac{x^3}{\log(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/ln(c\*x)\*\*2,x)

[Out] -x\*\*4/log(c\*x) + 4\*Integral(x\*\*3/log(c\*x), x)

---

**Giac [A]** time = 1.12186, size = 32, normalized size = 1.33

$$-\frac{x^4}{\log(cx)} + \frac{4 \operatorname{Ei}(4 \log(cx))}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(c\*x)^2,x, algorithm="giac")

[Out] -x^4/log(c\*x) + 4\*Ei(4\*log(c\*x))/c^4

$$3.30 \quad \int \frac{x^2}{\log^2(cx)} dx$$

**Optimal.** Leaf size=24

$$\frac{3\text{Ei}(3\log(cx))}{c^3} - \frac{x^3}{\log(cx)}$$

[Out] (3\*ExpIntegralEi[3\*Log[c\*x]])/c^3 - x^3/Log[c\*x]

**Rubi [A]** time = 0.0372768, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {2306, 2309, 2178}

$$\frac{3\text{Ei}(3\log(cx))}{c^3} - \frac{x^3}{\log(cx)}$$

Antiderivative was successfully verified.

[In] Int[x^2/Log[c\*x]^2,x]

[Out] (3\*ExpIntegralEi[3\*Log[c\*x]])/c^3 - x^3/Log[c\*x]

#### Rule 2306

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^(p + 1))/(b*d*n*(p + 1)), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

#### Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol]
:> Dist[1/c^(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x]
/; FreeQ[{a, b, c, p}, x] && IntegerQ[m]
```

#### Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x]
/; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

#### Rubi steps

$$\begin{aligned} \int \frac{x^2}{\log^2(cx)} dx &= -\frac{x^3}{\log(cx)} + 3 \int \frac{x^2}{\log(cx)} dx \\ &= -\frac{x^3}{\log(cx)} + \frac{3 \text{Subst}\left(\int \frac{e^{3x}}{x} dx, x, \log(cx)\right)}{c^3} \\ &= \frac{3\text{Ei}(3\log(cx))}{c^3} - \frac{x^3}{\log(cx)} \end{aligned}$$

**Mathematica [A]** time = 0.0154332, size = 24, normalized size = 1.

$$\frac{3\text{Ei}(3\log(cx))}{c^3} - \frac{x^3}{\log(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Log[c\*x]^2,x]

[Out] (3\*ExpIntegralEi[3\*Log[c\*x]])/c^3 - x^3/Log[c\*x]

**Maple [A]** time = 0.033, size = 26, normalized size = 1.1

$$-\frac{x^3}{\ln(cx)} - 3 \frac{\text{Ei}(1, -3 \ln(cx))}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/ln(c\*x)^2,x)

[Out] -x^3/ln(c\*x)-3/c^3\*Ei(1,-3\*ln(c\*x))

**Maxima [A]** time = 1.22859, size = 18, normalized size = 0.75

$$\frac{3\Gamma(-1, -3 \log(cx))}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/log(c\*x)^2,x, algorithm="maxima")

[Out] 3\*gamma(-1, -3\*log(c\*x))/c^3

**Fricas [A]** time = 0.756365, size = 84, normalized size = 3.5

$$\frac{c^3x^3 - 3 \log(cx) \log\_integral(c^3x^3)}{c^3 \log(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/log(c\*x)^2,x, algorithm="fricas")

[Out] -(c^3\*x^3 - 3\*log(c\*x)\*log\_integral(c^3\*x^3))/(c^3\*log(c\*x))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{x^3}{\log(cx)} + 3 \int \frac{x^2}{\log(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/ln(c\*x)\*\*2,x)

[Out] -x\*\*3/log(c\*x) + 3\*Integral(x\*\*2/log(c\*x), x)

---

**Giac [A]** time = 1.10352, size = 32, normalized size = 1.33

$$-\frac{x^3}{\log(cx)} + \frac{3 \operatorname{Ei}(3 \log(cx))}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/log(c\*x)^2,x, algorithm="giac")

[Out] -x^3/log(c\*x) + 3\*Ei(3\*log(c\*x))/c^3



$$3.31 \quad \int \frac{x}{\log^2(cx)} dx$$

**Optimal.** Leaf size=24

$$\frac{2\text{Ei}(2 \log(cx))}{c^2} - \frac{x^2}{\log(cx)}$$

[Out] (2\*ExpIntegralEi[2\*Log[c\*x]])/c^2 - x^2/Log[c\*x]

**Rubi [A]** time = 0.0263859, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {2306, 2309, 2178}

$$\frac{2\text{Ei}(2 \log(cx))}{c^2} - \frac{x^2}{\log(cx)}$$

Antiderivative was successfully verified.

[In] Int[x/Log[c\*x]^2,x]

[Out] (2\*ExpIntegralEi[2\*Log[c\*x]])/c^2 - x^2/Log[c\*x]

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2309

Int[((a\_.) + Log[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/c^(m + 1), Subst[Int[E^((m + 1)\*x)\*(a + b\*x)^p, x], x, Log[c\*x]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{x}{\log^2(cx)} dx &= -\frac{x^2}{\log(cx)} + 2 \int \frac{x}{\log(cx)} dx \\ &= -\frac{x^2}{\log(cx)} + \frac{2 \text{Subst}\left(\int \frac{e^{2x}}{x} dx, x, \log(cx)\right)}{c^2} \\ &= \frac{2\text{Ei}(2 \log(cx))}{c^2} - \frac{x^2}{\log(cx)} \end{aligned}$$

**Mathematica [A]** time = 0.013267, size = 24, normalized size = 1.

$$\frac{2\text{Ei}(2 \log(cx))}{c^2} - \frac{x^2}{\log(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[x/Log[c\*x]^2,x]

[Out] (2\*ExpIntegralEi[2\*Log[c\*x]])/c^2 - x^2/Log[c\*x]

**Maple [A]** time = 0.036, size = 26, normalized size = 1.1

$$-\frac{x^2}{\ln(cx)} - 2 \frac{\text{Ei}(1, -2 \ln(cx))}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/ln(c\*x)^2,x)

[Out] -x^2/ln(c\*x)-2/c^2\*Ei(1,-2\*ln(c\*x))

**Maxima [A]** time = 1.25397, size = 18, normalized size = 0.75

$$\frac{2\Gamma(-1, -2 \log(cx))}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(c\*x)^2,x, algorithm="maxima")

[Out] 2\*gamma(-1, -2\*log(c\*x))/c^2

**Fricas [A]** time = 0.887326, size = 84, normalized size = 3.5

$$\frac{c^2x^2 - 2 \log(cx) \log\_integral(c^2x^2)}{c^2 \log(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(c\*x)^2,x, algorithm="fricas")

[Out] -(c^2\*x^2 - 2\*log(c\*x)\*log\_integral(c^2\*x^2))/(c^2\*log(c\*x))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{x^2}{\log(cx)} + 2 \int \frac{x}{\log(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/ln(c\*x)\*\*2,x)

[Out] -x\*\*2/log(c\*x) + 2\*Integral(x/log(c\*x), x)

---

**Giac [A]** time = 1.14104, size = 32, normalized size = 1.33

$$-\frac{x^2}{\log(cx)} + \frac{2 \operatorname{Ei}(2 \log(cx))}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(c\*x)^2,x, algorithm="giac")

[Out] -x^2/log(c\*x) + 2\*Ei(2\*log(c\*x))/c^2

$$3.32 \quad \int \frac{1}{\log^2(cx)} dx$$

**Optimal.** Leaf size=18

$$\frac{\text{li}(cx)}{c} - \frac{x}{\log(cx)}$$

[Out]  $-(x/\text{Log}[c*x]) + \text{LogIntegral}[c*x]/c$

**Rubi [A]** time = 0.0050655, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2297, 2298}

$$\frac{\text{li}(cx)}{c} - \frac{x}{\log(cx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Log}[c*x]^{-2}, x]$

[Out]  $-(x/\text{Log}[c*x]) + \text{LogIntegral}[c*x]/c$

#### Rule 2297

$\text{Int}[(a + \text{Log}[c*x^n])^{p+1}/(b*n*(p+1)), x] - \text{Dist}[1/(b*n*(p+1)), \text{Int}[(a + b*\text{Log}[c*x^n])^{p+1}, x], x] /;$  FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && IntegerQ[2\*p]

#### Rule 2298

$\text{Int}[\text{Log}[c*x]^{-1}, x] /;$  FreeQ[c, x]

#### Rubi steps

$$\begin{aligned} \int \frac{1}{\log^2(cx)} dx &= -\frac{x}{\log(cx)} + \int \frac{1}{\log(cx)} dx \\ &= -\frac{x}{\log(cx)} + \frac{\text{li}(cx)}{c} \end{aligned}$$

**Mathematica [A]** time = 0.0040804, size = 18, normalized size = 1.

$$\frac{\text{li}(cx)}{c} - \frac{x}{\log(cx)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[\text{Log}[c*x]^{-2}, x]$

[Out]  $-(x/\text{Log}[c*x]) + \text{LogIntegral}[c*x]/c$

---

**Maple [A]** time = 0.036, size = 24, normalized size = 1.3

$$-\frac{x}{\ln(cx)} - \frac{\text{Ei}(1, -\ln(cx))}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/ln(c\*x)^2,x)

[Out] -x/ln(c\*x)-1/c\*Ei(1,-ln(c\*x))

---

**Maxima [A]** time = 1.23525, size = 16, normalized size = 0.89

$$\frac{\Gamma(-1, -\log(cx))}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(c\*x)^2,x, algorithm="maxima")

[Out] gamma(-1, -log(c\*x))/c

---

**Fricas [A]** time = 0.791876, size = 68, normalized size = 3.78

$$-\frac{cx - \log(cx) \log\_integral(cx)}{c \log(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(c\*x)^2,x, algorithm="fricas")

[Out] -(c\*x - log(c\*x)\*log\_integral(c\*x))/(c\*log(c\*x))

---

**Sympy [A]** time = 0.487873, size = 12, normalized size = 0.67

$$-\frac{x}{\log(cx)} + \frac{\text{li}(cx)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/ln(c\*x)\*\*2,x)

[Out] -x/log(c\*x) + li(c\*x)/c

---

**Giac [A]** time = 1.11997, size = 26, normalized size = 1.44

$$\frac{\text{Ei}(\log(cx))}{c} - \frac{x}{\log(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*x)^2,x, algorithm="giac")
```

```
[Out] Ei(log(c*x))/c - x/log(c*x)
```

$$3.33 \quad \int \frac{1}{x \log^2(cx)} dx$$

**Optimal.** Leaf size=8

$$-\frac{1}{\log(cx)}$$

[Out] -Log[c\*x]^(-1)

**Rubi [A]** time = 0.0118305, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2302, 30}

$$-\frac{1}{\log(cx)}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Log[c\*x]^2),x]

[Out] -Log[c\*x]^(-1)

Rule 2302

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] := Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{1}{x \log^2(cx)} dx &= \text{Subst} \left( \int \frac{1}{x^2} dx, x, \log(cx) \right) \\ &= -\frac{1}{\log(cx)} \end{aligned}$$

**Mathematica [A]** time = 0.0010445, size = 8, normalized size = 1.

$$-\frac{1}{\log(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Log[c\*x]^2),x]

[Out] -Log[c\*x]^(-1)

**Maple [A]** time = 0.033, size = 9, normalized size = 1.1

$$-(\ln(cx))^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/ln(c\*x)^2,x)

[Out] -1/ln(c\*x)

---

**Maxima [A]** time = 1.09388, size = 11, normalized size = 1.38

$$-\frac{1}{\log(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(c\*x)^2,x, algorithm="maxima")

[Out] -1/log(c\*x)

---

**Fricas [A]** time = 0.768227, size = 18, normalized size = 2.25

$$-\frac{1}{\log(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(c\*x)^2,x, algorithm="fricas")

[Out] -1/log(c\*x)

---

**Sympy [A]** time = 0.082636, size = 7, normalized size = 0.88

$$-\frac{1}{\log(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/ln(c\*x)\*\*2,x)

[Out] -1/log(c\*x)

---

**Giac [A]** time = 1.10535, size = 11, normalized size = 1.38

$$-\frac{1}{\log(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(1/x/log(c*x)^2,x, algorithm="giac")
```

```
[Out] -1/log(c*x)
```

$$3.34 \quad \int \frac{1}{x^2 \log^2(cx)} dx$$

**Optimal.** Leaf size=22

$$-c\text{Ei}(-\log(cx)) - \frac{1}{x \log(cx)}$$

[Out]  $-(c*\text{ExpIntegralEi}[-\text{Log}[c*x]]) - 1/(x*\text{Log}[c*x])$

**Rubi [A]** time = 0.034901, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {2306, 2309, 2178}

$$-c\text{Ei}(-\log(cx)) - \frac{1}{x \log(cx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^2*\text{Log}[c*x]^2), x]$

[Out]  $-(c*\text{ExpIntegralEi}[-\text{Log}[c*x]]) - 1/(x*\text{Log}[c*x])$

#### Rule 2306

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)]*(b_.)]^{(p_)}*((d_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*(a + b*\text{Log}[c*x^n])^{(p+1)}/(b*d*n*(p+1)), x] - \text{Dist}[(m+1)/(b*n*(p+1)), \text{Int}[(d*x)^m*(a + b*\text{Log}[c*x^n])^{(p+1)}, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x\} \&\& \text{NeQ}[m, -1] \&\& \text{LtQ}[p, -1]$

#### Rule 2309

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)]*(b_.)]^{(p_)}*(x_)^{(m_.)}, x\_Symbol] \rightarrow \text{Dist}[1/c^{(m+1)}, \text{Subst}[\text{Int}[E^{((m+1)*x)}*(a + b*x)^p, x], x, \text{Log}[c*x]], x] /; \text{FreeQ}\{a, b, c, p\}, x\} \&\& \text{IntegerQ}[m]$

#### Rule 2178

$\text{Int}[(F_.)^{(g_.)}*((e_.) + (f_.)*(x_))]/((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \text{Simp}[(F^{(g*(e - (c*f)/d)})*\text{ExpIntegralEi}[(f*g*(c + d*x)*\text{Log}[F])/d])/d, x] /; \text{FreeQ}\{F, c, d, e, f, g\}, x\} \&\& \text{!}\$UseGamma == \text{True}$

#### Rubi steps

$$\begin{aligned} \int \frac{1}{x^2 \log^2(cx)} dx &= -\frac{1}{x \log(cx)} - \int \frac{1}{x^2 \log(cx)} dx \\ &= -\frac{1}{x \log(cx)} - c \text{Subst} \left( \int \frac{e^{-x}}{x} dx, x, \log(cx) \right) \\ &= -c\text{Ei}(-\log(cx)) - \frac{1}{x \log(cx)} \end{aligned}$$

**Mathematica [A]** time = 0.0153269, size = 22, normalized size = 1.

$$-c\text{Ei}(-\log(cx)) - \frac{1}{x \log(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*Log[c\*x]^2),x]

[Out] -(c\*ExpIntegralEi[-Log[c\*x]]) - 1/(x\*Log[c\*x])

**Maple [A]** time = 0.036, size = 21, normalized size = 1.

$$-\frac{1}{x \ln(cx)} + c \operatorname{Ei}(1, \ln(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/ln(c\*x)^2,x)

[Out] -1/x/ln(c\*x)+c\*Ei(1,ln(c\*x))

**Maxima [A]** time = 1.24657, size = 12, normalized size = 0.55

$$-c \Gamma(-1, \log(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(c\*x)^2,x, algorithm="maxima")

[Out] -c\*gamma(-1, log(c\*x))

**Fricas [A]** time = 0.578465, size = 76, normalized size = 3.45

$$-\frac{cx \log(cx) \log\_integral\left(\frac{1}{cx}\right) + 1}{x \log(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(c\*x)^2,x, algorithm="fricas")

[Out] -(c\*x\*log(c\*x)\*log\_integral(1/(c\*x)) + 1)/(x\*log(c\*x))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{x^2 \log(cx)} dx - \frac{1}{x \log(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/ln(c\*x)\*\*2,x)

[Out] -Integral(1/(x\*\*2\*log(c\*x)), x) - 1/(x\*log(c\*x))

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \log(cx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(c\*x)^2,x, algorithm="giac")

[Out] integrate(1/(x^2\*log(c\*x)^2), x)

$$3.35 \quad \int \frac{1}{x^3 \log^2(cx)} dx$$

**Optimal.** Leaf size=24

$$-2c^2 \text{Ei}(-2 \log(cx)) - \frac{1}{x^2 \log(cx)}$$

[Out]  $-2*c^2*ExpIntegralEi[-2*Log[c*x]] - 1/(x^2*Log[c*x])$

**Rubi [A]** time = 0.0370607, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {2306, 2309, 2178}

$$-2c^2 \text{Ei}(-2 \log(cx)) - \frac{1}{x^2 \log(cx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^3*\text{Log}[c*x]^2), x]$

[Out]  $-2*c^2*ExpIntegralEi[-2*Log[c*x]] - 1/(x^2*Log[c*x])$

#### Rule 2306

$\text{Int}[(a + \text{Log}[c*x^n])*(b*x^m)^p, x\_Symbol] \rightarrow \text{Simp}[(d*x)^{m+1}*(a + b*\text{Log}[c*x^n])^{p+1}/(b*d*n*(p+1)), x] - \text{Dist}[(m+1)/(b*n*(p+1)), \text{Int}[(d*x)^m*(a + b*\text{Log}[c*x^n])^{p+1}, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ \text{LtQ}[p, -1]$

#### Rule 2309

$\text{Int}[(a + \text{Log}[c*x])*(b*x^m)^p, x\_Symbol] \rightarrow \text{Dist}[1/c^{m+1}, \text{Subst}[\text{Int}[E^{(m+1)*x}*(a + b*x)^p, x], x, \text{Log}[c*x]], x] /; \text{FreeQ}\{a, b, c, p\}, x \ \&\& \ \text{IntegerQ}[m]$

#### Rule 2178

$\text{Int}[(F)^{(g*(e + f*x))/(c + d*x)}, x\_Symbol] \rightarrow \text{Simp}[(F^{g*(e - (c*f)/d)} * \text{ExpIntegralEi}[(f*g*(c + d*x)*\text{Log}[F])/d])/d, x] /; \text{FreeQ}\{F, c, d, e, f, g\}, x \ \&\& \ \text{!UseGamma} == \text{True}$

#### Rubi steps

$$\begin{aligned} \int \frac{1}{x^3 \log^2(cx)} dx &= -\frac{1}{x^2 \log(cx)} - 2 \int \frac{1}{x^3 \log(cx)} dx \\ &= -\frac{1}{x^2 \log(cx)} - (2c^2) \text{Subst} \left( \int \frac{e^{-2x}}{x} dx, x, \log(cx) \right) \\ &= -2c^2 \text{Ei}(-2 \log(cx)) - \frac{1}{x^2 \log(cx)} \end{aligned}$$

**Mathematica [A]** time = 0.0145643, size = 24, normalized size = 1.

$$-2c^2 \text{Ei}(-2 \log(cx)) - \frac{1}{x^2 \log(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*Log[c\*x]^2),x]

[Out] -2\*c^2\*ExpIntegralEi[-2\*Log[c\*x]] - 1/(x^2\*Log[c\*x])

**Maple [A]** time = 0.035, size = 26, normalized size = 1.1

$$-\frac{1}{x^2 \ln(cx)} + 2c^2 \text{Ei}(1, 2 \ln(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/ln(c\*x)^2,x)

[Out] -1/x^2/ln(c\*x)+2\*c^2\*Ei(1,2\*ln(c\*x))

**Maxima [A]** time = 1.2607, size = 18, normalized size = 0.75

$$-2c^2\Gamma(-1, 2 \log(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(c\*x)^2,x, algorithm="maxima")

[Out] -2\*c^2\*gamma(-1, 2\*log(c\*x))

**Fricas [A]** time = 0.686952, size = 92, normalized size = 3.83

$$-\frac{2c^2x^2 \log(cx) \log\_integral\left(\frac{1}{c^2x^2}\right) + 1}{x^2 \log(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(c\*x)^2,x, algorithm="fricas")

[Out] -(2\*c^2\*x^2\*log(c\*x)\*log\_integral(1/(c^2\*x^2)) + 1)/(x^2\*log(c\*x))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-2 \int \frac{1}{x^3 \log(cx)} dx - \frac{1}{x^2 \log(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/ln(c\*x)\*\*2,x)

[Out] -2\*Integral(1/(x\*\*3\*log(c\*x)), x) - 1/(x\*\*2\*log(c\*x))

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \log(cx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(c*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^3*log(c*x)^2), x)
```

$$3.36 \quad \int \frac{x^3}{\log^3(cx)} dx$$

**Optimal.** Leaf size=37

$$\frac{8\text{Ei}(4 \log(cx))}{c^4} - \frac{x^4}{2 \log^2(cx)} - \frac{2x^4}{\log(cx)}$$

[Out] (8\*ExpIntegralEi[4\*Log[c\*x]])/c^4 - x^4/(2\*Log[c\*x]^2) - (2\*x^4)/Log[c\*x]

**Rubi [A]** time = 0.0520612, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {2306, 2309, 2178}

$$\frac{8\text{Ei}(4 \log(cx))}{c^4} - \frac{x^4}{2 \log^2(cx)} - \frac{2x^4}{\log(cx)}$$

Antiderivative was successfully verified.

[In] Int[x^3/Log[c\*x]^3,x]

[Out] (8\*ExpIntegralEi[4\*Log[c\*x]])/c^4 - x^4/(2\*Log[c\*x]^2) - (2\*x^4)/Log[c\*x]

#### Rule 2306

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[(((d*x)^(m + 1)*(a + b*Log[c*x^n])^(p + 1))/(b*d*n*(p + 1)), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

#### Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol]
:> Dist[1/c^(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

#### Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; FreeQ
[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

#### Rubi steps

$$\begin{aligned} \int \frac{x^3}{\log^3(cx)} dx &= -\frac{x^4}{2 \log^2(cx)} + 2 \int \frac{x^3}{\log^2(cx)} dx \\ &= -\frac{x^4}{2 \log^2(cx)} - \frac{2x^4}{\log(cx)} + 8 \int \frac{x^3}{\log(cx)} dx \\ &= -\frac{x^4}{2 \log^2(cx)} - \frac{2x^4}{\log(cx)} + \frac{8 \text{Subst}\left(\int \frac{e^{4x}}{x} dx, x, \log(cx)\right)}{c^4} \\ &= \frac{8\text{Ei}(4 \log(cx))}{c^4} - \frac{x^4}{2 \log^2(cx)} - \frac{2x^4}{\log(cx)} \end{aligned}$$



**Mathematica [A]** time = 0.0159373, size = 37, normalized size = 1.

$$\frac{8\text{Ei}(4\log(cx))}{c^4} - \frac{x^4}{2\log^2(cx)} - \frac{2x^4}{\log(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Log[c\*x]^3,x]

[Out] (8\*ExpIntegralEi[4\*Log[c\*x]])/c^4 - x^4/(2\*Log[c\*x]^2) - (2\*x^4)/Log[c\*x]

**Maple [A]** time = 0.046, size = 37, normalized size = 1.

$$-\frac{x^4}{2(\ln(cx))^2} - 2\frac{x^4}{\ln(cx)} - 8\frac{\text{Ei}(1, -4\ln(cx))}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/ln(c\*x)^3,x)

[Out] -1/2\*x^4/ln(c\*x)^2-2\*x^4/ln(c\*x)-8/c^4\*Ei(1,-4\*ln(c\*x))

**Maxima [A]** time = 1.25461, size = 18, normalized size = 0.49

$$\frac{16\Gamma(-2, -4\log(cx))}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(c\*x)^3,x, algorithm="maxima")

[Out] -16\*gamma(-2, -4\*log(c\*x))/c^4

**Fricas [A]** time = 0.857464, size = 124, normalized size = 3.35

$$\frac{4c^4x^4\log(cx) + c^4x^4 - 16\log(cx)^2\log\_integral(c^4x^4)}{2c^4\log(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(c\*x)^3,x, algorithm="fricas")

[Out] -1/2\*(4\*c^4\*x^4\*log(c\*x) + c^4\*x^4 - 16\*log(c\*x)^2\*log\_integral(c^4\*x^4))/(c^4\*log(c\*x)^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{-4x^4\log(cx) - x^4}{2\log(cx)^2} + 8\int\frac{x^3}{\log(cx)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/ln(c\*x)\*\*3,x)

[Out] (-4\*x\*\*4\*log(c\*x) - x\*\*4)/(2\*log(c\*x)\*\*2) + 8\*Integral(x\*\*3/log(c\*x), x)

**Giac [A]** time = 1.1246, size = 47, normalized size = 1.27

$$-\frac{2x^4}{\log(cx)} - \frac{x^4}{2\log(cx)^2} + \frac{8\operatorname{Ei}(4\log(cx))}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(c\*x)^3,x, algorithm="giac")

[Out] -2\*x^4/log(c\*x) - 1/2\*x^4/log(c\*x)^2 + 8\*Ei(4\*log(c\*x))/c^4

$$3.37 \quad \int \frac{x^2}{\log^3(cx)} dx$$

**Optimal.** Leaf size=41

$$\frac{9\text{Ei}(3\log(cx))}{2c^3} - \frac{x^3}{2\log^2(cx)} - \frac{3x^3}{2\log(cx)}$$

[Out] (9\*ExpIntegralEi[3\*Log[c\*x]])/(2\*c^3) - x^3/(2\*Log[c\*x]^2) - (3\*x^3)/(2\*Log[c\*x])

**Rubi [A]** time = 0.05251, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {2306, 2309, 2178}

$$\frac{9\text{Ei}(3\log(cx))}{2c^3} - \frac{x^3}{2\log^2(cx)} - \frac{3x^3}{2\log(cx)}$$

Antiderivative was successfully verified.

[In] Int[x^2/Log[c\*x]^3, x]

[Out] (9\*ExpIntegralEi[3\*Log[c\*x]])/(2\*c^3) - x^3/(2\*Log[c\*x]^2) - (3\*x^3)/(2\*Log[c\*x])

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2309

Int[((a\_.) + Log[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/c^(m + 1), Subst[Int[E^((m + 1)\*x)\*(a + b\*x)^p, x], x, Log[c\*x]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{x^2}{\log^3(cx)} dx &= -\frac{x^3}{2\log^2(cx)} + \frac{3}{2} \int \frac{x^2}{\log^2(cx)} dx \\ &= -\frac{x^3}{2\log^2(cx)} - \frac{3x^3}{2\log(cx)} + \frac{9}{2} \int \frac{x^2}{\log(cx)} dx \\ &= -\frac{x^3}{2\log^2(cx)} - \frac{3x^3}{2\log(cx)} + \frac{9 \text{Subst}\left(\int \frac{e^{3x}}{x} dx, x, \log(cx)\right)}{2c^3} \\ &= \frac{9\text{Ei}(3\log(cx))}{2c^3} - \frac{x^3}{2\log^2(cx)} - \frac{3x^3}{2\log(cx)} \end{aligned}$$

**Mathematica [A]** time = 0.0148196, size = 41, normalized size = 1.

$$\frac{9\text{Ei}(3\log(cx))}{2c^3} - \frac{x^3}{2\log^2(cx)} - \frac{3x^3}{2\log(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Log[c\*x]^3,x]

[Out] (9\*ExpIntegralEi[3\*Log[c\*x]])/(2\*c^3) - x^3/(2\*Log[c\*x]^2) - (3\*x^3)/(2\*Log[c\*x])

**Maple [A]** time = 0.034, size = 37, normalized size = 0.9

$$-\frac{x^3}{2(\ln(cx))^2} - \frac{3x^3}{2\ln(cx)} - \frac{9\text{Ei}(1, -3\ln(cx))}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/ln(c\*x)^3,x)

[Out] -1/2\*x^3/ln(c\*x)^2-3/2\*x^3/ln(c\*x)-9/2/c^3\*Ei(1,-3\*ln(c\*x))

**Maxima [A]** time = 1.12459, size = 18, normalized size = 0.44

$$-\frac{9\Gamma(-2, -3\log(cx))}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/log(c\*x)^3,x, algorithm="maxima")

[Out] -9\*gamma(-2, -3\*log(c\*x))/c^3

**Fricas [A]** time = 0.755635, size = 123, normalized size = 3.

$$-\frac{3c^3x^3\log(cx) + c^3x^3 - 9\log(cx)^2\log\_integral(c^3x^3)}{2c^3\log(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/log(c\*x)^3,x, algorithm="fricas")

[Out] -1/2\*(3\*c^3\*x^3\*log(c\*x) + c^3\*x^3 - 9\*log(c\*x)^2\*log\_integral(c^3\*x^3))/(c^3\*log(c\*x)^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{-3x^3\log(cx) - x^3}{2\log(cx)^2} + \frac{9\int\frac{x^2}{\log(cx)}dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/ln(c\*x)\*\*3,x)

[Out]  $(-3*x**3*\log(c*x) - x**3)/(2*\log(c*x)**2) + 9*\text{Integral}(x**2/\log(c*x), x)/2$

**Giac [A]** time = 1.12121, size = 47, normalized size = 1.15

$$-\frac{3x^3}{2\log(cx)} - \frac{x^3}{2\log(cx)^2} + \frac{9\text{Ei}(3\log(cx))}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/log(c\*x)^3,x, algorithm="giac")

[Out]  $-3/2*x^3/\log(c*x) - 1/2*x^3/\log(c*x)^2 + 9/2*\text{Ei}(3*\log(c*x))/c^3$

$$3.38 \quad \int \frac{x}{\log^3(cx)} dx$$

**Optimal.** Leaf size=37

$$\frac{2\text{Ei}(2 \log(cx))}{c^2} - \frac{x^2}{2 \log^2(cx)} - \frac{x^2}{\log(cx)}$$

[Out] (2\*ExpIntegralEi[2\*Log[c\*x]])/c^2 - x^2/(2\*Log[c\*x]^2) - x^2/Log[c\*x]

**Rubi [A]** time = 0.0314825, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {2306, 2309, 2178}

$$\frac{2\text{Ei}(2 \log(cx))}{c^2} - \frac{x^2}{2 \log^2(cx)} - \frac{x^2}{\log(cx)}$$

Antiderivative was successfully verified.

[In] Int[x/Log[c\*x]^3,x]

[Out] (2\*ExpIntegralEi[2\*Log[c\*x]])/c^2 - x^2/(2\*Log[c\*x]^2) - x^2/Log[c\*x]

#### Rule 2306

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol]
  := Simp[(((d*x)^(m + 1)*(a + b*Log[c*x^n])^(p + 1))/(b*d*n*(p + 1)), x] -
  Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
  /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

#### Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol]
  := Dist[1/c^(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x]
  /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]
```

#### Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol]
  := Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x]
  /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

#### Rubi steps

$$\begin{aligned} \int \frac{x}{\log^3(cx)} dx &= -\frac{x^2}{2 \log^2(cx)} + \int \frac{x}{\log^2(cx)} dx \\ &= -\frac{x^2}{2 \log^2(cx)} - \frac{x^2}{\log(cx)} + 2 \int \frac{x}{\log(cx)} dx \\ &= -\frac{x^2}{2 \log^2(cx)} - \frac{x^2}{\log(cx)} + \frac{2 \text{Subst}\left(\int \frac{e^{2x}}{x} dx, x, \log(cx)\right)}{c^2} \\ &= \frac{2\text{Ei}(2 \log(cx))}{c^2} - \frac{x^2}{2 \log^2(cx)} - \frac{x^2}{\log(cx)} \end{aligned}$$

**Mathematica [A]** time = 0.0049546, size = 37, normalized size = 1.

$$\frac{2\text{Ei}(2\log(cx))}{c^2} - \frac{x^2}{2\log^2(cx)} - \frac{x^2}{\log(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[x/Log[c\*x]^3,x]

[Out] (2\*ExpIntegralEi[2\*Log[c\*x]])/c^2 - x^2/(2\*Log[c\*x]^2) - x^2/Log[c\*x]

**Maple [A]** time = 0.045, size = 37, normalized size = 1.

$$-\frac{x^2}{2(\ln(cx))^2} - \frac{x^2}{\ln(cx)} - 2\frac{\text{Ei}(1, -2\ln(cx))}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/ln(c\*x)^3,x)

[Out] -1/2\*x^2/ln(c\*x)^2-x^2/ln(c\*x)-2/c^2\*Ei(1,-2\*ln(c\*x))

**Maxima [A]** time = 1.15327, size = 18, normalized size = 0.49

$$-\frac{4\Gamma(-2, -2\log(cx))}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(c\*x)^3,x, algorithm="maxima")

[Out] -4\*gamma(-2, -2\*log(c\*x))/c^2

**Fricas [A]** time = 0.852558, size = 123, normalized size = 3.32

$$-\frac{2c^2x^2\log(cx) + c^2x^2 - 4\log(cx)^2\log\_integral(c^2x^2)}{2c^2\log(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(c\*x)^3,x, algorithm="fricas")

[Out] -1/2\*(2\*c^2\*x^2\*log(c\*x) + c^2\*x^2 - 4\*log(c\*x)^2\*log\_integral(c^2\*x^2))/(c^2\*log(c\*x)^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{-2x^2\log(cx) - x^2}{2\log(cx)^2} + 2\int\frac{x}{\log(cx)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/ln(c\*x)\*\*3,x)

[Out]  $(-2*x**2*\log(c*x) - x**2)/(2*\log(c*x)**2) + 2*\text{Integral}(x/\log(c*x), x)$

**Giac [A]** time = 1.13404, size = 47, normalized size = 1.27

$$-\frac{x^2}{\log(cx)} - \frac{x^2}{2 \log(cx)^2} + \frac{2 \text{Ei}(2 \log(cx))}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(c\*x)^3,x, algorithm="giac")

[Out]  $-x^2/\log(c*x) - 1/2*x^2/\log(c*x)^2 + 2*\text{Ei}(2*\log(c*x))/c^2$



$$3.39 \quad \int \frac{1}{\log^3(cx)} dx$$

**Optimal.** Leaf size=34

$$\frac{\text{li}(cx)}{2c} - \frac{x}{2\log^2(cx)} - \frac{x}{2\log(cx)}$$

[Out]  $-x/(2*\text{Log}[c*x]^2) - x/(2*\text{Log}[c*x]) + \text{LogIntegral}[c*x]/(2*c)$

**Rubi [A]** time = 0.0087665, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2297, 2298}

$$\frac{\text{li}(cx)}{2c} - \frac{x}{2\log^2(cx)} - \frac{x}{2\log(cx)}$$

Antiderivative was successfully verified.

[In] Int[Log[c\*x]^(-3), x]

[Out]  $-x/(2*\text{Log}[c*x]^2) - x/(2*\text{Log}[c*x]) + \text{LogIntegral}[c*x]/(2*c)$

Rule 2297

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_), x\_Symbol] :> Simp[(x\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*n\*(p + 1)), x] - Dist[1/(b\*n\*(p + 1)), Int[(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && IntegerQ[2\*p]

Rule 2298

Int[Log[(c\_.)\*(x\_)]^(-1), x\_Symbol] :> Simp[LogIntegral[c\*x]/c, x] /; FreeQ[c, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\log^3(cx)} dx &= -\frac{x}{2\log^2(cx)} + \frac{1}{2} \int \frac{1}{\log^2(cx)} dx \\ &= -\frac{x}{2\log^2(cx)} - \frac{x}{2\log(cx)} + \frac{1}{2} \int \frac{1}{\log(cx)} dx \\ &= -\frac{x}{2\log^2(cx)} - \frac{x}{2\log(cx)} + \frac{\text{li}(cx)}{2c} \end{aligned}$$

**Mathematica [A]** time = 0.0051097, size = 34, normalized size = 1.

$$\frac{\text{li}(cx)}{2c} - \frac{x}{2\log^2(cx)} - \frac{x}{2\log(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[Log[c\*x]^(-3), x]

[Out]  $-x/(2*\text{Log}[c*x]^2) - x/(2*\text{Log}[c*x]) + \text{LogIntegral}[c*x]/(2*c)$

**Maple [A]** time = 0.035, size = 33, normalized size = 1.

$$-\frac{x}{2(\ln(cx))^2} - \frac{x}{2\ln(cx)} - \frac{\text{Ei}(1, -\ln(cx))}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/ln(c*x)^3,x)`

[Out]  $-1/2*x/\ln(c*x)^2 - 1/2*x/\ln(c*x) - 1/2/c*\text{Ei}(1, -\ln(c*x))$

**Maxima [A]** time = 1.1648, size = 18, normalized size = 0.53

$$-\frac{\Gamma(-2, -\log(cx))}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/log(c*x)^3,x, algorithm="maxima")`

[Out]  $-\text{gamma}(-2, -\log(c*x))/c$

**Fricas [A]** time = 0.663737, size = 99, normalized size = 2.91

$$\frac{cx \log(cx) - \log(cx)^2 \log\_integral(cx) + cx}{2c \log(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/log(c*x)^3,x, algorithm="fricas")`

[Out]  $-1/2*(c*x*\log(c*x) - \log(c*x)^2*\log\_integral(c*x) + c*x)/(c*\log(c*x)^2)$

**Sympy [A]** time = 0.519021, size = 26, normalized size = 0.76

$$\frac{-x \log(cx) - x}{2 \log(cx)^2} + \frac{\text{li}(cx)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/ln(c*x)**3,x)`

[Out]  $(-x*\log(c*x) - x)/(2*\log(c*x)**2) + \text{li}(c*x)/(2*c)$

**Giac [A]** time = 1.11764, size = 39, normalized size = 1.15

$$\frac{\text{Ei}(\log(cx))}{2c} - \frac{x}{2\log(cx)} - \frac{x}{2\log(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(c*x)^3,x, algorithm="giac")
```

```
[Out] 1/2*Ei(log(c*x))/c - 1/2*x/log(c*x) - 1/2*x/log(c*x)^2
```

$$3.40 \quad \int \frac{1}{x \log^3(cx)} dx$$

**Optimal.** Leaf size=10

$$-\frac{1}{2 \log^2(cx)}$$

[Out] -1/(2\*Log[c\*x]^2)

**Rubi [A]** time = 0.0121339, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2302, 30}

$$-\frac{1}{2 \log^2(cx)}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Log[c\*x]^3),x]

[Out] -1/(2\*Log[c\*x]^2)

#### Rule 2302

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] :> Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{1}{x \log^3(cx)} dx &= \text{Subst} \left( \int \frac{1}{x^3} dx, x, \log(cx) \right) \\ &= -\frac{1}{2 \log^2(cx)} \end{aligned}$$

**Mathematica [A]** time = 0.0010784, size = 10, normalized size = 1.

$$-\frac{1}{2 \log^2(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Log[c\*x]^3),x]

[Out] -1/(2\*Log[c\*x]^2)

**Maple [A]** time = 0.033, size = 9, normalized size = 0.9

$$-\frac{1}{2 (\ln (cx))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/ln(c\*x)^3,x)

[Out] -1/2/ln(c\*x)^2

---

**Maxima [A]** time = 1.10878, size = 11, normalized size = 1.1

$$-\frac{1}{2 \log (cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(c\*x)^3,x, algorithm="maxima")

[Out] -1/2/log(c\*x)^2

---

**Fricas [A]** time = 0.860774, size = 23, normalized size = 2.3

$$-\frac{1}{2 \log (cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(c\*x)^3,x, algorithm="fricas")

[Out] -1/2/log(c\*x)^2

---

**Sympy [A]** time = 0.090014, size = 10, normalized size = 1.

$$-\frac{1}{2 \log (cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/ln(c\*x)\*\*3,x)

[Out] -1/(2\*log(c\*x)\*\*2)

---

**Giac [A]** time = 1.11042, size = 11, normalized size = 1.1

$$-\frac{1}{2 \log (cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(c*x)^3,x, algorithm="giac")
```

```
[Out] -1/2/log(c*x)^2
```

$$3.41 \quad \int \frac{1}{x^2 \log^3(cx)} dx$$

**Optimal.** Leaf size=39

$$\frac{1}{2}c\text{Ei}(-\log(cx)) - \frac{1}{2x \log^2(cx)} + \frac{1}{2x \log(cx)}$$

[Out] (c\*ExpIntegralEi[-Log[c\*x]])/2 - 1/(2\*x\*Log[c\*x]^2) + 1/(2\*x\*Log[c\*x])

**Rubi [A]** time = 0.0495498, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {2306, 2309, 2178}

$$\frac{1}{2}c\text{Ei}(-\log(cx)) - \frac{1}{2x \log^2(cx)} + \frac{1}{2x \log(cx)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*Log[c\*x]^3),x]

[Out] (c\*ExpIntegralEi[-Log[c\*x]])/2 - 1/(2\*x\*Log[c\*x]^2) + 1/(2\*x\*Log[c\*x])

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2309

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/c^(m + 1), Subst[Int[E^((m + 1)\*x)\*(a + b\*x)^p, x], x, Log[c\*x]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{1}{x^2 \log^3(cx)} dx &= -\frac{1}{2x \log^2(cx)} - \frac{1}{2} \int \frac{1}{x^2 \log^2(cx)} dx \\ &= -\frac{1}{2x \log^2(cx)} + \frac{1}{2x \log(cx)} + \frac{1}{2} \int \frac{1}{x^2 \log(cx)} dx \\ &= -\frac{1}{2x \log^2(cx)} + \frac{1}{2x \log(cx)} + \frac{1}{2}c \text{Subst}\left(\int \frac{e^{-x}}{x} dx, x, \log(cx)\right) \\ &= \frac{1}{2}c\text{Ei}(-\log(cx)) - \frac{1}{2x \log^2(cx)} + \frac{1}{2x \log(cx)} \end{aligned}$$

**Mathematica [A]** time = 0.0145465, size = 39, normalized size = 1.

$$\frac{1}{2}c\text{Ei}(-\log(cx)) - \frac{1}{2x \log^2(cx)} + \frac{1}{2x \log(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*Log[c\*x]^3),x]

[Out] (c\*ExpIntegralEi[-Log[c\*x]])/2 - 1/(2\*x\*Log[c\*x]^2) + 1/(2\*x\*Log[c\*x])

**Maple [A]** time = 0.034, size = 33, normalized size = 0.9

$$-\frac{1}{2x(\ln(cx))^2} + \frac{1}{2x \ln(cx)} - \frac{c\text{Ei}(1, \ln(cx))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/ln(c\*x)^3,x)

[Out] -1/2/x/ln(c\*x)^2+1/2/x/ln(c\*x)-1/2\*c\*Ei(1,ln(c\*x))

**Maxima [A]** time = 1.16639, size = 12, normalized size = 0.31

$$-c\Gamma(-2, \log(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(c\*x)^3,x, algorithm="maxima")

[Out] -c\*gamma(-2, log(c\*x))

**Fricas [A]** time = 0.846329, size = 100, normalized size = 2.56

$$\frac{cx \log(cx)^2 \log\_integral\left(\frac{1}{cx}\right) + \log(cx) - 1}{2x \log(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(c\*x)^3,x, algorithm="fricas")

[Out] 1/2\*(c\*x\*log(c\*x)^2\*log\_integral(1/(c\*x)) + log(c\*x) - 1)/(x\*log(c\*x)^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{x^2 \log(cx)} dx}{2} + \frac{\log(cx) - 1}{2x \log(cx)^2}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/ln(c\*x)\*\*3,x)

[Out] Integral(1/(x\*\*2\*log(c\*x)), x)/2 + (log(c\*x) - 1)/(2\*x\*log(c\*x)\*\*2)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \log(cx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(c\*x)^3,x, algorithm="giac")

[Out] integrate(1/(x^2\*log(c\*x)^3), x)

$$3.42 \quad \int \frac{1}{x^3 \log^3(cx)} dx$$

**Optimal.** Leaf size=36

$$2c^2 \text{Ei}(-2 \log(cx)) - \frac{1}{2x^2 \log^2(cx)} + \frac{1}{x^2 \log(cx)}$$

[Out]  $2*c^2*ExpIntegralEi[-2*Log[c*x]] - 1/(2*x^2*Log[c*x]^2) + 1/(x^2*Log[c*x])$

**Rubi [A]** time = 0.049051, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {2306, 2309, 2178}

$$2c^2 \text{Ei}(-2 \log(cx)) - \frac{1}{2x^2 \log^2(cx)} + \frac{1}{x^2 \log(cx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^3*\text{Log}[c*x]^3), x]$

[Out]  $2*c^2*ExpIntegralEi[-2*Log[c*x]] - 1/(2*x^2*Log[c*x]^2) + 1/(x^2*Log[c*x])$

#### Rule 2306

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]^{(p_.)}*((d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*(a + b*\text{Log}[c*x^n])^{(p+1)}/(b*d*n*(p+1)), x] - \text{Dist}[(m+1)/(b*n*(p+1)), \text{Int}[(d*x)^m*(a + b*\text{Log}[c*x^n])^{(p+1)}, x], x] /;$   $\text{FreeQ}\{a, b, c, d, m, n\}, x\} \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ \text{LtQ}[p, -1]$

#### Rule 2309

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_.)]*(b_.)]^{(p_.)}*(x_.)^{(m_.)}, x\_Symbol] \rightarrow \text{Dist}[1/c^{(m+1)}, \text{Subst}[\text{Int}[E^{((m+1)*x)}*(a + b*x)^p, x], x, \text{Log}[c*x]], x] /;$   $\text{FreeQ}\{a, b, c, p\}, x\} \ \&\& \ \text{IntegerQ}[m]$

#### Rule 2178

$\text{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_.)))/((c_.) + (d_.)*(x_.))}, x\_Symbol] \rightarrow \text{Simp}[(F^{(g*(e - (c*f)/d)})*ExpIntegralEi[(f*g*(c + d*x)*\text{Log}[F])/d])/d, x] /;$   $\text{FreeQ}\{F, c, d, e, f, g\}, x\} \ \&\& \ \text{!}\$UseGamma == \text{True}$

#### Rubi steps

$$\begin{aligned} \int \frac{1}{x^3 \log^3(cx)} dx &= -\frac{1}{2x^2 \log^2(cx)} - \int \frac{1}{x^3 \log^2(cx)} dx \\ &= -\frac{1}{2x^2 \log^2(cx)} + \frac{1}{x^2 \log(cx)} + 2 \int \frac{1}{x^3 \log(cx)} dx \\ &= -\frac{1}{2x^2 \log^2(cx)} + \frac{1}{x^2 \log(cx)} + (2c^2) \text{Subst} \left( \int \frac{e^{-2x}}{x} dx, x, \log(cx) \right) \\ &= 2c^2 \text{Ei}(-2 \log(cx)) - \frac{1}{2x^2 \log^2(cx)} + \frac{1}{x^2 \log(cx)} \end{aligned}$$

**Mathematica [A]** time = 0.0159106, size = 36, normalized size = 1.

$$2c^2\text{Ei}(-2\log(cx)) - \frac{1}{2x^2\log^2(cx)} + \frac{1}{x^2\log(cx)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*Log[c\*x]^3),x]

[Out] 2\*c^2\*ExpIntegralEi[-2\*Log[c\*x]] - 1/(2\*x^2\*Log[c\*x]^2) + 1/(x^2\*Log[c\*x])

**Maple [A]** time = 0.033, size = 36, normalized size = 1.

$$-\frac{1}{2x^2(\ln(cx))^2} + \frac{1}{x^2\ln(cx)} - 2c^2\text{Ei}(1, 2\ln(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/ln(c\*x)^3,x)

[Out] -1/2/x^2/ln(c\*x)^2+1/x^2/ln(c\*x)-2\*c^2\*Ei(1,2\*ln(c\*x))

**Maxima [A]** time = 1.24094, size = 18, normalized size = 0.5

$$-4c^2\Gamma(-2, 2\log(cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(c\*x)^3,x, algorithm="maxima")

[Out] -4\*c^2\*gamma(-2, 2\*log(c\*x))

**Fricas [A]** time = 0.717287, size = 119, normalized size = 3.31

$$\frac{4c^2x^2\log(cx)^2\log\_integral\left(\frac{1}{c^2x^2}\right) + 2\log(cx) - 1}{2x^2\log(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(c\*x)^3,x, algorithm="fricas")

[Out] 1/2\*(4\*c^2\*x^2\*log(c\*x)^2\*log\_integral(1/(c^2\*x^2)) + 2\*log(c\*x) - 1)/(x^2\*log(c\*x)^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$2 \int \frac{1}{x^3 \log(cx)} dx + \frac{2 \log(cx) - 1}{2x^2 \log(cx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/ln(c\*x)\*\*3,x)

[Out] 2\*Integral(1/(x\*\*3\*log(c\*x)), x) + (2\*log(c\*x) - 1)/(2\*x\*\*2\*log(c\*x)\*\*2)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \log(cx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(c\*x)^3,x, algorithm="giac")

[Out] integrate(1/(x^3\*log(c\*x)^3), x)

### 3.43 $\int x^3 (a + b \log(cx^n)) dx$

**Optimal.** Leaf size=27

$$\frac{1}{4}x^4(a + b \log(cx^n)) - \frac{1}{16}bnx^4$$

[Out]  $-(b*n*x^4)/16 + (x^4*(a + b*Log[c*x^n]))/4$

**Rubi [A]** time = 0.0125367, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {2304}

$$\frac{1}{4}x^4(a + b \log(cx^n)) - \frac{1}{16}bnx^4$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*Log[c\*x^n]),x]

[Out]  $-(b*n*x^4)/16 + (x^4*(a + b*Log[c*x^n]))/4$

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :=  
Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\int x^3 (a + b \log(cx^n)) dx = -\frac{1}{16}bnx^4 + \frac{1}{4}x^4(a + b \log(cx^n))$$

**Mathematica [A]** time = 0.0020941, size = 32, normalized size = 1.19

$$\frac{ax^4}{4} + \frac{1}{4}bx^4 \log(cx^n) - \frac{1}{16}bnx^4$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*Log[c\*x^n]),x]

[Out]  $(a*x^4)/4 - (b*n*x^4)/16 + (b*x^4*Log[c*x^n])/4$

**Maple [C]** time = 0.21, size = 112, normalized size = 4.2

$$\frac{bx^4 \ln(x^n)}{4} + \frac{x^4 (2ib\pi \operatorname{csgn}(ix^n) (\operatorname{csgn}(icx^n))^2 - 2ib\pi \operatorname{csgn}(ix^n) \operatorname{csgn}(icx^n) \operatorname{csgn}(ic) - 2ib\pi (\operatorname{csgn}(icx^n))^3 + 2ib\pi)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*ln(c\*x^n)),x)

[Out]  $\frac{1}{4}bx^4\ln(x^n)+\frac{1}{16}x^4(2Ib\pi\operatorname{csgn}(Ix^n)\operatorname{csgn}(Icx^n)^2-2Ib\pi\operatorname{csgn}(Ix^n)\operatorname{csgn}(Icx^n)\operatorname{csgn}(Ic)-2Ib\pi\operatorname{csgn}(Icx^n)^3+2Ib\pi\operatorname{csgn}(Icx^n)^2\operatorname{csgn}(Ic)+4b\ln(c)-bn+4a)$

**Maxima [A]** time = 1.10864, size = 35, normalized size = 1.3

$$-\frac{1}{16}bnx^4 + \frac{1}{4}bx^4\log(cx^n) + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*log(c*x^n)),x, algorithm="maxima")`

[Out]  $-1/16*b*n*x^4 + 1/4*b*x^4*\log(c*x^n) + 1/4*a*x^4$

**Fricas [A]** time = 0.787481, size = 84, normalized size = 3.11

$$\frac{1}{4}bnx^4\log(x) + \frac{1}{4}bx^4\log(c) - \frac{1}{16}(bn - 4a)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*log(c*x^n)),x, algorithm="fricas")`

[Out]  $1/4*b*n*x^4*\log(x) + 1/4*b*x^4*\log(c) - 1/16*(b*n - 4*a)*x^4$

**Sympy [A]** time = 1.39695, size = 36, normalized size = 1.33

$$\frac{ax^4}{4} + \frac{bnx^4\log(x)}{4} - \frac{bnx^4}{16} + \frac{bx^4\log(c)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(a+b*ln(c*x**n)),x)`

[Out]  $a*x**4/4 + b*n*x**4*\log(x)/4 - b*n*x**4/16 + b*x**4*\log(c)/4$

**Giac [A]** time = 1.11166, size = 42, normalized size = 1.56

$$\frac{1}{4}bnx^4\log(x) - \frac{1}{16}bnx^4 + \frac{1}{4}bx^4\log(c) + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*log(c*x^n)),x, algorithm="giac")`

[Out]  $1/4*b*n*x^4*\log(x) - 1/16*b*n*x^4 + 1/4*b*x^4*\log(c) + 1/4*a*x^4$

### 3.44 $\int x^2 (a + b \log(cx^n)) dx$

**Optimal.** Leaf size=27

$$\frac{1}{3}x^3 (a + b \log(cx^n)) - \frac{1}{9}bnx^3$$

[Out]  $-(b*n*x^3)/9 + (x^3*(a + b*Log[c*x^n]))/3$

**Rubi [A]** time = 0.0121256, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {2304}

$$\frac{1}{3}x^3 (a + b \log(cx^n)) - \frac{1}{9}bnx^3$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*Log[c\*x^n]),x]

[Out]  $-(b*n*x^3)/9 + (x^3*(a + b*Log[c*x^n]))/3$

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :=  
Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\int x^2 (a + b \log(cx^n)) dx = -\frac{1}{9}bnx^3 + \frac{1}{3}x^3 (a + b \log(cx^n))$$

**Mathematica [A]** time = 0.001165, size = 32, normalized size = 1.19

$$\frac{ax^3}{3} + \frac{1}{3}bx^3 \log(cx^n) - \frac{1}{9}bnx^3$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*Log[c\*x^n]),x]

[Out]  $(a*x^3)/3 - (b*n*x^3)/9 + (b*x^3*Log[c*x^n])/3$

**Maple [C]** time = 0.167, size = 112, normalized size = 4.2

$$\frac{bx^3 \ln(x^n)}{3} + \frac{x^3 (3 ib\pi \operatorname{csgn}(ix^n) (\operatorname{csgn}(icx^n))^2 - 3 ib\pi \operatorname{csgn}(ix^n) \operatorname{csgn}(icx^n) \operatorname{csgn}(ic) - 3 ib\pi (\operatorname{csgn}(icx^n))^3 + 3 ib\pi)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*ln(c\*x^n)),x)

[Out]  $\frac{1}{3}bx^3 \ln(x^n) + \frac{1}{18}x^3 (3Ib\pi \operatorname{csgn}(Ix^n) \operatorname{csgn}(Icx^n)^2 - 3Ib\pi \operatorname{csgn}(Ix^n) \operatorname{csgn}(Icx^n) \operatorname{csgn}(Ic) - 3Ib\pi \operatorname{csgn}(Icx^n)^3 + 3Ib\pi \operatorname{csgn}(Icx^n)^2 \operatorname{csgn}(Ic) + 6b \ln(c) - 2bn + 6a)$

**Maxima [A]** time = 1.02609, size = 35, normalized size = 1.3

$$-\frac{1}{9}bnx^3 + \frac{1}{3}bx^3 \log(cx^n) + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*log(c*x^n)),x, algorithm="maxima")`

[Out]  $-1/9*b*n*x^3 + 1/3*b*x^3*\log(c*x^n) + 1/3*a*x^3$

**Fricas [A]** time = 0.862332, size = 82, normalized size = 3.04

$$\frac{1}{3}bnx^3 \log(x) + \frac{1}{3}bx^3 \log(c) - \frac{1}{9}(bn - 3a)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*log(c*x^n)),x, algorithm="fricas")`

[Out]  $1/3*b*n*x^3*\log(x) + 1/3*b*x^3*\log(c) - 1/9*(b*n - 3*a)*x^3$

**Sympy [A]** time = 0.818564, size = 36, normalized size = 1.33

$$\frac{ax^3}{3} + \frac{bnx^3 \log(x)}{3} - \frac{bnx^3}{9} + \frac{bx^3 \log(c)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*ln(c*x**n)),x)`

[Out]  $a*x**3/3 + b*n*x**3*\log(x)/3 - b*n*x**3/9 + b*x**3*\log(c)/3$

**Giac [A]** time = 1.09674, size = 42, normalized size = 1.56

$$\frac{1}{3}bnx^3 \log(x) - \frac{1}{9}bnx^3 + \frac{1}{3}bx^3 \log(c) + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*log(c*x^n)),x, algorithm="giac")`

[Out]  $1/3*b*n*x^3*\log(x) - 1/9*b*n*x^3 + 1/3*b*x^3*\log(c) + 1/3*a*x^3$



### 3.45 $\int x (a + b \log (cx^n)) dx$

**Optimal.** Leaf size=27

$$\frac{1}{2}x^2 (a + b \log (cx^n)) - \frac{1}{4}bnx^2$$

[Out]  $-(b*n*x^2)/4 + (x^2*(a + b*Log[c*x^n]))/2$

**Rubi [A]** time = 0.0072916, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {2304}

$$\frac{1}{2}x^2 (a + b \log (cx^n)) - \frac{1}{4}bnx^2$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*Log[c\*x^n]),x]

[Out]  $-(b*n*x^2)/4 + (x^2*(a + b*Log[c*x^n]))/2$

**Rule 2304**

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :=  
Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

**Rubi steps**

$$\int x (a + b \log (cx^n)) dx = -\frac{1}{4}bnx^2 + \frac{1}{2}x^2 (a + b \log (cx^n))$$

**Mathematica [A]** time = 0.0010958, size = 32, normalized size = 1.19

$$\frac{ax^2}{2} + \frac{1}{2}bx^2 \log (cx^n) - \frac{1}{4}bnx^2$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*Log[c\*x^n]),x]

[Out]  $(a*x^2)/2 - (b*n*x^2)/4 + (b*x^2*Log[c*x^n])/2$

**Maple [A]** time = 0.052, size = 29, normalized size = 1.1

$$\frac{ax^2}{2} + \frac{x^2 b \ln (ce^{n \ln(x)})}{2} - \frac{bnx^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*ln(c\*x^n)),x)

[Out]  $1/2*a*x^2+1/2*x^2*b*\ln(c*\exp(n*\ln(x)))-1/4*b*n*x^2$

**Maxima [A]** time = 1.1695, size = 35, normalized size = 1.3

$$-\frac{1}{4} b n x^2 + \frac{1}{2} b x^2 \log (c x^n) + \frac{1}{2} a x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*log(c*x^n)),x, algorithm="maxima")`

[Out]  $-1/4*b*n*x^2 + 1/2*b*x^2*\log(c*x^n) + 1/2*a*x^2$

**Fricas [A]** time = 0.803031, size = 82, normalized size = 3.04

$$\frac{1}{2} b n x^2 \log (x) + \frac{1}{2} b x^2 \log (c) - \frac{1}{4} (b n - 2 a) x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*log(c*x^n)),x, algorithm="fricas")`

[Out]  $1/2*b*n*x^2*\log(x) + 1/2*b*x^2*\log(c) - 1/4*(b*n - 2*a)*x^2$

**Sympy [A]** time = 0.460422, size = 36, normalized size = 1.33

$$\frac{a x^2}{2} + \frac{b n x^2 \log (x)}{2} - \frac{b n x^2}{4} + \frac{b x^2 \log (c)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*ln(c*x**n)),x)`

[Out]  $a*x**2/2 + b*n*x**2*\log(x)/2 - b*n*x**2/4 + b*x**2*\log(c)/2$

**Giac [A]** time = 1.17518, size = 42, normalized size = 1.56

$$\frac{1}{2} b n x^2 \log (x) - \frac{1}{4} b n x^2 + \frac{1}{2} b x^2 \log (c) + \frac{1}{2} a x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*log(c*x^n)),x, algorithm="giac")`

[Out]  $1/2*b*n*x^2*\log(x) - 1/4*b*n*x^2 + 1/2*b*x^2*\log(c) + 1/2*a*x^2$

### 3.46 $\int (a + b \log(cx^n)) dx$

**Optimal.** Leaf size=18

$$ax + bx \log(cx^n) - bnx$$

[Out] a\*x - b\*n\*x + b\*x\*Log[c\*x^n]

**Rubi [A]** time = 0.005548, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2295}

$$ax + bx \log(cx^n) - bnx$$

Antiderivative was successfully verified.

[In] Int[a + b\*Log[c\*x^n], x]

[Out] a\*x - b\*n\*x + b\*x\*Log[c\*x^n]

Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] :> Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

Rubi steps

$$\begin{aligned} \int (a + b \log(cx^n)) dx &= ax + b \int \log(cx^n) dx \\ &= ax - bnx + bx \log(cx^n) \end{aligned}$$

**Mathematica [A]** time = 0.000665, size = 18, normalized size = 1.

$$ax + bx \log(cx^n) - bnx$$

Antiderivative was successfully verified.

[In] Integrate[a + b\*Log[c\*x^n], x]

[Out] a\*x - b\*n\*x + b\*x\*Log[c\*x^n]

**Maple [A]** time = 0.038, size = 19, normalized size = 1.1

$$ax - bnx + bx \ln(cx^n)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a+b\*ln(c\*x^n), x)

[Out] a\*x-b\*n\*x+b\*x\*ln(c\*x^n)

---

**Maxima [A]** time = 1.10327, size = 24, normalized size = 1.33

$$-bnx + bx \log(cx^n) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*log(c\*x^n),x, algorithm="maxima")

[Out] -b\*n\*x + b\*x\*log(c\*x^n) + a\*x

---

**Fricas [A]** time = 0.793783, size = 55, normalized size = 3.06

$$bnx \log(x) + bx \log(c) - (bn - a)x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*log(c\*x^n),x, algorithm="fricas")

[Out] b\*n\*x\*log(x) + b\*x\*log(c) - (b\*n - a)\*x

---

**Sympy [A]** time = 0.239296, size = 19, normalized size = 1.06

$$ax + b(nx \log(x) - nx + x \log(c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*ln(c\*x\*\*n),x)

[Out] a\*x + b\*(n\*x\*log(x) - n\*x + x\*log(c))

---

**Giac [A]** time = 1.17849, size = 27, normalized size = 1.5

$$(nx \log(x) - nx + x \log(c))b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*log(c\*x^n),x, algorithm="giac")

[Out] (n\*x\*log(x) - n\*x + x\*log(c))\*b + a\*x

$$3.47 \quad \int \frac{a+b \log(cx^n)}{x} dx$$

**Optimal.** Leaf size=22

$$\frac{(a + b \log(cx^n))^2}{2bn}$$

[Out] (a + b\*Log[c\*x^n])^2/(2\*b\*n)

**Rubi [A]** time = 0.0122896, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {2301}

$$\frac{(a + b \log(cx^n))^2}{2bn}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])/x,x]

[Out] (a + b\*Log[c\*x^n])^2/(2\*b\*n)

**Rule 2301**

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

**Rubi steps**

$$\int \frac{a + b \log(cx^n)}{x} dx = \frac{(a + b \log(cx^n))^2}{2bn}$$

**Mathematica [A]** time = 0.0010606, size = 21, normalized size = 0.95

$$a \log(x) + \frac{b \log^2(cx^n)}{2n}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])/x,x]

[Out] a\*Log[x] + (b\*Log[c\*x^n]^2)/(2\*n)

**Maple [A]** time = 0.035, size = 27, normalized size = 1.2

$$\frac{(\ln(cx^n))^2 b}{2n} + \frac{a \ln(cx^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^n))/x,x)

[Out]  $1/2/n*\ln(c*x^n)^2*b+1/n*a*\ln(c*x^n)$

**Maxima [A]** time = 1.11712, size = 27, normalized size = 1.23

$$\frac{(b \log(cx^n) + a)^2}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))/x,x, algorithm="maxima")`

[Out]  $1/2*(b*\log(c*x^n) + a)^2/(b*n)$

**Fricas [A]** time = 0.87814, size = 57, normalized size = 2.59

$$\frac{1}{2}bn \log(x)^2 + (b \log(c) + a) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))/x,x, algorithm="fricas")`

[Out]  $1/2*b*n*\log(x)^2 + (b*\log(c) + a)*\log(x)$

**Sympy [A]** time = 6.94959, size = 36, normalized size = 1.64

$$- \begin{cases} -a \log(x) & \text{for } b = 0 \\ -(a + b \log(c)) \log(x) & \text{for } n = 0 \\ -\frac{(a+b \log(cx^n))^2}{2bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*ln(c*x**n))/x,x)`

[Out] `-Piecewise((-a*log(x), Eq(b, 0)), (-(a + b*log(c))*log(x), Eq(n, 0)), (-(a + b*log(c*x**n))**2/(2*b*n), True))`

**Giac [A]** time = 1.13865, size = 26, normalized size = 1.18

$$\frac{1}{2}bn \log(x)^2 + b \log(c) \log(x) + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))/x,x, algorithm="giac")`

[Out]  $1/2*b*n*\log(x)^2 + b*\log(c)*\log(x) + a*\log(x)$

$$3.48 \quad \int \frac{a+b \log(cx^n)}{x^2} dx$$

**Optimal.** Leaf size=23

$$-\frac{a+b \log(cx^n)}{x} - \frac{bn}{x}$$

[Out]  $-\frac{(b*n)}{x} - (a + b*\text{Log}[c*x^n])/x$

**Rubi [A]** time = 0.0129638, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {2304}

$$-\frac{a+b \log(cx^n)}{x} - \frac{bn}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])/x^2,x]

[Out]  $-\frac{(b*n)}{x} - (a + b*\text{Log}[c*x^n])/x$

**Rule 2304**

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :=  
Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

**Rubi steps**

$$\int \frac{a+b \log(cx^n)}{x^2} dx = -\frac{bn}{x} - \frac{a+b \log(cx^n)}{x}$$

**Mathematica [A]** time = 0.001195, size = 26, normalized size = 1.13

$$-\frac{a}{x} - \frac{b \log(cx^n)}{x} - \frac{bn}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])/x^2,x]

[Out]  $-(a/x) - (b*n)/x - (b*\text{Log}[c*x^n])/x$

**Maple [C]** time = 0.082, size = 112, normalized size = 4.9

$$\frac{b \ln(x^n)}{x} - \frac{ib\pi \operatorname{csgn}(ix^n) (\operatorname{csgn}(icx^n))^2 - ib\pi \operatorname{csgn}(ix^n) \operatorname{csgn}(icx^n) \operatorname{csgn}(ic) - ib\pi (\operatorname{csgn}(icx^n))^3 + ib\pi (\operatorname{csgn}(icx^n))}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^n))/x^2,x)

[Out]  $-b/x \cdot \ln(x^n) - 1/2 \cdot (I \cdot b \cdot \text{Pi} \cdot \text{csgn}(I \cdot x^n) \cdot \text{csgn}(I \cdot c \cdot x^n)^2 - I \cdot b \cdot \text{Pi} \cdot \text{csgn}(I \cdot x^n) \cdot \text{csgn}(I \cdot c \cdot x^n) \cdot \text{csgn}(I \cdot c) - I \cdot b \cdot \text{Pi} \cdot \text{csgn}(I \cdot c \cdot x^n)^3 + I \cdot b \cdot \text{Pi} \cdot \text{csgn}(I \cdot c \cdot x^n)^2 \cdot \text{csgn}(I \cdot c) + 2 \cdot b \cdot \ln(c) + 2 \cdot b \cdot n + 2 \cdot a) / x$

**Maxima [A]** time = 1.18859, size = 35, normalized size = 1.52

$$-\frac{bn}{x} - \frac{b \log(cx^n)}{x} - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))/x^2,x, algorithm="maxima")`

[Out]  $-b \cdot n / x - b \cdot \log(c \cdot x^n) / x - a / x$

**Fricas [A]** time = 0.961065, size = 51, normalized size = 2.22

$$-\frac{bn \log(x) + bn + b \log(c) + a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))/x^2,x, algorithm="fricas")`

[Out]  $-(b \cdot n \cdot \log(x) + b \cdot n + b \cdot \log(c) + a) / x$

**Sympy [A]** time = 0.473747, size = 24, normalized size = 1.04

$$-\frac{a}{x} - \frac{bn \log(x)}{x} - \frac{bn}{x} - \frac{b \log(c)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*ln(c*x**n))/x**2,x)`

[Out]  $-a/x - b \cdot n \cdot \log(x) / x - b \cdot n / x - b \cdot \log(c) / x$

**Giac [A]** time = 1.17494, size = 32, normalized size = 1.39

$$-\frac{bn \log(x)}{x} - \frac{bn + b \log(c) + a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))/x^2,x, algorithm="giac")`

[Out]  $-b \cdot n \cdot \log(x) / x - (b \cdot n + b \cdot \log(c) + a) / x$



$$3.49 \quad \int \frac{a+b \log(cx^n)}{x^3} dx$$

**Optimal.** Leaf size=27

$$-\frac{a+b \log(cx^n)}{2x^2} - \frac{bn}{4x^2}$$

[Out]  $-(b*n)/(4*x^2) - (a + b*Log[c*x^n])/(2*x^2)$

**Rubi [A]** time = 0.0125244, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {2304}

$$-\frac{a+b \log(cx^n)}{2x^2} - \frac{bn}{4x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])/x^3, x]

[Out]  $-(b*n)/(4*x^2) - (a + b*Log[c*x^n])/(2*x^2)$

**Rule 2304**

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :=  
Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

**Rubi steps**

$$\int \frac{a+b \log(cx^n)}{x^3} dx = -\frac{bn}{4x^2} - \frac{a+b \log(cx^n)}{2x^2}$$

**Mathematica [A]** time = 0.0012246, size = 32, normalized size = 1.19

$$-\frac{a}{2x^2} - \frac{b \log(cx^n)}{2x^2} - \frac{bn}{4x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])/x^3, x]

[Out]  $-a/(2*x^2) - (b*n)/(4*x^2) - (b*Log[c*x^n])/(2*x^2)$

**Maple [C]** time = 0.073, size = 111, normalized size = 4.1

$$-\frac{b \ln(x^n)}{2x^2} - \frac{ib\pi \operatorname{csgn}(ix^n) (\operatorname{csgn}(icx^n))^2 - ib\pi \operatorname{csgn}(ix^n) \operatorname{csgn}(icx^n) \operatorname{csgn}(ic) - ib\pi (\operatorname{csgn}(icx^n))^3 + ib\pi (\operatorname{csgn}(icx^n))}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^n))/x^3, x)

[Out]  $-1/2*b/x^2*\ln(x^n)-1/4*(I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*b*Pi*csgn(I*c*x^n)^3+I*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+2*b*\ln(c)+b*n+2*a)/x^2$

**Maxima [A]** time = 1.13792, size = 35, normalized size = 1.3

$$-\frac{bn}{4x^2} - \frac{b \log(cx^n)}{2x^2} - \frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))/x^3,x, algorithm="maxima")

[Out]  $-1/4*b*n/x^2 - 1/2*b*\log(c*x^n)/x^2 - 1/2*a/x^2$

**Fricas [A]** time = 0.834715, size = 68, normalized size = 2.52

$$-\frac{2bn \log(x) + bn + 2b \log(c) + 2a}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))/x^3,x, algorithm="fricas")

[Out]  $-1/4*(2*b*n*\log(x) + b*n + 2*b*\log(c) + 2*a)/x^2$

**Sympy [A]** time = 1.2403, size = 37, normalized size = 1.37

$$-\frac{a}{2x^2} - \frac{bn \log(x)}{2x^2} - \frac{bn}{4x^2} - \frac{b \log(c)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))/x\*\*3,x)

[Out]  $-a/(2*x**2) - b*n*\log(x)/(2*x**2) - b*n/(4*x**2) - b*\log(c)/(2*x**2)$

**Giac [A]** time = 1.14973, size = 36, normalized size = 1.33

$$-\frac{bn \log(x)}{2x^2} - \frac{bn + 2b \log(c) + 2a}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))/x^3,x, algorithm="giac")

[Out]  $-1/2*b*n*\log(x)/x^2 - 1/4*(b*n + 2*b*\log(c) + 2*a)/x^2$

### 3.50 $\int x^3 (a + b \log(cx^n))^2 dx$

**Optimal.** Leaf size=52

$$\frac{1}{4}x^4(a + b \log(cx^n))^2 - \frac{1}{8}bnx^4(a + b \log(cx^n)) + \frac{1}{32}b^2n^2x^4$$

[Out]  $(b^2n^2x^4)/32 - (bnx^4(a + b \log[cx^n]))/8 + (x^4(a + b \log[cx^n])^2)/4$

**Rubi [A]** time = 0.0362843, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2305, 2304}

$$\frac{1}{4}x^4(a + b \log(cx^n))^2 - \frac{1}{8}bnx^4(a + b \log(cx^n)) + \frac{1}{32}b^2n^2x^4$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*Log[c\*x^n])^2,x]

[Out]  $(b^2n^2x^4)/32 - (bnx^4(a + b \log[cx^n]))/8 + (x^4(a + b \log[cx^n])^2)/4$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*(d\*x)^p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int x^3 (a + b \log(cx^n))^2 dx &= \frac{1}{4}x^4(a + b \log(cx^n))^2 - \frac{1}{2}(bn) \int x^3 (a + b \log(cx^n)) dx \\ &= \frac{1}{32}b^2n^2x^4 - \frac{1}{8}bnx^4(a + b \log(cx^n)) + \frac{1}{4}x^4(a + b \log(cx^n))^2 \end{aligned}$$

**Mathematica [A]** time = 0.0161885, size = 43, normalized size = 0.83

$$\frac{1}{32}x^4(-4bn(a + b \log(cx^n)) + 8(a + b \log(cx^n))^2 + b^2n^2)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*Log[c\*x^n])^2,x]

[Out]  $(x^4*(b^2n^2 - 4*bn*(a + b \log[cx^n]) + 8*(a + b \log[cx^n])^2))/32$

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**Maple [C]** time = 0.191, size = 691, normalized size = 13.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^3*(a+b*\ln(c*x^n))^2,x)$

[Out]  $\frac{1}{4}b^2x^4\ln(x^n)^2 + \frac{1}{8}b^2x^4(2Ib\pi\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^2 - 2Ib\pi\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)\text{csgn}(I*c) - 2Ib\pi\text{csgn}(I*c*x^n)^3 + 2Ib\pi\text{csgn}(I*c*x^n)^2\text{csgn}(I*c) + 4b\ln(c) - b^n + 4a)*\ln(x^n) + \frac{1}{32}x^4(8\ln(c)^2b^2 - 2\pi^2b^2\text{csgn}(I*c*x^n)^4\text{csgn}(I*c)^2 - 4a*b*n + b^2*n^2 + 8a^2 - 2I\pi*b^2*n*\text{csgn}(I*c*x^n)^2\text{csgn}(I*c) + 8I*\ln(c)*\pi*b^2*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^2 + 8I\pi*a*b*\text{csgn}(I*c*x^n)^2\text{csgn}(I*c) - 2I\pi*b^2*n*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^2 + 8I*\ln(c)*\pi*b^2*\text{csgn}(I*c*x^n)^2\text{csgn}(I*c) + 8I\pi*a*b*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^2 + 4\pi^2*b^2*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^3\text{csgn}(I*c)^2 + 4\pi^2*b^2*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^3*\text{csgn}(I*c) - 2\pi^2*b^2*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^2*\text{csgn}(I*c)^2 - 8\pi^2*b^2*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^4*\text{csgn}(I*c) - 8I*\ln(c)*\pi*b^2*\text{csgn}(I*c*x^n)^3 - 8I\pi*a*b*\text{csgn}(I*c*x^n)^3 + 2I\pi*b^2*n*\text{csgn}(I*c*x^n)^3 - 8I*\ln(c)*\pi*b^2*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)\text{csgn}(I*c) - 8I\pi*a*b*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)\text{csgn}(I*c) - 2\pi^2*b^2*\text{csgn}(I*c*x^n)^6 + 16*\ln(c)*a*b - 4*\ln(c)*b^2*n + 2I\pi*b^2*n*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)\text{csgn}(I*c) + 4\pi^2*b^2*\text{csgn}(I*c*x^n)^5*\text{csgn}(I*c) + 4\pi^2*b^2*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^5 - 2\pi^2*b^2*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^4)$

---

**Maxima [A]** time = 1.12839, size = 96, normalized size = 1.85

$$\frac{1}{4}b^2x^4\log(cx^n)^2 - \frac{1}{8}abnx^4 + \frac{1}{2}abx^4\log(cx^n) + \frac{1}{4}a^2x^4 + \frac{1}{32}(n^2x^4 - 4nx^4\log(cx^n))b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(x^3*(a+b*\log(c*x^n))^2,x, \text{algorithm}="maxima")$

[Out]  $\frac{1}{4}b^2x^4\log(c*x^n)^2 - \frac{1}{8}a*b*n*x^4 + \frac{1}{2}a*b*x^4*\log(c*x^n) + \frac{1}{4}a^2*x^4 + \frac{1}{32}(n^2*x^4 - 4*n*x^4*\log(c*x^n))*b^2$

---

**Fricas [B]** time = 0.890636, size = 244, normalized size = 4.69

$$\frac{1}{4}b^2n^2x^4\log(x)^2 + \frac{1}{4}b^2x^4\log(c)^2 - \frac{1}{8}(b^2n - 4ab)x^4\log(c) + \frac{1}{32}(b^2n^2 - 4abn + 8a^2)x^4 + \frac{1}{8}(4b^2nx^4\log(c) - (b^2n^2 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(x^3*(a+b*\log(c*x^n))^2,x, \text{algorithm}="fricas")$

[Out]  $\frac{1}{4}b^2n^2x^4\log(x)^2 + \frac{1}{4}b^2x^4\log(c)^2 - \frac{1}{8}(b^2n - 4a*b)*x^4*\log(c) + \frac{1}{32}(b^2n^2 - 4a*b*n + 8a^2)*x^4 + \frac{1}{8}(4*b^2*n*x^4*\log(c) - (b^2n^2 - 4a*b*n)*x^4)*\log(x)$

---

**Sympy [B]** time = 2.9114, size = 131, normalized size = 2.52

$$\frac{a^2x^4}{4} + \frac{abnx^4 \log(x)}{2} - \frac{abnx^4}{8} + \frac{abx^4 \log(c)}{2} + \frac{b^2n^2x^4 \log(x)^2}{4} - \frac{b^2n^2x^4 \log(x)}{8} + \frac{b^2n^2x^4}{32} + \frac{b^2nx^4 \log(c) \log(x)}{2} - \frac{b^2nx^4 \log(c)}{8} + \frac{b^2nx^4 \log(c)^2}{4} + \frac{abnx^4 \log(c)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] a\*\*2\*x\*\*4/4 + a\*b\*n\*x\*\*4\*log(x)/2 - a\*b\*n\*x\*\*4/8 + a\*b\*x\*\*4\*log(c)/2 + b\*\*2\*n\*\*2\*x\*\*4\*log(x)\*\*2/4 - b\*\*2\*n\*\*2\*x\*\*4\*log(x)/8 + b\*\*2\*n\*\*2\*x\*\*4/32 + b\*\*2\*n\*x\*\*4\*log(c)\*log(x)/2 - b\*\*2\*n\*x\*\*4\*log(c)/8 + b\*\*2\*x\*\*4\*log(c)\*\*2/4

**Giac [B]** time = 1.15678, size = 150, normalized size = 2.88

$$\frac{1}{4} b^2 n^2 x^4 \log(x)^2 - \frac{1}{8} b^2 n^2 x^4 \log(x) + \frac{1}{2} b^2 n x^4 \log(c) \log(x) + \frac{1}{32} b^2 n^2 x^4 - \frac{1}{8} b^2 n x^4 \log(c) + \frac{1}{4} b^2 x^4 \log(c)^2 + \frac{1}{2} abnx^4 \log(c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] 1/4\*b^2\*n^2\*x^4\*log(x)^2 - 1/8\*b^2\*n^2\*x^4\*log(x) + 1/2\*b^2\*n\*x^4\*log(c)\*log(x) + 1/32\*b^2\*n^2\*x^4 - 1/8\*b^2\*n\*x^4\*log(c) + 1/4\*b^2\*x^4\*log(c)^2 + 1/2\*a\*b\*n\*x^4\*log(c) - 1/8\*a\*b\*n\*x^4 + 1/2\*a\*b\*x^4\*log(c) + 1/4\*a^2\*x^4

### 3.51 $\int x^2 (a + b \log(cx^n))^2 dx$

**Optimal.** Leaf size=52

$$\frac{1}{3}x^3 (a + b \log(cx^n))^2 - \frac{2}{9}bnx^3 (a + b \log(cx^n)) + \frac{2}{27}b^2n^2x^3$$

[Out]  $(2*b^2*n^2*x^3)/27 - (2*b*n*x^3*(a + b*Log[c*x^n]))/9 + (x^3*(a + b*Log[c*x^n])^2)/3$

**Rubi [A]** time = 0.0367015, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2305, 2304}

$$\frac{1}{3}x^3 (a + b \log(cx^n))^2 - \frac{2}{9}bnx^3 (a + b \log(cx^n)) + \frac{2}{27}b^2n^2x^3$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*Log[c\*x^n])^2,x]

[Out]  $(2*b^2*n^2*x^3)/27 - (2*b*n*x^3*(a + b*Log[c*x^n]))/9 + (x^3*(a + b*Log[c*x^n])^2)/3$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int x^2 (a + b \log(cx^n))^2 dx &= \frac{1}{3}x^3 (a + b \log(cx^n))^2 - \frac{1}{3}(2bn) \int x^2 (a + b \log(cx^n)) dx \\ &= \frac{2}{27}b^2n^2x^3 - \frac{2}{9}bnx^3 (a + b \log(cx^n)) + \frac{1}{3}x^3 (a + b \log(cx^n))^2 \end{aligned}$$

**Mathematica [A]** time = 0.0188488, size = 46, normalized size = 0.88

$$\frac{1}{3} \left( x^3 (a + b \log(cx^n))^2 + \frac{2}{9}bnx^3 (-3a - 3b \log(cx^n) + bn) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*Log[c\*x^n])^2,x]

[Out]  $((2*b*n*x^3*(-3*a + b*n - 3*b*Log[c*x^n]))/9 + x^3*(a + b*Log[c*x^n])^2)/3$

---

**Maple [C]** time = 0.204, size = 692, normalized size = 13.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^2*(a+b*\ln(cx^n))^2,x)$

[Out]  $\frac{1}{3}b^2x^3\ln(x^n)^2 + \frac{1}{9}b^2x^3(3Ib\pi\text{csgn}(Ix^n)\text{csgn}(Icx^n)^2 - 3Ib\pi\text{csgn}(Ix^n)\text{csgn}(Icx^n)\text{csgn}(Ic) - 3Ib\pi\text{csgn}(Icx^n)^3 + 3Ib\pi\text{csgn}(Icx^n)^2\text{csgn}(Ic) + 6b^2\ln(c) - 2bn + 6a)\ln(x^n) + \frac{1}{108}x^3(-12I\pi b^2n\text{csgn}(Icx^n)^2\text{csgn}(Ic) + 36I\ln(c)\pi b^2\text{csgn}(Ix^n)\text{csgn}(Icx^n)^2 + 36I\ln(c)\pi b^2\text{csgn}(Icx^n)^2\text{csgn}(Ic) + 36I\pi a b\text{csgn}(Ix^n)\text{csgn}(Icx^n)^2 + 36I\pi a b\text{csgn}(Icx^n)^2\text{csgn}(Ic) - 12I\pi b^2n\text{csgn}(Ix^n)\text{csgn}(Icx^n)^2 + 36\ln(c)^2b^2 - 9\pi^2b^2\text{csgn}(Icx^n)^4\text{csgn}(Ic)^2 - 24a b^2n + 8b^2n^2 + 36a^2 + 18\pi^2b^2\text{csgn}(Ix^n)\text{csgn}(Icx^n)^3\text{csgn}(Ic)^2 + 18\pi^2b^2\text{csgn}(Ix^n)^2\text{csgn}(Icx^n)^3\text{csgn}(Ic) - 9\pi^2b^2\text{csgn}(Ix^n)^2\text{csgn}(Icx^n)^2\text{csgn}(Ic)^2 - 36\pi^2b^2\text{csgn}(Ix^n)\text{csgn}(Icx^n)^4\text{csgn}(Ic) - 36I\ln(c)\pi b^2\text{csgn}(Icx^n)^3 - 36I\pi a b\text{csgn}(Icx^n)^3 + 12I\pi b^2n\text{csgn}(Icx^n)^3 - 36I\ln(c)\pi b^2\text{csgn}(Ix^n)\text{csgn}(Icx^n)\text{csgn}(Ic) - 36I\pi a b\text{csgn}(Ix^n)\text{csgn}(Icx^n)\text{csgn}(Ic) - 9\pi^2b^2\text{csgn}(Icx^n)^6 + 72\ln(c)ab - 24\ln(c)b^2n + 12I\pi b^2n\text{csgn}(Ix^n)\text{csgn}(Icx^n)\text{csgn}(Ic) + 18\pi^2b^2\text{csgn}(Icx^n)^5\text{csgn}(Ic) + 18\pi^2b^2\text{csgn}(Ix^n)\text{csgn}(Icx^n)^5 - 9\pi^2b^2\text{csgn}(Ix^n)^2\text{csgn}(Icx^n)^4)$

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**Maxima [A]** time = 1.08595, size = 96, normalized size = 1.85

$$\frac{1}{3}b^2x^3\log(cx^n)^2 - \frac{2}{9}abnx^3 + \frac{2}{3}abx^3\log(cx^n) + \frac{1}{3}a^2x^3 + \frac{2}{27}(n^2x^3 - 3nx^3\log(cx^n))b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(x^2*(a+b*\log(cx^n))^2,x, \text{algorithm}="maxima")$

[Out]  $\frac{1}{3}b^2x^3\log(cx^n)^2 - \frac{2}{9}a^2b^2n^2x^3 + \frac{2}{3}a^2b^2x^3\log(cx^n) + \frac{1}{3}a^2x^3 + \frac{2}{27}(n^2x^3 - 3n^2x^3\log(cx^n))b^2$

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**Fricas [B]** time = 0.848358, size = 247, normalized size = 4.75

$$\frac{1}{3}b^2n^2x^3\log(x)^2 + \frac{1}{3}b^2x^3\log(c)^2 - \frac{2}{9}(b^2n - 3ab)x^3\log(c) + \frac{1}{27}(2b^2n^2 - 6abn + 9a^2)x^3 + \frac{2}{9}(3b^2nx^3\log(c) - (b^2n^2 - 3a^2b^2n)x^3)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(x^2*(a+b*\log(cx^n))^2,x, \text{algorithm}="fricas")$

[Out]  $\frac{1}{3}b^2n^2x^3\log(x)^2 + \frac{1}{3}b^2x^3\log(c)^2 - \frac{2}{9}(b^2n - 3a^2b^2n)x^3\log(c) + \frac{1}{27}(2b^2n^2 - 6a^2b^2n + 9a^2)x^3 + \frac{2}{9}(3b^2n^2x^3\log(c) - (b^2n^2 - 3a^2b^2n)x^3)\log(x)$

---

**Sympy [B]** time = 1.73516, size = 143, normalized size = 2.75

$$\frac{a^2x^3}{3} + \frac{2abnx^3 \log(x)}{3} - \frac{2abnx^3}{9} + \frac{2abx^3 \log(c)}{3} + \frac{b^2n^2x^3 \log(x)^2}{3} - \frac{2b^2n^2x^3 \log(x)}{9} + \frac{2b^2n^2x^3}{27} + \frac{2b^2nx^3 \log(c) \log(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] a\*\*2\*x\*\*3/3 + 2\*a\*b\*n\*x\*\*3\*log(x)/3 - 2\*a\*b\*n\*x\*\*3/9 + 2\*a\*b\*x\*\*3\*log(c)/3 + b\*\*2\*n\*\*2\*x\*\*3\*log(x)\*\*2/3 - 2\*b\*\*2\*n\*\*2\*x\*\*3\*log(x)/9 + 2\*b\*\*2\*n\*\*2\*x\*\*3/27 + 2\*b\*\*2\*n\*x\*\*3\*log(c)\*log(x)/3 - 2\*b\*\*2\*n\*x\*\*3\*log(c)/9 + b\*\*2\*x\*\*3\*log(c)\*\*2/3

**Giac [B]** time = 1.18623, size = 150, normalized size = 2.88

$$\frac{1}{3}b^2n^2x^3 \log(x)^2 - \frac{2}{9}b^2n^2x^3 \log(x) + \frac{2}{3}b^2nx^3 \log(c) \log(x) + \frac{2}{27}b^2n^2x^3 - \frac{2}{9}b^2nx^3 \log(c) + \frac{1}{3}b^2x^3 \log(c)^2 + \frac{2}{3}abnx^3 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] 1/3\*b^2\*n^2\*x^3\*log(x)^2 - 2/9\*b^2\*n^2\*x^3\*log(x) + 2/3\*b^2\*n\*x^3\*log(c)\*log(x) + 2/27\*b^2\*n^2\*x^3 - 2/9\*b^2\*n\*x^3\*log(c) + 1/3\*b^2\*x^3\*log(c)^2 + 2/3\*a\*b\*n\*x^3\*log(x) - 2/9\*a\*b\*n\*x^3 + 2/3\*a\*b\*x^3\*log(c) + 1/3\*a^2\*x^3



### 3.52 $\int x (a + b \log (cx^n))^2 dx$

**Optimal.** Leaf size=52

$$\frac{1}{2}x^2 (a + b \log (cx^n))^2 - \frac{1}{2}bnx^2 (a + b \log (cx^n)) + \frac{1}{4}b^2n^2x^2$$

[Out]  $(b^2n^2x^2)/4 - (bnx^2(a + b\text{Log}[c*x^n]))/2 + (x^2*(a + b\text{Log}[c*x^n])^2)/2$

**Rubi [A]** time = 0.0230684, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2305, 2304}

$$\frac{1}{2}x^2 (a + b \log (cx^n))^2 - \frac{1}{2}bnx^2 (a + b \log (cx^n)) + \frac{1}{4}b^2n^2x^2$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*Log[c\*x^n])^2,x]

[Out]  $(b^2n^2x^2)/4 - (bnx^2(a + b\text{Log}[c*x^n]))/2 + (x^2*(a + b\text{Log}[c*x^n])^2)/2$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int x (a + b \log (cx^n))^2 dx &= \frac{1}{2}x^2 (a + b \log (cx^n))^2 - (bn) \int x (a + b \log (cx^n)) dx \\ &= \frac{1}{4}b^2n^2x^2 - \frac{1}{2}bnx^2 (a + b \log (cx^n)) + \frac{1}{2}x^2 (a + b \log (cx^n))^2 \end{aligned}$$

**Mathematica [A]** time = 0.0130538, size = 41, normalized size = 0.79

$$\frac{1}{4}x^2 (2(a + b \log (cx^n))^2 + bn(-2a - 2b \log (cx^n) + bn))$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*Log[c\*x^n])^2,x]

[Out]  $(x^2*(b*n*(-2*a + b*n - 2*b*Log[c*x^n]) + 2*(a + b*Log[c*x^n])^2))/4$

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**Maple [C]** time = 0.213, size = 692, normalized size = 13.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a+b*ln(c*x^n))^2,x)`

[Out]  $\frac{1}{2}b^2x^2\ln(x^n)^2 + \frac{1}{2}b^2x^2(Ib\pi\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^2 - Ib\pi\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)*\operatorname{csgn}(I*c) - Ib\pi\operatorname{csgn}(I*c*x^n)^3 + Ib\pi\operatorname{csgn}(I*c*x^n)^2*\operatorname{csgn}(I*c) + 2*b*\ln(c) - b*n + 2*a)*\ln(x^n) + \frac{1}{8}x^2*(4*I*\ln(c)*\pi*b^2*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^2 + 4*\ln(c)^2*b^2 - \pi^2*b^2*\operatorname{csgn}(I*c*x^n)^4*\operatorname{csgn}(I*c)^2 - 4*a*b*n + 2*b^2*n^2 + 4*a^2 - 2*I*\pi*b^2*n*\operatorname{csgn}(I*c*x^n)^2*\operatorname{csgn}(I*c) - 2*I*\pi*b^2*n*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^2 + 2*\pi^2*b^2*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^3*\operatorname{csgn}(I*c)^2 + 2*\pi^2*b^2*\operatorname{csgn}(I*x^n)^2*\operatorname{csgn}(I*c*x^n)^3*\operatorname{csgn}(I*c) - \pi^2*b^2*\operatorname{csgn}(I*x^n)^2*\operatorname{csgn}(I*c*x^n)^2*\operatorname{csgn}(I*c)^2 - 4*\pi^2*b^2*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^4*\operatorname{csgn}(I*c) + 2*I*\pi*b^2*n*\operatorname{csgn}(I*c*x^n)^3 + 4*I*\ln(c)*\pi*b^2*\operatorname{csgn}(I*c*x^n)^2*\operatorname{csgn}(I*c) + 4*I*\pi*a*b*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^2 + 4*I*\pi*a*b*\operatorname{csgn}(I*c*x^n)^2*\operatorname{csgn}(I*c) - 4*I*\pi*a*b*\operatorname{csgn}(I*c*x^n)^3 - 4*I*\ln(c)*\pi*b^2*\operatorname{csgn}(I*c*x^n)^3 - 4*I*\ln(c)*\pi*b^2*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)*\operatorname{csgn}(I*c) - 4*I*\pi*a*b*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)*\operatorname{csgn}(I*c) - \pi^2*b^2*\operatorname{csgn}(I*c*x^n)^6 + 8*\ln(c)*a*b - 4*\ln(c)*b^2*n + 2*I*\pi*b^2*n*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)*\operatorname{csgn}(I*c) + 2*\pi^2*b^2*\operatorname{csgn}(I*c*x^n)^5*\operatorname{csgn}(I*c) + 2*\pi^2*b^2*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^5 - \pi^2*b^2*\operatorname{csgn}(I*x^n)^2*\operatorname{csgn}(I*c*x^n)^4)$

---

**Maxima [A]** time = 1.11882, size = 95, normalized size = 1.83

$$\frac{1}{2}b^2x^2\log(cx^n)^2 - \frac{1}{2}abnx^2 + abx^2\log(cx^n) + \frac{1}{2}a^2x^2 + \frac{1}{4}(n^2x^2 - 2nx^2\log(cx^n))b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*log(c*x^n))^2,x, algorithm="maxima")`

[Out]  $\frac{1}{2}b^2x^2\log(c*x^n)^2 - \frac{1}{2}a*b*n*x^2 + a*b*x^2*\log(c*x^n) + \frac{1}{2}a^2*x^2 + \frac{1}{4}(n^2*x^2 - 2*n*x^2*\log(c*x^n))*b^2$

---

**Fricas [B]** time = 0.87917, size = 243, normalized size = 4.67

$$\frac{1}{2}b^2n^2x^2\log(x)^2 + \frac{1}{2}b^2x^2\log(c)^2 - \frac{1}{2}(b^2n - 2ab)x^2\log(c) + \frac{1}{4}(b^2n^2 - 2abn + 2a^2)x^2 + \frac{1}{2}(2b^2nx^2\log(c) - (b^2n^2 - 2a^2)x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*log(c*x^n))^2,x, algorithm="fricas")`

[Out]  $\frac{1}{2}b^2n^2x^2*\log(x)^2 + \frac{1}{2}b^2x^2*\log(c)^2 - \frac{1}{2}(b^2n - 2*a*b)*x^2*\log(c) + \frac{1}{4}(b^2n^2 - 2*a*b*n + 2*a^2)*x^2 + \frac{1}{2}(2*b^2n*x^2*\log(c) - (b^2n^2 - 2*a*b*n)*x^2)*\log(x)$

---

**Sympy [B]** time = 1.02012, size = 126, normalized size = 2.42

$$\frac{a^2x^2}{2} + abnx^2 \log(x) - \frac{abnx^2}{2} + abx^2 \log(c) + \frac{b^2n^2x^2 \log(x)^2}{2} - \frac{b^2n^2x^2 \log(x)}{2} + \frac{b^2n^2x^2}{4} + b^2nx^2 \log(c) \log(x) - \frac{b^2n^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] a\*\*2\*x\*\*2/2 + a\*b\*n\*x\*\*2\*log(x) - a\*b\*n\*x\*\*2/2 + a\*b\*x\*\*2\*log(c) + b\*\*2\*n\*\*2\*x\*\*2\*log(x)\*\*2/2 - b\*\*2\*n\*\*2\*x\*\*2\*log(x)/2 + b\*\*2\*n\*\*2\*x\*\*2/4 + b\*\*2\*n\*x\*\*2\*log(c)\*log(x) - b\*\*2\*n\*x\*\*2\*log(c)/2 + b\*\*2\*x\*\*2\*log(c)\*\*2/2

**Giac [B]** time = 1.15036, size = 146, normalized size = 2.81

$$\frac{1}{2} b^2 n^2 x^2 \log(x)^2 - \frac{1}{2} b^2 n^2 x^2 \log(x) + b^2 n x^2 \log(c) \log(x) + \frac{1}{4} b^2 n^2 x^2 - \frac{1}{2} b^2 n x^2 \log(c) + \frac{1}{2} b^2 x^2 \log(c)^2 + abnx^2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] 1/2\*b^2\*n^2\*x^2\*log(x)^2 - 1/2\*b^2\*n^2\*x^2\*log(x) + b^2\*n\*x^2\*log(c)\*log(x) + 1/4\*b^2\*n^2\*x^2 - 1/2\*b^2\*n\*x^2\*log(c) + 1/2\*b^2\*x^2\*log(c)^2 + a\*b\*n\*x^2\*log(x) - 1/2\*a\*b\*n\*x^2 + a\*b\*x^2\*log(c) + 1/2\*a^2\*x^2

### 3.53 $\int (a + b \log(cx^n))^2 dx$

**Optimal.** Leaf size=43

$$x(a + b \log(cx^n))^2 - 2abnx - 2b^2nx \log(cx^n) + 2b^2n^2x$$

[Out]  $-2*a*b*n*x + 2*b^2*n^2*x - 2*b^2*n*x*Log[c*x^n] + x*(a + b*Log[c*x^n])^2$

**Rubi [A]** time = 0.0128483, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2296, 2295}

$$x(a + b \log(cx^n))^2 - 2abnx - 2b^2nx \log(cx^n) + 2b^2n^2x$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^2,x]

[Out]  $-2*a*b*n*x + 2*b^2*n^2*x - 2*b^2*n*x*Log[c*x^n] + x*(a + b*Log[c*x^n])^2$

#### Rule 2296

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Simp[x\*(a + b\*Log[c\*x^n])^p, x] - Dist[b\*n\*p, Int[(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2\*p]

#### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

#### Rubi steps

$$\begin{aligned} \int (a + b \log(cx^n))^2 dx &= x(a + b \log(cx^n))^2 - (2bn) \int (a + b \log(cx^n)) dx \\ &= -2abnx + x(a + b \log(cx^n))^2 - (2b^2n) \int \log(cx^n) dx \\ &= -2abnx + 2b^2n^2x - 2b^2nx \log(cx^n) + x(a + b \log(cx^n))^2 \end{aligned}$$

**Mathematica [A]** time = 0.008462, size = 33, normalized size = 0.77

$$x((a + b \log(cx^n))^2 - 2bn(a + b \log(cx^n) - bn))$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^2,x]

[Out]  $x*((a + b*Log[c*x^n])^2 - 2*b*n*(a - b*n + b*Log[c*x^n]))$

**Maple [A]** time = 0.05, size = 63, normalized size = 1.5

$$xa^2 + b^2x(\ln(ce^{n \ln(x)}))^2 + 2b^2n^2x - 2b^2nx \ln(ce^{n \ln(x)}) + 2xab \ln(cx^n) - 2abnx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*ln(c*x^n))^2,x)`

[Out]  $x*a^2+b^2*x*ln(c*exp(n*ln(x)))^2+2*b^2*n^2*x-2*b^2*n*x*ln(c*exp(n*ln(x)))+2*x*a*b*ln(c*x^n)-2*a*b*n*x$

**Maxima [A]** time = 1.08919, size = 77, normalized size = 1.79

$$b^2x \log(cx^n)^2 - 2abnx + 2abx \log(cx^n) + 2(n^2x - nx \log(cx^n))b^2 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))^2,x, algorithm="maxima")`

[Out]  $b^2*x*log(c*x^n)^2 - 2*a*b*n*x + 2*a*b*x*log(c*x^n) + 2*(n^2*x - n*x*log(c*x^n))*b^2 + a^2*x$

**Fricas [A]** time = 0.792463, size = 197, normalized size = 4.58

$$b^2n^2x \log(x)^2 + b^2x \log(c)^2 - 2(b^2n - ab)x \log(c) + (2b^2n^2 - 2abn + a^2)x + 2(b^2nx \log(c) - (b^2n^2 - abn)x) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))^2,x, algorithm="fricas")`

[Out]  $b^2*n^2*x*log(x)^2 + b^2*x*log(c)^2 - 2*(b^2*n - a*b)*x*log(c) + (2*b^2*n^2 - 2*a*b*n + a^2)*x + 2*(b^2*n*x*log(c) - (b^2*n^2 - a*b*n)*x)*log(x)$

**Sympy [B]** time = 0.607413, size = 109, normalized size = 2.53

$$a^2x + 2abnx \log(x) - 2abnx + 2abx \log(c) + b^2n^2x \log(x)^2 - 2b^2n^2x \log(x) + 2b^2n^2x + 2b^2nx \log(c) \log(x) - 2b^2nx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*ln(c*x**n))**2,x)`

[Out]  $a**2*x + 2*a*b*n*x*log(x) - 2*a*b*n*x + 2*a*b*x*log(c) + b**2*n**2*x*log(x)**2 - 2*b**2*n**2*x*log(x) + 2*b**2*n**2*x + 2*b**2*n*x*log(c)*log(x) - 2*b**2*n*x*log(c) + b**2*x*log(c)**2$

**Giac [B]** time = 1.1966, size = 119, normalized size = 2.77

$$b^2n^2x \log(x)^2 - 2b^2n^2x \log(x) + 2b^2nx \log(c) \log(x) + 2b^2n^2x - 2b^2nx \log(c) + b^2x \log(c)^2 + 2abnx \log(x) - 2ab$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))^2,x, algorithm="giac")`

```
[Out] b^2*n^2*x*log(x)^2 - 2*b^2*n^2*x*log(x) + 2*b^2*n*x*log(c)*log(x) + 2*b^2*n^2*x - 2*b^2*n*x*log(c) + b^2*x*log(c)^2 + 2*a*b*n*x*log(x) - 2*a*b*n*x + 2*a*b*x*log(c) + a^2*x
```

$$3.54 \quad \int \frac{(a+b \log(cx^n))^2}{x} dx$$

**Optimal.** Leaf size=22

$$\frac{(a + b \log(cx^n))^3}{3bn}$$

[Out] (a + b\*Log[c\*x^n])^3/(3\*b\*n)

**Rubi [A]** time = 0.0239641, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2302, 30}

$$\frac{(a + b \log(cx^n))^3}{3bn}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^2/x, x]

[Out] (a + b\*Log[c\*x^n])^3/(3\*b\*n)

#### Rule 2302

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] := Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{(a + b \log(cx^n))^2}{x} dx &= \frac{\text{Subst}\left(\int x^2 dx, x, a + b \log(cx^n)\right)}{bn} \\ &= \frac{(a + b \log(cx^n))^3}{3bn} \end{aligned}$$

**Mathematica [A]** time = 0.0030663, size = 22, normalized size = 1.

$$\frac{(a + b \log(cx^n))^3}{3bn}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^2/x, x]

[Out] (a + b\*Log[c\*x^n])^3/(3\*b\*n)

**Maple [B]** time = 0.037, size = 56, normalized size = 2.6

$$\frac{b^2 (\ln(cx^n))^3}{3n} + \frac{b (\ln(cx^n))^2 a}{n} + \frac{\ln(cx^n) a^2}{n} + \frac{a^3}{3bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^n))^2/x,x)

[Out] 1/3\*b^2/n\*ln(c\*x^n)^3+b/n\*ln(c\*x^n)^2\*a+1/n\*ln(c\*x^n)\*a^2+1/3/b/n\*a^3

**Maxima [A]** time = 1.18812, size = 27, normalized size = 1.23

$$\frac{(b \log(cx^n) + a)^3}{3bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^2/x,x, algorithm="maxima")

[Out] 1/3\*(b\*log(c\*x^n) + a)^3/(b\*n)

**Fricas [B]** time = 0.837986, size = 136, normalized size = 6.18

$$\frac{1}{3} b^2 n^2 \log(x)^3 + (b^2 n \log(c) + abn) \log(x)^2 + (b^2 \log(c)^2 + 2ab \log(c) + a^2) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^2/x,x, algorithm="fricas")

[Out] 1/3\*b^2\*n^2\*log(x)^3 + (b^2\*n\*log(c) + a\*b\*n)\*log(x)^2 + (b^2\*log(c)^2 + 2\*a\*b\*log(c) + a^2)\*log(x)

**Sympy [A]** time = 22.3419, size = 60, normalized size = 2.73

$$\begin{cases} \frac{a^2 \log(cx^n) + ab \log(cx^n)^2 + \frac{b^2 \log(cx^n)^3}{3}}{n} & \text{for } n \neq 0 \\ (a^2 + 2ab \log(c) + b^2 \log(c)^2) \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))\*\*2/x,x)

[Out] Piecewise(((a\*\*2\*log(c\*x\*\*n) + a\*b\*log(c\*x\*\*n)\*\*2 + b\*\*2\*log(c\*x\*\*n)\*\*3)/3)/n, Ne(n, 0)), ((a\*\*2 + 2\*a\*b\*log(c) + b\*\*2\*log(c)\*\*2)\*log(x), True)

**Giac [B]** time = 1.24202, size = 76, normalized size = 3.45

$$\frac{1}{3} b^2 n^2 \log(x)^3 + b^2 n \log(c) \log(x)^2 + b^2 \log(c)^2 \log(x) + abn \log(x)^2 + 2ab \log(c) \log(x) + a^2 \log(x)$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

```
[Out] 1/3*b^2*n^2*log(x)^3 + b^2*n*log(c)*log(x)^2 + b^2*log(c)^2*log(x) + a*b*n*  
log(x)^2 + 2*a*b*log(c)*log(x) + a^2*log(x)
```

$$3.55 \quad \int \frac{(a+b \log(cx^n))^2}{x^2} dx$$

**Optimal.** Leaf size=46

$$-\frac{2bn(a+b \log(cx^n))}{x} - \frac{(a+b \log(cx^n))^2}{x} - \frac{2b^2n^2}{x}$$

[Out]  $(-2*b^2*n^2)/x - (2*b*n*(a + b*Log[c*x^n]))/x - (a + b*Log[c*x^n])^2/x$

**Rubi [A]** time = 0.0348372, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2305, 2304}

$$-\frac{2bn(a+b \log(cx^n))}{x} - \frac{(a+b \log(cx^n))^2}{x} - \frac{2b^2n^2}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^2/x^2,x]

[Out]  $(-2*b^2*n^2)/x - (2*b*n*(a + b*Log[c*x^n]))/x - (a + b*Log[c*x^n])^2/x$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{(a+b \log(cx^n))^2}{x^2} dx &= -\frac{(a+b \log(cx^n))^2}{x} + (2bn) \int \frac{a+b \log(cx^n)}{x^2} dx \\ &= -\frac{2b^2n^2}{x} - \frac{2bn(a+b \log(cx^n))}{x} - \frac{(a+b \log(cx^n))^2}{x} \end{aligned}$$

**Mathematica [A]** time = 0.0106017, size = 35, normalized size = 0.76

$$-\frac{(a+b \log(cx^n))^2 + 2bn(a+b \log(cx^n) + bn)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^2/x^2,x]

[Out]  $-(((a + b*Log[c*x^n])^2 + 2*b*n*(a + b*n + b*Log[c*x^n]))/x)$

---

**Maple [C]** time = 0.122, size = 704, normalized size = 15.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^n))^2/x^2,x)

[Out] 
$$-b^2/x*\ln(x^n)^2-(I*\pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2-I*\pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*\pi*b^2*csgn(I*c*x^n)^3+I*\pi*b^2*csgn(I*c*x^n)^2*csgn(I*c)+2*\ln(c)*b^2+2*b^2*n+2*a*b)/x*\ln(x^n)-1/4*(4*I*\ln(c)*\pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2+4*\ln(c)^2*b^2-\pi^2*b^2*csgn(I*c*x^n)^4*csgn(I*c)^2+8*a*b*n+8*b^2*n^2+4*a^2-4*I*\pi*b^2*n*csgn(I*c*x^n)^3+4*I*\pi*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)^2+4*I*\pi*b^2*n*csgn(I*c*x^n)^2*csgn(I*c)+2*\pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2+2*\pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-\pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2-4*\pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)+4*I*\ln(c)*\pi*b^2*csgn(I*c*x^n)^2*csgn(I*c)+4*I*\pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)^2+4*I*\pi*a*b*csgn(I*c*x^n)^2*csgn(I*c)-4*I*\pi*a*b*csgn(I*c*x^n)^3-4*I*\ln(c)*\pi*b^2*csgn(I*c*x^n)^3-4*I*\ln(c)*\pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-4*I*\pi*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-4*I*\pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-\pi^2*b^2*csgn(I*c*x^n)^6+8*\ln(c)*a*b+8*\ln(c)*b^2*n+2*\pi^2*b^2*csgn(I*c*x^n)^5*csgn(I*c)+2*\pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^5-\pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4)/x$$

---

**Maxima [A]** time = 1.11781, size = 95, normalized size = 2.07

$$-2b^2\left(\frac{n^2}{x} + \frac{n \log(cx^n)}{x}\right) - \frac{b^2 \log(cx^n)^2}{x} - \frac{2abn}{x} - \frac{2ab \log(cx^n)}{x} - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^2/x^2,x, algorithm="maxima")

[Out] 
$$-2*b^2*(n^2/x + n*\log(c*x^n)/x) - b^2*\log(c*x^n)^2/x - 2*a*b*n/x - 2*a*b*\log(c*x^n)/x - a^2/x$$

---

**Fricas [A]** time = 0.859764, size = 182, normalized size = 3.96

$$\frac{b^2 n^2 \log(x)^2 + 2 b^2 n^2 + b^2 \log(c)^2 + 2 abn + a^2 + 2 (b^2 n + ab) \log(c) + 2 (b^2 n^2 + b^2 n \log(c) + abn) \log(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^2/x^2,x, algorithm="fricas")

[Out] 
$$-(b^2*n^2*\log(x)^2 + 2*b^2*n^2 + b^2*\log(c)^2 + 2*a*b*n + a^2 + 2*(b^2*n + a*b)*\log(c) + 2*(b^2*n^2 + b^2*n*\log(c) + a*b*n)*\log(x))/x$$

---

**Sympy [B]** time = 1.07547, size = 110, normalized size = 2.39

$$\frac{a^2}{x} - \frac{2abn \log(x)}{x} - \frac{2abn}{x} - \frac{2ab \log(c)}{x} - \frac{b^2 n^2 \log(x)^2}{x} - \frac{2b^2 n^2 \log(x)}{x} - \frac{2b^2 n^2}{x} - \frac{2b^2 n \log(c) \log(x)}{x} - \frac{2b^2 n \log(c)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))\*\*2/x\*\*2,x)

[Out] -a\*\*2/x - 2\*a\*b\*n\*log(x)/x - 2\*a\*b\*n/x - 2\*a\*b\*log(c)/x - b\*\*2\*n\*\*2\*log(x)\*  
\*2/x - 2\*b\*\*2\*n\*\*2\*log(x)/x - 2\*b\*\*2\*n\*\*2/x - 2\*b\*\*2\*n\*log(c)\*log(x)/x - 2\*  
b\*\*2\*n\*log(c)/x - b\*\*2\*log(c)\*\*2/x

**Giac [A]** time = 1.15635, size = 116, normalized size = 2.52

$$\frac{b^2 n^2 \log(x)^2}{x} - \frac{2(b^2 n^2 + b^2 n \log(c) + abn) \log(x)}{x} - \frac{2b^2 n^2 + 2b^2 n \log(c) + b^2 \log(c)^2 + 2abn + 2ab \log(c) + a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^2/x^2,x, algorithm="giac")

[Out] -b^2\*n^2\*log(x)^2/x - 2\*(b^2\*n^2 + b^2\*n\*log(c) + a\*b\*n)\*log(x)/x - (2\*b^2\*  
n^2 + 2\*b^2\*n\*log(c) + b^2\*log(c)^2 + 2\*a\*b\*n + 2\*a\*b\*log(c) + a^2)/x

$$3.56 \quad \int \frac{(a+b \log(cx^n))^2}{x^3} dx$$

**Optimal.** Leaf size=52

$$-\frac{bn(a+b \log(cx^n))}{2x^2} - \frac{(a+b \log(cx^n))^2}{2x^2} - \frac{b^2n^2}{4x^2}$$

[Out]  $-(b^2n^2)/(4x^2) - (bn*(a + b*Log[c*x^n]))/(2*x^2) - (a + b*Log[c*x^n])^2/(2*x^2)$

**Rubi [A]** time = 0.0352318, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2305, 2304}

$$-\frac{bn(a+b \log(cx^n))}{2x^2} - \frac{(a+b \log(cx^n))^2}{2x^2} - \frac{b^2n^2}{4x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^2/x^3, x]

[Out]  $-(b^2n^2)/(4*x^2) - (bn*(a + b*Log[c*x^n]))/(2*x^2) - (a + b*Log[c*x^n])^2/(2*x^2)$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{(a+b \log(cx^n))^2}{x^3} dx &= -\frac{(a+b \log(cx^n))^2}{2x^2} + (bn) \int \frac{a+b \log(cx^n)}{x^3} dx \\ &= -\frac{b^2n^2}{4x^2} - \frac{bn(a+b \log(cx^n))}{2x^2} - \frac{(a+b \log(cx^n))^2}{2x^2} \end{aligned}$$

**Mathematica [A]** time = 0.0129237, size = 41, normalized size = 0.79

$$-\frac{2(a+b \log(cx^n))^2 + bn(2a + 2b \log(cx^n) + bn)}{4x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^2/x^3, x]

[Out]  $-(2*(a + b*\text{Log}[c*x^n])^2 + b*n*(2*a + b*n + 2*b*\text{Log}[c*x^n]))/(4*x^2)$

**Maple [C]** time = 0.122, size = 703, normalized size = 13.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a+b*\ln(c*x^n))^2/x^3,x)$

[Out]  $-1/2*b^2/x^2*\ln(x^n)^2-1/2*(I*\text{Pi}*b^2*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^2-I*\text{Pi}*b^2*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)*\text{csgn}(I*c)-I*\text{Pi}*b^2*\text{csgn}(I*c*x^n)^3+I*\text{Pi}*b^2*\text{csgn}(I*c*x^n)^2*\text{csgn}(I*c)+2*\ln(c)*b^2+b^2*n+2*a*b)/x^2*\ln(x^n)-1/8*(4*I*\ln(c)*\text{Pi}*b^2*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^2+4*\ln(c)^2*b^2-\text{Pi}^2*b^2*\text{csgn}(I*c*x^n)^4*\text{csgn}(I*c)^2+4*a*b*n+2*b^2*n^2+4*a^2+2*\text{Pi}^2*b^2*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^3*\text{csgn}(I*c)^2+2*\text{Pi}^2*b^2*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^3*\text{csgn}(I*c)-\text{Pi}^2*b^2*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^2*\text{csgn}(I*c)^2-4*\text{Pi}^2*b^2*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^4*\text{csgn}(I*c)-2*I*\text{Pi}*b^2*n*\text{csgn}(I*c*x^n)^3+4*I*\ln(c)*\text{Pi}*b^2*\text{csgn}(I*c*x^n)^2*\text{csgn}(I*c)+4*I*\text{Pi}*a*b*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^2+4*I*\text{Pi}*a*b*\text{csgn}(I*c*x^n)^2*\text{csgn}(I*c)-2*I*\text{Pi}*b^2*n*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)*\text{csgn}(I*c)-4*I*\text{Pi}*a*b*\text{csgn}(I*c*x^n)^3-4*I*\ln(c)*\text{Pi}*b^2*\text{csgn}(I*c*x^n)^3-4*I*\ln(c)*\text{Pi}*b^2*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)*\text{csgn}(I*c)-4*I*\text{Pi}*a*b*\text{csgn}(I*c*x^n)^4*\text{csgn}(I*c)-\text{Pi}^2*b^2*\text{csgn}(I*c*x^n)^6+8*\ln(c)*a*b+4*\ln(c)*b^2*n+2*\text{Pi}^2*b^2*\text{csgn}(I*c*x^n)^5*\text{csgn}(I*c)+2*\text{Pi}^2*b^2*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^5+2*I*\text{Pi}*b^2*n*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^2+2*I*\text{Pi}*b^2*n*\text{csgn}(I*c*x^n)^2*\text{csgn}(I*c)-\text{Pi}^2*b^2*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^4)/x^2$

**Maxima [A]** time = 1.15863, size = 96, normalized size = 1.85

$$-\frac{1}{4}b^2\left(\frac{n^2}{x^2} + \frac{2n \log(cx^n)}{x^2}\right) - \frac{b^2 \log(cx^n)^2}{2x^2} - \frac{abn}{2x^2} - \frac{ab \log(cx^n)}{x^2} - \frac{a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a+b*\log(c*x^n))^2/x^3,x, \text{algorithm}="maxima")$

[Out]  $-1/4*b^2*(n^2/x^2 + 2*n*\log(c*x^n)/x^2) - 1/2*b^2*\log(c*x^n)^2/x^2 - 1/2*a*b*n/x^2 - a*b*\log(c*x^n)/x^2 - 1/2*a^2/x^2$

**Fricas [A]** time = 0.863724, size = 204, normalized size = 3.92

$$\frac{2b^2n^2 \log(x)^2 + b^2n^2 + 2b^2 \log(c)^2 + 2abn + 2a^2 + 2(b^2n + 2ab) \log(c) + 2(b^2n^2 + 2b^2n \log(c) + 2abn) \log(x)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a+b*\log(c*x^n))^2/x^3,x, \text{algorithm}="fricas")$

[Out]  $-1/4*(2*b^2*n^2*\log(x)^2 + b^2*n^2 + 2*b^2*\log(c)^2 + 2*a*b*n + 2*a^2 + 2*(b^2*n + 2*a*b)*\log(c) + 2*(b^2*n^2 + 2*b^2*n*\log(c) + 2*a*b*n)*\log(x))/x^2$

---

**Sympy [B]** time = 1.74155, size = 128, normalized size = 2.46

$$\frac{a^2}{2x^2} - \frac{abn \log(x)}{x^2} - \frac{abn}{2x^2} - \frac{ab \log(c)}{x^2} - \frac{b^2 n^2 \log(x)^2}{2x^2} - \frac{b^2 n^2 \log(x)}{2x^2} - \frac{b^2 n^2}{4x^2} - \frac{b^2 n \log(c) \log(x)}{x^2} - \frac{b^2 n \log(c)}{2x^2} - \frac{b^2 n}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))\*\*2/x\*\*3,x)

[Out] -a\*\*2/(2\*x\*\*2) - a\*b\*n\*log(x)/x\*\*2 - a\*b\*n/(2\*x\*\*2) - a\*b\*log(c)/x\*\*2 - b\*\*2\*n\*\*2\*log(x)\*\*2/(2\*x\*\*2) - b\*\*2\*n\*\*2\*log(x)/(2\*x\*\*2) - b\*\*2\*n\*\*2/(4\*x\*\*2) - b\*\*2\*n\*log(c)\*log(x)/x\*\*2 - b\*\*2\*n\*log(c)/(2\*x\*\*2) - b\*\*2\*log(c)\*\*2/(2\*x\*\*2)

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**Giac [A]** time = 1.23047, size = 122, normalized size = 2.35

$$\frac{b^2 n^2 \log(x)^2}{2x^2} - \frac{(b^2 n^2 + 2b^2 n \log(c) + 2abn) \log(x)}{2x^2} - \frac{b^2 n^2 + 2b^2 n \log(c) + 2b^2 \log(c)^2 + 2abn + 4ab \log(c) + 2a^2}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^2/x^3,x, algorithm="giac")

[Out] -1/2\*b^2\*n^2\*log(x)^2/x^2 - 1/2\*(b^2\*n^2 + 2\*b^2\*n\*log(c) + 2\*a\*b\*n)\*log(x)/x^2 - 1/4\*(b^2\*n^2 + 2\*b^2\*n\*log(c) + 2\*b^2\*log(c)^2 + 2\*a\*b\*n + 4\*a\*b\*log(c) + 2\*a^2)/x^2

### 3.57 $\int x^3 (a + b \log(cx^n))^3 dx$

**Optimal.** Leaf size=77

$$\frac{3}{32}b^2n^2x^4(a + b \log(cx^n)) + \frac{1}{4}x^4(a + b \log(cx^n))^3 - \frac{3}{16}bnx^4(a + b \log(cx^n))^2 - \frac{3}{128}b^3n^3x^4$$

[Out]  $(-3*b^3*n^3*x^4)/128 + (3*b^2*n^2*x^4*(a + b*Log[c*x^n]))/32 - (3*b*n*x^4*(a + b*Log[c*x^n])^2)/16 + (x^4*(a + b*Log[c*x^n])^3)/4$

**Rubi [A]** time = 0.0626123, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2305, 2304}

$$\frac{3}{32}b^2n^2x^4(a + b \log(cx^n)) + \frac{1}{4}x^4(a + b \log(cx^n))^3 - \frac{3}{16}bnx^4(a + b \log(cx^n))^2 - \frac{3}{128}b^3n^3x^4$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*Log[c\*x^n])^3,x]

[Out]  $(-3*b^3*n^3*x^4)/128 + (3*b^2*n^2*x^4*(a + b*Log[c*x^n]))/32 - (3*b*n*x^4*(a + b*Log[c*x^n])^2)/16 + (x^4*(a + b*Log[c*x^n])^3)/4$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int x^3 (a + b \log(cx^n))^3 dx &= \frac{1}{4}x^4 (a + b \log(cx^n))^3 - \frac{1}{4}(3bn) \int x^3 (a + b \log(cx^n))^2 dx \\ &= -\frac{3}{16}bnx^4 (a + b \log(cx^n))^2 + \frac{1}{4}x^4 (a + b \log(cx^n))^3 + \frac{1}{8}(3b^2n^2) \int x^3 (a + b \log(cx^n)) dx \\ &= -\frac{3}{128}b^3n^3x^4 + \frac{3}{32}b^2n^2x^4 (a + b \log(cx^n)) - \frac{3}{16}bnx^4 (a + b \log(cx^n))^2 + \frac{1}{4}x^4 (a + b \log(cx^n)) \end{aligned}$$

**Mathematica [A]** time = 0.0292536, size = 66, normalized size = 0.86

$$\frac{1}{4} \left( x^4 (a + b \log(cx^n))^3 - \frac{3}{32}bnx^4 (-4bn(a + b \log(cx^n)) + 8(a + b \log(cx^n))^2 + b^2n^2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*Log[c\*x^n])^3,x]



[Out]  $(x^4*(a + b*\text{Log}[c*x^n])^3 - (3*b*n*x^4*(b^2*n^2 - 4*b*n*(a + b*\text{Log}[c*x^n]) + 8*(a + b*\text{Log}[c*x^n])^2))/32)/4$

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**Maple [C]** time = 0.318, size = 2649, normalized size = 34.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^3*(a+b*\ln(c*x^n))^3, x)$

[Out]  $\frac{1}{4}b^3x^4\ln(x^n)^3 + \frac{3}{16}b^2x^4(2Ib\pi\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^2 - 2Ib\pi\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)\text{csgn}(I*c) - 2Ib\pi\text{csgn}(I*c*x^n)^3 + 2Ib\pi\text{csgn}(I*c*x^n)^2\text{csgn}(I*c) + 4b*\ln(c) - b*n + 4a)*\ln(x^n)^2 + \frac{3}{32}b*x^4(8*\ln(c)^2*b^2 - 2\pi^2*b^2\text{csgn}(I*c*x^n)^4\text{csgn}(I*c)^2 - 4a*b*n + b^2*n^2 + 8a^2 - 2I\pi*b^2*n*\text{csgn}(I*c*x^n)^2\text{csgn}(I*c) + 8I*\ln(c)*\pi*b^2*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^2 + 8I\pi*a*b*\text{csgn}(I*c*x^n)^2\text{csgn}(I*c) - 2I\pi*b^2*n*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^2 + 8I*\ln(c)*\pi*b^2*\text{csgn}(I*c*x^n)^2\text{csgn}(I*c) + 8I\pi*a*b*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^2 + 4\pi^2*b^2*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^3\text{csgn}(I*c)^2 + 4\pi^2*b^2*\text{csgn}(I*x^n)^2\text{csgn}(I*c*x^n)^3\text{csgn}(I*c) - 2\pi^2*b^2*\text{csgn}(I*x^n)^2\text{csgn}(I*c*x^n)^2\text{csgn}(I*c)^2 - 8\pi^2*b^2*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^4\text{csgn}(I*c) - 8I*\ln(c)*\pi*b^2*\text{csgn}(I*c*x^n)^3 - 8I\pi*a*b*\text{csgn}(I*c*x^n)^3 + 2I\pi*b^2*n*\text{csgn}(I*c*x^n)^3 - 8I*\ln(c)*\pi*b^2*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)\text{csgn}(I*c) - 8I\pi*a*b*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)\text{csgn}(I*c) - 2\pi^2*b^2*\text{csgn}(I*c*x^n)^6 + 16*\ln(c)*a*b - 4*\ln(c)*b^2*n + 2I\pi*b^2*n*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)\text{csgn}(I*c) + 4\pi^2*b^2*\text{csgn}(I*c*x^n)^5\text{csgn}(I*c) + 4\pi^2*b^2*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^5 - 2\pi^2*b^2*\text{csgn}(I*x^n)^2\text{csgn}(I*c*x^n)^4)*\ln(x^n) + \frac{1}{128}x^4(32*a^3 + 12*a*b^2*n^2 - 24*a^2*b*n - 6*I\pi*b^3*n^2*\text{csgn}(I*c*x^n)^3 - 4I\pi^3*b^3*\text{csgn}(I*x^n)^3*\text{csgn}(I*c*x^n)^6 + 12I\pi^3*b^3*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^7 - 24\pi^2*a*b^2*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^4 + 24I*\ln(c)*\pi*b^3*n*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)\text{csgn}(I*c) + 32*\ln(c)^3*b^3 - 96\pi^2*a*b^2*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^4\text{csgn}(I*c) + 24I*\ln(c)*\pi*b^3*n*\text{csgn}(I*c*x^n)^3 + 6I\pi*b^3*n^2*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^2 + 6I\pi*b^3*n^2*\text{csgn}(I*c*x^n)^2*\text{csgn}(I*c) + 24I\pi*a*b^2*n*\text{csgn}(I*c*x^n)^3 - 12\pi^2*b^3*n*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^3*\text{csgn}(I*c) + 6\pi^2*b^3*n*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^2*\text{csgn}(I*c)^2 + 24\pi^2*b^3*n*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^4*\text{csgn}(I*c) + 12I\pi^3*b^3*\text{csgn}(I*c*x^n)^7*\text{csgn}(I*c)^2 - 4I\pi^3*b^3*\text{csgn}(I*c*x^n)^6*\text{csgn}(I*c)^3 - 48I*\ln(c)^2*\pi*b^3*\text{csgn}(I*c*x^n)^3 - 48I\pi*a^2*b*\text{csgn}(I*c*x^n)^3 - 24*\ln(c)*\pi^2*b^3*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^4 + 48*\ln(c)*\pi^2*b^3*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^5 + 48*\ln(c)*\pi^2*b^3*\text{csgn}(I*c*x^n)^5*\text{csgn}(I*c) - 12I\pi^3*b^3*\text{csgn}(I*c*x^n)^8*\text{csgn}(I*c) - 12\pi^2*b^3*n*\text{csgn}(I*c*x^n)^5*\text{csgn}(I*c) + 6\pi^2*b^3*n*\text{csgn}(I*c*x^n)^4*\text{csgn}(I*c)^2 - 3*b^3*n^3 + 24I\pi*a*b^2*n*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)\text{csgn}(I*c) - 36I\pi^3*b^3*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^6*\text{csgn}(I*c)^2 + 12I\pi^3*b^3*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^5*\text{csgn}(I*c)^3 + 48I*\ln(c)^2*\pi*b^3*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^2 + 48I*\ln(c)^2*\pi*b^3*\text{csgn}(I*c*x^n)^2*\text{csgn}(I*c) - 12I\pi^3*b^3*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^8 + 48\pi^2*a*b^2*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^5 - 24*\ln(c)^2*b^3*n + 12*\ln(c)*b^3*n^2 + 96*\ln(c)*a^2*b + 96*\ln(c)^2*a*b^2 + 48\pi^2*a*b^2*\text{csgn}(I*c*x^n)^5*\text{csgn}(I*c) - 24\pi^2*a*b^2*\text{csgn}(I*c*x^n)^4*\text{csgn}(I*c)^2 - 24*\ln(c)*\pi^2*b^3*\text{csgn}(I*c*x^n)^4*\text{csgn}(I*c)^2 + 6\pi^2*b^3*n*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^4 - 12\pi^2*b^3*n*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^5 + 6\pi^2*b^3*n*\text{csgn}(I*c*x^n)^6 + 4I\pi^3*b^3*\text{csgn}(I*c*x^n)^9 - 24*\ln(c)*\pi^2*b^3*\text{csgn}(I*c*x^n)^6 - 24\pi^2*a*b^2*\text{csgn}(I*c*x^n)^6 - 96I*\ln(c)*\pi*a*b^2*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)\text{csgn}(I*c) - 36I\pi^3*b^3*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^6*\text{csgn}(I*c) + 36I\pi^3*b^3*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^5*\text{csgn}(I*c)^2 - 12I\pi^3*b^3*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^4*\text{csgn}(I*c)^3 + 36I\pi^3*b^3*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^7*\text{csgn}(I*c) - 96I*\ln(c)*\pi*a*b^2*\text{csgn}(I*c*x^n)^3 + 48*\ln(c)*\pi^2*b^3*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^3*\text{csgn}(I*c)^2 + 48I\pi*a^2*b*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^2 + 48I\pi*a^2*b*\text{csgn}(I*c*x^n)^2*\text{csgn}(I*c) + 48\pi^2*a*b^2*\text{csgn}(I*x^n)\text{csgn}(I*c*x^n)^3*c$

```
sgn(I*c)^2+12*I*Pi^3*b^3*csgn(I*x^n)^3*csgn(I*c*x^n)^5*csgn(I*c)-12*I*Pi^3*
b^3*csgn(I*x^n)^3*csgn(I*c*x^n)^4*csgn(I*c)^2+4*I*Pi^3*b^3*csgn(I*x^n)^3*cs
gn(I*c*x^n)^3*csgn(I*c)^3-24*ln(c)*Pi^2*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^2*c
sgn(I*c)^2-96*ln(c)*Pi^2*b^3*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)-12*Pi^2*
b^3*n*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2+48*ln(c)*Pi^2*b^3*csgn(I*x^n)
^2*csgn(I*c*x^n)^3*csgn(I*c)+48*Pi^2*a*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^3*cs
gn(I*c)-24*Pi^2*a*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2-48*ln(c)*a*
b^2*n-24*I*ln(c)*Pi*b^3*n*csgn(I*c*x^n)^2*csgn(I*c)+96*I*ln(c)*Pi*a*b^2*csg
n(I*c*x^n)^2*csgn(I*c)-48*I*Pi*a^2*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-24
*I*Pi*a*b^2*n*csgn(I*c*x^n)^2*csgn(I*c)-48*I*ln(c)^2*Pi*b^3*csgn(I*x^n)*csg
n(I*c*x^n)*csgn(I*c)+96*I*ln(c)*Pi*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2-6*I*Pi
*b^3*n^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-24*I*Pi*a*b^2*n*csgn(I*x^n)*cs
gn(I*c*x^n)^2-24*I*ln(c)*Pi*b^3*n*csgn(I*x^n)*csgn(I*c*x^n)^2)
```

**Maxima [A]** time = 1.13463, size = 182, normalized size = 2.36

$$\frac{1}{4}b^3x^4 \log(cx^n)^3 + \frac{3}{4}ab^2x^4 \log(cx^n)^2 - \frac{3}{16}a^2bnx^4 + \frac{3}{4}a^2bx^4 \log(cx^n) + \frac{1}{4}a^3x^4 + \frac{3}{32}(n^2x^4 - 4nx^4 \log(cx^n))ab^2 - \frac{3}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4*log(c*x^n)^3 + 3/4*a*b^2*x^4*log(c*x^n)^2 - 3/16*a^2*b*n*x^4 +
3/4*a^2*b*x^4*log(c*x^n) + 1/4*a^3*x^4 + 3/32*(n^2*x^4 - 4*n*x^4*log(c*x^n)
)*a*b^2 - 3/128*(8*n*x^4*log(c*x^n)^2 + (n^2*x^4 - 4*n*x^4*log(c*x^n))*n)*b
^3
```

**Fricas [B]** time = 0.866165, size = 517, normalized size = 6.71

$$\frac{1}{4}b^3n^3x^4 \log(x)^3 + \frac{1}{4}b^3x^4 \log(c)^3 - \frac{3}{16}(b^3n - 4ab^2)x^4 \log(c)^2 + \frac{3}{32}(b^3n^2 - 4ab^2n + 8a^2b)x^4 \log(c) - \frac{1}{128}(3b^3n^3 - 12$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] 1/4*b^3*n^3*x^4*log(x)^3 + 1/4*b^3*x^4*log(c)^3 - 3/16*(b^3*n - 4*a*b^2)*x^
4*log(c)^2 + 3/32*(b^3*n^2 - 4*a*b^2*n + 8*a^2*b)*x^4*log(c) - 1/128*(3*b^3
*n^3 - 12*a*b^2*n^2 + 24*a^2*b*n - 32*a^3)*x^4 + 3/16*(4*b^3*n^2*x^4*log(c)
- (b^3*n^3 - 4*a*b^2*n^2)*x^4)*log(x)^2 + 3/32*(8*b^3*n*x^4*log(c)^2 - 4*(
b^3*n^2 - 4*a*b^2*n)*x^4*log(c) + (b^3*n^3 - 4*a*b^2*n^2 + 8*a^2*b*n)*x^4)*
log(x)
```

**Sympy [B]** time = 7.59318, size = 338, normalized size = 4.39

$$\frac{a^3x^4}{4} + \frac{3a^2bnx^4 \log(x)}{4} - \frac{3a^2bnx^4}{16} + \frac{3a^2bx^4 \log(c)}{4} + \frac{3ab^2n^2x^4 \log(x)^2}{4} - \frac{3ab^2n^2x^4 \log(x)}{8} + \frac{3ab^2n^2x^4}{32} + \frac{3ab^2nx^4 \log(c)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*ln(c*x**n))**3,x)
```

```
[Out] a**3*x**4/4 + 3*a**2*b*n*x**4*log(x)/4 - 3*a**2*b*n*x**4/16 + 3*a**2*b*x**4
*log(c)/4 + 3*a*b**2*n**2*x**4*log(x)**2/4 - 3*a*b**2*n**2*x**4*log(x)/8 +
3*a*b**2*n**2*x**4/32 + 3*a*b**2*n*x**4*log(c)*log(x)/2 - 3*a*b**2*n*x**4*log(c)/8 +
3*a*b**2*x**4*log(c)**2/4 + b**3*n**3*x**4*log(x)**3/4 - 3*b**3*n
**3*x**4*log(x)**2/16 + 3*b**3*n**3*x**4*log(x)/32 - 3*b**3*n**3*x**4/128 +
3*b**3*n**2*x**4*log(c)*log(x)**2/4 - 3*b**3*n**2*x**4*log(c)*log(x)/8 + 3
*b**3*n**2*x**4*log(c)/32 + 3*b**3*n*x**4*log(c)**2*log(x)/4 - 3*b**3*n*x**
4*log(c)**2/16 + b**3*x**4*log(c)**3/4
```

**Giac [B]** time = 1.26319, size = 354, normalized size = 4.6

$$\frac{1}{4} b^3 n^3 x^4 \log(x)^3 - \frac{3}{16} b^3 n^3 x^4 \log(x)^2 + \frac{3}{4} b^3 n^2 x^4 \log(c) \log(x)^2 + \frac{3}{32} b^3 n^3 x^4 \log(x) - \frac{3}{8} b^3 n^2 x^4 \log(c) \log(x) + \frac{3}{4} b^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] 1/4*b^3*n^3*x^4*log(x)^3 - 3/16*b^3*n^3*x^4*log(x)^2 + 3/4*b^3*n^2*x^4*log(
c)*log(x)^2 + 3/32*b^3*n^3*x^4*log(x) - 3/8*b^3*n^2*x^4*log(c)*log(x) + 3/4
*b^3*n*x^4*log(c)^2*log(x) + 3/4*a*b^2*n^2*x^4*log(x)^2 - 3/128*b^3*n^3*x^4
+ 3/32*b^3*n^2*x^4*log(c) - 3/16*b^3*n*x^4*log(c)^2 + 1/4*b^3*x^4*log(c)^3
- 3/8*a*b^2*n^2*x^4*log(x) + 3/2*a*b^2*n*x^4*log(c)*log(x) + 3/32*a*b^2*n^
2*x^4 - 3/8*a*b^2*n*x^4*log(c) + 3/4*a*b^2*x^4*log(c)^2 + 3/4*a^2*b*n*x^4*log(x)
- 3/16*a^2*b*n*x^4 + 3/4*a^2*b*x^4*log(c) + 1/4*a^3*x^4
```

### 3.58 $\int x^2 (a + b \log(cx^n))^3 dx$

**Optimal.** Leaf size=77

$$\frac{2}{9}b^2n^2x^3(a + b \log(cx^n)) + \frac{1}{3}x^3(a + b \log(cx^n))^3 - \frac{1}{3}bnx^3(a + b \log(cx^n))^2 - \frac{2}{27}b^3n^3x^3$$

[Out]  $(-2*b^3*n^3*x^3)/27 + (2*b^2*n^2*x^3*(a + b*Log[c*x^n]))/9 - (b*n*x^3*(a + b*Log[c*x^n])^2)/3 + (x^3*(a + b*Log[c*x^n])^3)/3$

**Rubi [A]** time = 0.0600692, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2305, 2304}

$$\frac{2}{9}b^2n^2x^3(a + b \log(cx^n)) + \frac{1}{3}x^3(a + b \log(cx^n))^3 - \frac{1}{3}bnx^3(a + b \log(cx^n))^2 - \frac{2}{27}b^3n^3x^3$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*Log[c\*x^n])^3,x]

[Out]  $(-2*b^3*n^3*x^3)/27 + (2*b^2*n^2*x^3*(a + b*Log[c*x^n]))/9 - (b*n*x^3*(a + b*Log[c*x^n])^2)/3 + (x^3*(a + b*Log[c*x^n])^3)/3$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int x^2 (a + b \log(cx^n))^3 dx &= \frac{1}{3}x^3 (a + b \log(cx^n))^3 - (bn) \int x^2 (a + b \log(cx^n))^2 dx \\ &= -\frac{1}{3}bnx^3 (a + b \log(cx^n))^2 + \frac{1}{3}x^3 (a + b \log(cx^n))^3 + \frac{1}{3}(2b^2n^2) \int x^2 (a + b \log(cx^n)) dx \\ &= -\frac{2}{27}b^3n^3x^3 + \frac{2}{9}b^2n^2x^3 (a + b \log(cx^n)) - \frac{1}{3}bnx^3 (a + b \log(cx^n))^2 + \frac{1}{3}x^3 (a + b \log(cx^n))^3 \end{aligned}$$

**Mathematica [A]** time = 0.0142996, size = 67, normalized size = 0.87

$$\frac{1}{3} \left( x^3 (a + b \log(cx^n))^3 - bn \left( x^3 (a + b \log(cx^n))^2 + \frac{2}{9}bnx^3 (-3a - 3b \log(cx^n) + bn) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*Log[c\*x^n])^3,x]

```
[Out] (x^3*(a + b*Log[c*x^n])^3 - b*n*((2*b*n*x^3*(-3*a + b*n - 3*b*Log[c*x^n]))/
9 + x^3*(a + b*Log[c*x^n])^2))/3
```

---

**Maple [C]** time = 0.332, size = 2650, normalized size = 34.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*x^n))^3,x)
```

```
[Out] 1/3*b^3*x^3*ln(x^n)^3+1/6*b^2*x^3*(3*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-3*I
*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-3*I*b*Pi*csgn(I*c*x^n)^3+3*I*b*Pi
*csgn(I*c*x^n)^2*csgn(I*c)+6*b*ln(c)-2*b*n+6*a)*ln(x^n)^2+1/36*b*x^3*(36*ln
(c)^2*b^2-9*Pi^2*b^2*csgn(I*c*x^n)^4*csgn(I*c)^2-24*a*b*n-36*I*ln(c)*Pi*b^2
*csgn(I*c*x^n)^3-36*I*Pi*a*b*csgn(I*c*x^n)^3+12*I*Pi*b^2*n*csgn(I*c*x^n)^3+
8*b^2*n^2+36*a^2+18*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2+18*Pi^
2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-9*Pi^2*b^2*csgn(I*x^n)^2*csgn
(I*c*x^n)^2*csgn(I*c)^2-36*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)-9
*Pi^2*b^2*csgn(I*c*x^n)^6+72*ln(c)*a*b-24*ln(c)*b^2*n+18*Pi^2*b^2*csgn(I*c*
x^n)^5*csgn(I*c)+18*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^5+36*I*ln(c)*Pi*b^2*
csgn(I*c*x^n)^2*csgn(I*c)+36*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)^2+36*I*Pi*a
*b*csgn(I*c*x^n)^2*csgn(I*c)-12*I*Pi*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)^2-12*I
*Pi*b^2*n*csgn(I*c*x^n)^2*csgn(I*c)+36*I*ln(c)*Pi*b^2*csgn(I*x^n)*csgn(I*c*
x^n)^2-9*Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4-36*I*Pi*a*b*csgn(I*x^n)*csg
n(I*c*x^n)*csgn(I*c)+12*I*Pi*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-36*I
*ln(c)*Pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c))*ln(x^n)+1/216*x^3*(72*a^
3+48*a*b^2*n^2-72*a^2*b*n-54*Pi^2*a*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4+72*ln
(c)^3*b^3-216*Pi^2*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)-36*Pi^2*b^3*
n*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)+18*Pi^2*b^3*n*csgn(I*x^n)^2*csgn(
I*c*x^n)^2*csgn(I*c)^2+72*Pi^2*b^3*n*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)-
54*ln(c)*Pi^2*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^4+108*ln(c)*Pi^2*b^3*csgn(I*x
^n)*csgn(I*c*x^n)^5+108*ln(c)*Pi^2*b^3*csgn(I*c*x^n)^5*csgn(I*c)-36*Pi^2*b^
3*n*csgn(I*c*x^n)^5*csgn(I*c)+18*Pi^2*b^3*n*csgn(I*c*x^n)^4*csgn(I*c)^2+72*
I*ln(c)*Pi*b^3*n*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+72*I*Pi*a*b^2*n*csgn(I
*x^n)*csgn(I*c*x^n)*csgn(I*c)-216*I*ln(c)*Pi*a*b^2*csgn(I*x^n)*csgn(I*c*x^n
)*csgn(I*c)-16*b^3*n^3+9*I*Pi^3*b^3*csgn(I*c*x^n)^9+108*Pi^2*a*b^2*csgn(I*x
^n)*csgn(I*c*x^n)^5-108*I*ln(c)^2*Pi*b^3*csgn(I*c*x^n)^3-108*I*Pi*a^2*b*csg
n(I*c*x^n)^3-24*I*Pi*b^3*n^2*csgn(I*c*x^n)^3-9*I*Pi^3*b^3*csgn(I*x^n)^3*csg
n(I*c*x^n)^6-27*I*Pi^3*b^3*csgn(I*x^n)*csgn(I*c*x^n)^8-27*I*Pi^3*b^3*csgn(I
*c*x^n)^8*csgn(I*c)+27*I*Pi^3*b^3*csgn(I*c*x^n)^7*csgn(I*c)^2-9*I*Pi^3*b^3*
csgn(I*c*x^n)^6*csgn(I*c)^3-72*ln(c)^2*b^3*n+48*ln(c)*b^3*n^2+216*ln(c)*a^2
*b+216*ln(c)^2*a*b^2+108*Pi^2*a*b^2*csgn(I*c*x^n)^5*csgn(I*c)-54*Pi^2*a*b^2
*csgn(I*c*x^n)^4*csgn(I*c)^2-54*ln(c)*Pi^2*b^3*csgn(I*c*x^n)^4*csgn(I*c)^2+
18*Pi^2*b^3*n*csgn(I*x^n)^2*csgn(I*c*x^n)^4-36*Pi^2*b^3*n*csgn(I*x^n)*csgn(
I*c*x^n)^5+18*Pi^2*b^3*n*csgn(I*c*x^n)^6-54*ln(c)*Pi^2*b^3*csgn(I*c*x^n)^6-
54*Pi^2*a*b^2*csgn(I*c*x^n)^6+27*I*Pi^3*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^7+2
7*I*Pi^3*b^3*csgn(I*x^n)^3*csgn(I*c*x^n)^5*csgn(I*c)-27*I*Pi^3*b^3*csgn(I*x
^n)^3*csgn(I*c*x^n)^4*csgn(I*c)^2+9*I*Pi^3*b^3*csgn(I*x^n)^3*csgn(I*c*x^n)^
3*csgn(I*c)^3-81*I*Pi^3*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^6*csgn(I*c)+108*ln(
c)*Pi^2*b^3*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2+108*Pi^2*a*b^2*csgn(I*x
^n)*csgn(I*c*x^n)^3*csgn(I*c)^2-54*ln(c)*Pi^2*b^3*csgn(I*x^n)^2*csgn(I*c*x^
n)^2*csgn(I*c)^2-216*ln(c)*Pi^2*b^3*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)-3
6*Pi^2*b^3*n*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2+108*ln(c)*Pi^2*b^3*csg
n(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)+108*Pi^2*a*b^2*csgn(I*x^n)^2*csgn(I*c*
x^n)^3*csgn(I*c)-54*Pi^2*a*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2-24
*I*Pi*b^3*n^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-72*I*Pi*a*b^2*n*csgn(I*x^
```

$n) * \text{csgn}(I * c * x^n)^2 + 24 * I * \text{Pi} * b^3 * n^2 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^2 + 24 * I * \text{Pi} * b^3 * n^2 * \text{csgn}(I * c * x^n)^2 * \text{csgn}(I * c) + 72 * I * \text{Pi} * a * b^2 * n * \text{csgn}(I * c * x^n)^3 - 216 * I * \ln(c) * \text{Pi} * a * b^2 * \text{csgn}(I * c * x^n)^3 + 108 * I * \text{Pi} * a^2 * b * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^2 + 108 * I * \text{Pi} * a^2 * b * \text{csgn}(I * c * x^n)^2 * \text{csgn}(I * c) + 72 * I * \ln(c) * \text{Pi} * b^3 * n * \text{csgn}(I * c * x^n)^3 - 81 * I * \text{Pi} * i^3 * b^3 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^6 * \text{csgn}(I * c)^2 + 27 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^5 * \text{csgn}(I * c)^3 + 108 * I * \ln(c)^2 * \text{Pi} * b^3 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^2 + 108 * I * \ln(c)^2 * \text{Pi} * b^3 * \text{csgn}(I * c * x^n)^2 * \text{csgn}(I * c) - 144 * \ln(c) * a * b^2 * n + 81 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^5 * \text{csgn}(I * c)^2 - 27 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^4 * \text{csgn}(I * c)^3 + 81 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^7 * \text{csgn}(I * c) - 72 * I * \ln(c) * \text{Pi} * b^3 * n * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^2 - 108 * I * \ln(c)^2 * \text{Pi} * b^3 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n) * \text{csgn}(I * c) - 108 * I * \text{Pi} * a^2 * b * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n) * \text{csgn}(I * c) - 72 * I * \ln(c) * \text{Pi} * b^3 * n * \text{csgn}(I * c * x^n)^2 * \text{csgn}(I * c) + 216 * I * \ln(c) * \text{Pi} * a * b^2 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^2 + 216 * I * \ln(c) * \text{Pi} * a * b^2 * \text{csgn}(I * c * x^n)^2 * \text{csgn}(I * c) - 72 * I * \text{Pi} * a * b^2 * n * \text{csgn}(I * c * x^n)^2 * \text{csgn}(I * c)$

**Maxima [A]** time = 1.18369, size = 181, normalized size = 2.35

$$\frac{1}{3} b^3 x^3 \log(cx^n)^3 + ab^2 x^3 \log(cx^n)^2 - \frac{1}{3} a^2 b n x^3 + a^2 b x^3 \log(cx^n) + \frac{1}{3} a^3 x^3 + \frac{2}{9} (n^2 x^3 - 3 n x^3 \log(cx^n)) a b^2 - \frac{1}{27} (9 n x^3 \log(cx^n)^2 - 6 n^2 x^3 \log(cx^n) + 2 n^3) a b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out] 1/3\*b^3\*x^3\*log(c\*x^n)^3 + a\*b^2\*x^3\*log(c\*x^n)^2 - 1/3\*a^2\*b\*n\*x^3 + a^2\*b\*x^3\*log(c\*x^n) + 1/3\*a^3\*x^3 + 2/9\*(n^2\*x^3 - 3\*n\*x^3\*log(c\*x^n))\*a\*b^2 - 1/27\*(9\*n\*x^3\*log(c\*x^n)^2 + 2\*(n^2\*x^3 - 3\*n\*x^3\*log(c\*x^n))\*n)\*b^3

**Fricas [B]** time = 0.954472, size = 512, normalized size = 6.65

$$\frac{1}{3} b^3 n^3 x^3 \log(x)^3 + \frac{1}{3} b^3 x^3 \log(c)^3 - \frac{1}{3} (b^3 n - 3 a b^2) x^3 \log(c)^2 + \frac{1}{9} (2 b^3 n^2 - 6 a b^2 n + 9 a^2 b) x^3 \log(c) - \frac{1}{27} (2 b^3 n^3 - 6 a b^2 n^2 + 9 a^2 b n - 9 a^3) x^3 \log(c) - \frac{1}{27} (2 b^3 n^3 - 6 a b^2 n^2 + 9 a^2 b n - 9 a^3) x^3 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out] 1/3\*b^3\*n^3\*x^3\*log(x)^3 + 1/3\*b^3\*x^3\*log(c)^3 - 1/3\*(b^3\*n - 3\*a\*b^2)\*x^3\*log(c)^2 + 1/9\*(2\*b^3\*n^2 - 6\*a\*b^2\*n + 9\*a^2\*b)\*x^3\*log(c) - 1/27\*(2\*b^3\*n^3 - 6\*a\*b^2\*n^2 + 9\*a^2\*b\*n - 9\*a^3)\*x^3 + 1/3\*(3\*b^3\*n^2\*x^3\*log(c) - (b^3\*n^3 - 3\*a\*b^2\*n^2)\*x^3)\*log(x)^2 + 1/9\*(9\*b^3\*n\*x^3\*log(c)^2 - 6\*(b^3\*n^2 - 3\*a\*b^2\*n)\*x^3\*log(c) + (2\*b^3\*n^3 - 6\*a\*b^2\*n^2 + 9\*a^2\*b\*n)\*x^3)\*log(x)

**Sympy [B]** time = 4.03597, size = 311, normalized size = 4.04

$$\frac{a^3 x^3}{3} + a^2 b n x^3 \log(x) - \frac{a^2 b n x^3}{3} + a^2 b x^3 \log(c) + a b^2 n^2 x^3 \log(x)^2 - \frac{2 a b^2 n^2 x^3 \log(x)}{3} + \frac{2 a b^2 n^2 x^3}{9} + 2 a b^2 n x^3 \log(c) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*ln(c\*x\*\*n))\*\*3,x)

```
[Out] a**3*x**3/3 + a**2*b*n*x**3*log(x) - a**2*b*n*x**3/3 + a**2*b*x**3*log(c) +
a*b**2*n**2*x**3*log(x)**2 - 2*a*b**2*n**2*x**3*log(x)/3 + 2*a*b**2*n**2*x
**3/9 + 2*a*b**2*n*x**3*log(c)*log(x) - 2*a*b**2*n*x**3*log(c)/3 + a*b**2*x
**3*log(c)**2 + b**3*n**3*x**3*log(x)**3/3 - b**3*n**3*x**3*log(x)**2/3 + 2
*b**3*n**3*x**3*log(x)/9 - 2*b**3*n**3*x**3/27 + b**3*n**2*x**3*log(c)*log(
x)**2 - 2*b**3*n**2*x**3*log(c)*log(x)/3 + 2*b**3*n**2*x**3*log(c)/9 + b**3
*n*x**3*log(c)**2*log(x) - b**3*n*x**3*log(c)**2/3 + b**3*x**3*log(c)**3/3
```

**Giac [B]** time = 1.29104, size = 346, normalized size = 4.49

$$\frac{1}{3} b^3 n^3 x^3 \log(x)^3 - \frac{1}{3} b^3 n^3 x^3 \log(x)^2 + b^3 n^2 x^3 \log(c) \log(x)^2 + \frac{2}{9} b^3 n^3 x^3 \log(x) - \frac{2}{3} b^3 n^2 x^3 \log(c) \log(x) + b^3 n x^3 \log(c)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] 1/3*b^3*n^3*x^3*log(x)^3 - 1/3*b^3*n^3*x^3*log(x)^2 + b^3*n^2*x^3*log(c)*lo
g(x)^2 + 2/9*b^3*n^3*x^3*log(x) - 2/3*b^3*n^2*x^3*log(c)*log(x) + b^3*n*x^3
*log(c)^2*log(x) + a*b^2*n^2*x^3*log(x)^2 - 2/27*b^3*n^3*x^3 + 2/9*b^3*n^2*
x^3*log(c) - 1/3*b^3*n*x^3*log(c)^2 + 1/3*b^3*x^3*log(c)^3 - 2/3*a*b^2*n^2*
x^3*log(x) + 2*a*b^2*n*x^3*log(c)*log(x) + 2/9*a*b^2*n^2*x^3 - 2/3*a*b^2*n*
x^3*log(c) + a*b^2*x^3*log(c)^2 + a^2*b*n*x^3*log(x) - 1/3*a^2*b*n*x^3 + a^
2*b*x^3*log(c) + 1/3*a^3*x^3
```

### 3.59 $\int x (a + b \log (cx^n))^3 dx$

**Optimal.** Leaf size=77

$$\frac{3}{4}b^2n^2x^2(a + b \log (cx^n)) - \frac{3}{4}bnx^2(a + b \log (cx^n))^2 + \frac{1}{2}x^2(a + b \log (cx^n))^3 - \frac{3}{8}b^3n^3x^2$$

[Out]  $(-3*b^3*n^3*x^2)/8 + (3*b^2*n^2*x^2*(a + b*Log[c*x^n]))/4 - (3*b*n*x^2*(a + b*Log[c*x^n])^2)/4 + (x^2*(a + b*Log[c*x^n])^3)/2$

**Rubi [A]** time = 0.038703, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2305, 2304}

$$\frac{3}{4}b^2n^2x^2(a + b \log (cx^n)) - \frac{3}{4}bnx^2(a + b \log (cx^n))^2 + \frac{1}{2}x^2(a + b \log (cx^n))^3 - \frac{3}{8}b^3n^3x^2$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*Log[c\*x^n])^3,x]

[Out]  $(-3*b^3*n^3*x^2)/8 + (3*b^2*n^2*x^2*(a + b*Log[c*x^n]))/4 - (3*b*n*x^2*(a + b*Log[c*x^n])^2)/4 + (x^2*(a + b*Log[c*x^n])^3)/2$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int x (a + b \log (cx^n))^3 dx &= \frac{1}{2}x^2 (a + b \log (cx^n))^3 - \frac{1}{2}(3bn) \int x (a + b \log (cx^n))^2 dx \\ &= -\frac{3}{4}bnx^2 (a + b \log (cx^n))^2 + \frac{1}{2}x^2 (a + b \log (cx^n))^3 + \frac{1}{2}(3b^2n^2) \int x (a + b \log (cx^n)) dx \\ &= -\frac{3}{8}b^3n^3x^2 + \frac{3}{4}b^2n^2x^2 (a + b \log (cx^n)) - \frac{3}{4}bnx^2 (a + b \log (cx^n))^2 + \frac{1}{2}x^2 (a + b \log (cx^n))^3 \end{aligned}$$

**Mathematica [A]** time = 0.0295424, size = 60, normalized size = 0.78

$$\frac{1}{8}x^2 \left( 4(a + b \log (cx^n))^3 - 3bn \left( 2(a + b \log (cx^n))^2 + bn(-2a - 2b \log (cx^n) + bn) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*Log[c\*x^n])^3,x]



[Out]  $(x^2*(4*(a + b*\text{Log}[c*x^n])^3 - 3*b*n*(b*n*(-2*a + b*n - 2*b*\text{Log}[c*x^n]) + 2*(a + b*\text{Log}[c*x^n])^2)))/8$

**Maple [C]** time = 0.334, size = 2650, normalized size = 34.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x*(a+b*\ln(c*x^n))^3, x)$

[Out]  $1/2*x^2*b^3*\ln(x^n)^3+3/4*b^2*x^2*(I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*b*Pi*csgn(I*c*x^n)^3+I*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+2*b*\ln(c)-b*n+2*a)*\ln(x^n)^2+3/8*b*x^2*(4*I*\ln(c)*Pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2+4*\ln(c)^2*b^2-Pi^2*b^2*csgn(I*c*x^n)^4*csgn(I*c)^2-4*a*b*n+2*b^2*n^2+4*a^2-2*I*Pi*b^2*n*csgn(I*c*x^n)^2*csgn(I*c)-2*I*Pi*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)^2+2*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2+2*Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2-4*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)+2*I*Pi*b^2*n*csgn(I*c*x^n)^3+4*I*\ln(c)*Pi*b^2*csgn(I*c*x^n)^2*csgn(I*c)+4*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)^2+4*I*Pi*a*b*csgn(I*c*x^n)^2*csgn(I*c)-4*I*Pi*a*b*csgn(I*c*x^n)^3-4*I*\ln(c)*Pi*b^2*csgn(I*c*x^n)^3-4*I*\ln(c)*Pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-4*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-Pi^2*b^2*csgn(I*c*x^n)^6+8*\ln(c)*a*b-4*\ln(c)*b^2*n+2*I*Pi*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+2*Pi^2*b^2*csgn(I*c*x^n)^5*csgn(I*c)+2*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^5-Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4)*\ln(x^n)+1/16*x^2*(8*a^3+12*a*b^2*n^2-12*a^2*b*n-6*I*Pi*b^3*n^2*csgn(I*c*x^n)^3-6*Pi^2*a*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4+8*\ln(c)^3*b^3-24*Pi^2*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)-3*I*Pi^3*b^3*csgn(I*x^n)*csgn(I*c*x^n)^8-3*I*Pi^3*b^3*csgn(I*c*x^n)^8*csgn(I*c)+3*I*Pi^3*b^3*csgn(I*c*x^n)^7*csgn(I*c)^2-I*Pi^3*b^3*csgn(I*c*x^n)^6*csgn(I*c)^3-12*I*\ln(c)^2*Pi*b^3*csgn(I*c*x^n)^3+6*I*Pi*b^3*n^2*csgn(I*x^n)*csgn(I*c*x^n)^2+6*I*Pi*b^3*n^2*csgn(I*c*x^n)^2*csgn(I*c)-6*Pi^2*b^3*n*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)+12*I*Pi*a*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+12*I*\ln(c)*Pi*b^3*n*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+3*Pi^2*b^3*n*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2+12*Pi^2*b^3*n*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)-6*\ln(c)*Pi^2*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^4+12*\ln(c)*Pi^2*b^3*csgn(I*x^n)*csgn(I*c*x^n)^5+12*\ln(c)*Pi^2*b^3*csgn(I*c*x^n)^5*csgn(I*c)-6*Pi^2*b^3*n*csgn(I*c*x^n)^5*csgn(I*c)+3*Pi^2*b^3*n*csgn(I*c*x^n)^4*csgn(I*c)^2-24*I*\ln(c)*Pi*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-6*b^3*n^3+12*Pi^2*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)^5-12*I*Pi*a^2*b*csgn(I*c*x^n)^3-I*Pi^3*b^3*csgn(I*x^n)^3*csgn(I*c*x^n)^6+3*I*Pi^3*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^7-12*\ln(c)^2*b^3*n+12*\ln(c)*b^3*n^2+24*\ln(c)*a^2*b+24*\ln(c)^2*a*b^2+I*Pi^3*b^3*csgn(I*c*x^n)^9+12*Pi^2*a*b^2*csgn(I*c*x^n)^5*csgn(I*c)-6*Pi^2*a*b^2*csgn(I*c*x^n)^4*csgn(I*c)^2-6*\ln(c)*Pi^2*b^3*csgn(I*c*x^n)^4*csgn(I*c)^2+3*Pi^2*b^3*n*csgn(I*x^n)^2*csgn(I*c*x^n)^4-6*Pi^2*b^3*n*csgn(I*x^n)*csgn(I*c*x^n)^5+3*Pi^2*b^3*n*csgn(I*c*x^n)^6-6*\ln(c)*Pi^2*b^3*csgn(I*c*x^n)^6-6*Pi^2*a*b^2*csgn(I*c*x^n)^6+12*\ln(c)*Pi^2*b^3*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2+12*Pi^2*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2-6*\ln(c)*Pi^2*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2-24*\ln(c)*Pi^2*b^3*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)-6*Pi^2*b^3*n*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2+12*\ln(c)*Pi^2*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-12*I*\ln(c)^2*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+12*Pi^2*a*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-6*Pi^2*a*b^2*csgn(I*c*x^n)^2*csgn(I*c)^2+12*I*Pi*a^2*b*csgn(I*c*x^n)^2*csgn(I*c)+12*I*\ln(c)*Pi*b^3*n*csgn(I*c*x^n)^3+12*I*Pi*a*b^2*n*csgn(I*c*x^n)^3+I*Pi^3*b^3*csgn(I*x^n)^3*csgn(I*c*x^n)^3*csgn(I*c)^3+3*I*Pi^3*b^3*csgn(I*x^n)*csgn(I*c*x^n)^5*csgn(I*c)^3+12*I*\ln(c)^2*Pi*b^3*csgn(I*x^n)$

$n) * \text{csgn}(I * c * x^n)^2 + 12 * I * \ln(c)^2 * \text{Pi} * b^3 * \text{csgn}(I * c * x^n)^2 * \text{csgn}(I * c) - 24 * I * \ln(c) * \text{Pi} * a * b^2 * \text{csgn}(I * c * x^n)^3 + 12 * I * \text{Pi} * a^2 * b * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^2 - 3 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^4 * \text{csgn}(I * c)^3 + 9 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^7 * \text{csgn}(I * c) - 9 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^6 * \text{csgn}(I * c)^2 + 3 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * x^n)^3 * \text{csgn}(I * c * x^n)^5 * \text{csgn}(I * c) - 3 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * x^n)^3 * \text{csgn}(I * c * x^n)^4 * \text{csgn}(I * c)^2 - 9 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^6 * \text{csgn}(I * c) + 9 * I * \text{Pi}^3 * b^3 * \text{csgn}(I * x^n)^2 * \text{csgn}(I * c * x^n)^5 * \text{csgn}(I * c)^2 - 24 * \ln(c) * a * b^2 * n + 24 * I * \ln(c) * \text{Pi} * a * b^2 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^2 + 24 * I * \ln(c) * \text{Pi} * a * b^2 * \text{csgn}(I * c * x^n)^2 * \text{csgn}(I * c) - 12 * I * \text{Pi} * a * b^2 * n * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^2 - 12 * I * \ln(c) * \text{Pi} * b^3 * n * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n)^2 - 6 * I * \text{Pi} * b^3 * n^2 * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n) * \text{csgn}(I * c) - 12 * I * \text{Pi} * a^2 * b * \text{csgn}(I * x^n) * \text{csgn}(I * c * x^n) * \text{csgn}(I * c) - 12 * I * \text{Pi} * a * b^2 * n * \text{csgn}(I * c * x^n)^2 * \text{csgn}(I * c) - 12 * I * \ln(c) * \text{Pi} * b^3 * n * \text{csgn}(I * c * x^n)^2 * \text{csgn}(I * c)$

**Maxima [A]** time = 1.12363, size = 182, normalized size = 2.36

$$\frac{1}{2} b^3 x^2 \log(cx^n)^3 + \frac{3}{2} ab^2 x^2 \log(cx^n)^2 - \frac{3}{4} a^2 b n x^2 + \frac{3}{2} a^2 b x^2 \log(cx^n) + \frac{1}{2} a^3 x^2 + \frac{3}{4} (n^2 x^2 - 2 n x^2 \log(cx^n)) ab^2 - \frac{3}{8} (2 n x^2 \log(cx^n)^2 - 2 n x^2 \log(cx^n) \log(c) + n^2 x^2 \log(c)^2) b^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out]  $\frac{1}{2} b^3 x^2 \log(c * x^n)^3 + \frac{3}{2} a * b^2 x^2 \log(c * x^n)^2 - \frac{3}{4} a^2 b n x^2 + \frac{3}{2} a^2 b x^2 \log(c * x^n) + \frac{1}{2} a^3 x^2 + \frac{3}{4} (n^2 x^2 - 2 n x^2 \log(c * x^n)) a b^2 - \frac{3}{8} (2 n x^2 \log(c * x^n)^2 + (n^2 x^2 - 2 n x^2 \log(c * x^n)) n) b^3$

**Fricas [B]** time = 0.814465, size = 505, normalized size = 6.56

$$\frac{1}{2} b^3 n^3 x^2 \log(x)^3 + \frac{1}{2} b^3 x^2 \log(c)^3 - \frac{3}{4} (b^3 n - 2 a b^2) x^2 \log(c)^2 + \frac{3}{4} (b^3 n^2 - 2 a b^2 n + 2 a^2 b) x^2 \log(c) - \frac{1}{8} (3 b^3 n^3 - 6 a b^2 n^2 + 3 a^2 b n^2) x^2 \log(c)^2 - \frac{3}{4} (b^3 n^2 - 2 a b^2 n + 2 a^2 b) x^2 \log(c) - \frac{1}{8} (3 b^3 n^3 - 6 a b^2 n^2 + 3 a^2 b n^2) x^2 \log(c) - \frac{1}{8} (3 b^3 n^3 - 6 a b^2 n^2 + 3 a^2 b n^2) x^2 \log(c)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out]  $\frac{1}{2} b^3 n^3 x^2 \log(x)^3 + \frac{1}{2} b^3 x^2 \log(c)^3 - \frac{3}{4} (b^3 n - 2 a b^2) x^2 \log(c)^2 + \frac{3}{4} (b^3 n^2 - 2 a b^2 n + 2 a^2 b) x^2 \log(c) - \frac{1}{8} (3 b^3 n^3 - 6 a b^2 n^2 + 3 a^2 b n^2) x^2 \log(c)^2 - \frac{3}{4} (b^3 n^2 - 2 a b^2 n + 2 a^2 b) x^2 \log(c) - \frac{1}{8} (3 b^3 n^3 - 6 a b^2 n^2 + 3 a^2 b n^2) x^2 \log(c) - \frac{1}{8} (3 b^3 n^3 - 6 a b^2 n^2 + 3 a^2 b n^2) x^2 \log(c)^2$

**Sympy [B]** time = 2.49179, size = 337, normalized size = 4.38

$$\frac{a^3 x^2}{2} + \frac{3 a^2 b n x^2 \log(x)}{2} - \frac{3 a^2 b n x^2}{4} + \frac{3 a^2 b x^2 \log(c)}{2} + \frac{3 a b^2 n^2 x^2 \log(x)^2}{2} - \frac{3 a b^2 n^2 x^2 \log(x)}{2} + \frac{3 a b^2 n^2 x^2}{4} + 3 a b^2 n x^2 \log(c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*ln(c\*x\*\*n))\*\*3,x)

[Out]  $a^{**3} x^{**2} / 2 + 3 a^{**2} b n x^{**2} \log(x) / 2 - 3 a^{**2} b n x^{**2} / 4 + 3 a^{**2} b x^{**2} \log(c) / 2 + 3 a b^{**2} n^{**2} x^{**2} \log(x)^2 / 2 - 3 a b^{**2} n^{**2} x^{**2} \log(x) / 2 + 3 a b^{**2} n x^{**2} \log(c)$

```
*a*b**2*n**2*x**2/4 + 3*a*b**2*n*x**2*log(c)*log(x) - 3*a*b**2*n*x**2*log(c)
)/2 + 3*a*b**2*x**2*log(c)**2/2 + b**3*n**3*x**2*log(x)**3/2 - 3*b**3*n**3*
x**2*log(x)**2/4 + 3*b**3*n**3*x**2*log(x)/4 - 3*b**3*n**3*x**2/8 + 3*b**3*
n**2*x**2*log(c)*log(x)**2/2 - 3*b**3*n**2*x**2*log(c)*log(x)/2 + 3*b**3*n*
*2*x**2*log(c)/4 + 3*b**3*n*x**2*log(c)**2*log(x)/2 - 3*b**3*n*x**2*log(c)*
*2/4 + b**3*x**2*log(c)**3/2
```

**Giac [B]** time = 1.18497, size = 354, normalized size = 4.6

$$\frac{1}{2} b^3 n^3 x^2 \log(x)^3 - \frac{3}{4} b^3 n^3 x^2 \log(x)^2 + \frac{3}{2} b^3 n^2 x^2 \log(c) \log(x)^2 + \frac{3}{4} b^3 n^3 x^2 \log(x) - \frac{3}{2} b^3 n^2 x^2 \log(c) \log(x) + \frac{3}{2} b^3 n x^2 \log(c)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*n^3*x^2*log(x)^3 - 3/4*b^3*n^3*x^2*log(x)^2 + 3/2*b^3*n^2*x^2*log(c)
)*log(x)^2 + 3/4*b^3*n^3*x^2*log(x) - 3/2*b^3*n^2*x^2*log(c)*log(x) + 3/2*b
^3*n*x^2*log(c)^2*log(x) + 3/2*a*b^2*n^2*x^2*log(x)^2 - 3/8*b^3*n^3*x^2 + 3
/4*b^3*n^2*x^2*log(c) - 3/4*b^3*n*x^2*log(c)^2 + 1/2*b^3*x^2*log(c)^3 - 3/2
*a*b^2*n^2*x^2*log(x) + 3*a*b^2*n*x^2*log(c)*log(x) + 3/4*a*b^2*n^2*x^2 - 3
/2*a*b^2*n*x^2*log(c) + 3/2*a*b^2*x^2*log(c)^2 + 3/2*a^2*b*n*x^2*log(x) - 3
/4*a^2*b*n*x^2 + 3/2*a^2*b*x^2*log(c) + 1/2*a^3*x^2
```

### 3.60 $\int (a + b \log(cx^n))^3 dx$

**Optimal.** Leaf size=66

$$6ab^2n^2x - 3bnx(a + b \log(cx^n))^2 + x(a + b \log(cx^n))^3 + 6b^3n^2x \log(cx^n) - 6b^3n^3x$$

[Out]  $6a^2b^2n^2x - 6b^3n^3x + 6b^3n^2x \log(cx^n) - 3b^2n^2x(a + b \log(cx^n))^2 + x(a + b \log(cx^n))^3$

**Rubi [A]** time = 0.0238015, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2296, 2295}

$$6ab^2n^2x - 3bnx(a + b \log(cx^n))^2 + x(a + b \log(cx^n))^3 + 6b^3n^2x \log(cx^n) - 6b^3n^3x$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^3, x]

[Out]  $6a^2b^2n^2x - 6b^3n^3x + 6b^3n^2x \log(cx^n) - 3b^2n^2x(a + b \log(cx^n))^2 + x(a + b \log(cx^n))^3$

#### Rule 2296

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] :> Simp[x\*(a + b\*Log[c\*x^n])^p, x] - Dist[b\*n\*p, Int[(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2\*p]

#### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] :> Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

#### Rubi steps

$$\begin{aligned} \int (a + b \log(cx^n))^3 dx &= x(a + b \log(cx^n))^3 - (3bn) \int (a + b \log(cx^n))^2 dx \\ &= -3bnx(a + b \log(cx^n))^2 + x(a + b \log(cx^n))^3 + (6b^2n^2) \int (a + b \log(cx^n)) dx \\ &= 6ab^2n^2x - 3bnx(a + b \log(cx^n))^2 + x(a + b \log(cx^n))^3 + (6b^3n^2) \int \log(cx^n) dx \\ &= 6ab^2n^2x - 6b^3n^3x + 6b^3n^2x \log(cx^n) - 3bnx(a + b \log(cx^n))^2 + x(a + b \log(cx^n))^3 \end{aligned}$$

**Mathematica [A]** time = 0.0087649, size = 50, normalized size = 0.76

$$x((a + b \log(cx^n))^3 - 3bn((a + b \log(cx^n))^2 - 2bn(a + b \log(cx^n) - bn)))$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^3, x]

[Out]  $x((a + b \log(cx^n))^3 - 3b^2n^2x \log(cx^n) - 2b^2n^2x(a + b \log(cx^n))^2 - 2b^2n^2x(a - b \log(cx^n) + b \log^2(cx^n)))$

**Maple [C]** time = 0.299, size = 2641, normalized size = 40.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a+b*\ln(c*x^n))^3, x)$

[Out]  $x*b^3*\ln(x^n)^3+3/2*b^2*(I*b*\text{Pi}*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)^2-I*b*\text{Pi}*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)*c\text{sgn}(I*c)-I*b*\text{Pi}*c\text{sgn}(I*c*x^n)^3+I*b*\text{Pi}*c\text{sgn}(I*c*x^n)^2*c\text{sgn}(I*c)+2*b*\ln(c)-2*b*n+2*a)*x*\ln(x^n)^2+3/4*b*(4*I*\ln(c)*\text{Pi}*b^2*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)^2+4*\ln(c)^2*b^2-\text{Pi}^2*b^2*c\text{sgn}(I*c*x^n)^4*c\text{sgn}(I*c)^2-8*a*b*n+4*I*\text{Pi}*b^2*n*c\text{sgn}(I*c*x^n)^3+8*b^2*n^2+4*a^2-4*I*\text{Pi}*b^2*n*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)^2-4*I*\text{Pi}*b^2*n*c\text{sgn}(I*c*x^n)^2*c\text{sgn}(I*c)+2*\text{Pi}^2*b^2*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)^3*c\text{sgn}(I*c)^2+2*\text{Pi}^2*b^2*c\text{sgn}(I*x^n)^2*c\text{sgn}(I*c*x^n)^3*c\text{sgn}(I*c)-\text{Pi}^2*b^2*c\text{sgn}(I*x^n)^2*c\text{sgn}(I*c*x^n)^2*c\text{sgn}(I*c)^2-4*\text{Pi}^2*b^2*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)^4*c\text{sgn}(I*c)+4*I*\ln(c)*\text{Pi}*b^2*c\text{sgn}(I*c*x^n)^2*c\text{sgn}(I*c)+4*I*\text{Pi}*a*b*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)^2+4*I*\text{Pi}*a*b*c\text{sgn}(I*c*x^n)^2*c\text{sgn}(I*c)-4*I*\text{Pi}*a*b*c\text{sgn}(I*c*x^n)^3-4*I*\ln(c)*\text{Pi}*b^2*c\text{sgn}(I*c*x^n)^3-4*I*\ln(c)*\text{Pi}*b^2*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)*c\text{sgn}(I*c)-4*I*\text{Pi}*a*b*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)*c\text{sgn}(I*c)-\text{Pi}^2*b^2*c\text{sgn}(I*c*x^n)^6+8*\ln(c)*a*b-8*\ln(c)*b^2*n+2*\text{Pi}^2*b^2*c\text{sgn}(I*c*x^n)^5*c\text{sgn}(I*c)+2*\text{Pi}^2*b^2*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)^5-\text{Pi}^2*b^2*c\text{sgn}(I*x^n)^2*c\text{sgn}(I*c*x^n)^4+4*I*\text{Pi}*b^2*n*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)*c\text{sgn}(I*c))*x*\ln(x^n)+1/8*(8*a^3+48*a*b^2*n^2-24*a^2*b*n-6*\text{Pi}^2*a*b^2*c\text{sgn}(I*x^n)^2*c\text{sgn}(I*c*x^n)^4+24*I*\ln(c)*\text{Pi}*b^3*n*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)*c\text{sgn}(I*c)+8*\ln(c)^3*b^3-24*\text{Pi}^2*a*b^2*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)^4*c\text{sgn}(I*c)-3*I*\text{Pi}^3*b^3*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)^8-3*I*\text{Pi}^3*b^3*c\text{sgn}(I*c*x^n)^8*c\text{sgn}(I*c)+3*I*\text{Pi}^3*b^3*c\text{sgn}(I*c*x^n)^7*c\text{sgn}(I*c)^2-I*\text{Pi}^3*b^3*c\text{sgn}(I*c*x^n)^6*c\text{sgn}(I*c)^3-12*I*\ln(c)^2*\text{Pi}*b^3*c\text{sgn}(I*c*x^n)^3+24*I*\ln(c)*\text{Pi}*b^3*n*c\text{sgn}(I*c*x^n)^3+24*I*\text{Pi}*a*b^2*n*c\text{sgn}(I*c*x^n)^3-12*\text{Pi}^2*b^3*n*c\text{sgn}(I*x^n)^2*c\text{sgn}(I*c*x^n)^3*c\text{sgn}(I*c)+6*\text{Pi}^2*b^3*n*c\text{sgn}(I*x^n)^2*c\text{sgn}(I*c*x^n)^2*c\text{sgn}(I*c)^2+24*\text{Pi}^2*b^3*n*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)^4*c\text{sgn}(I*c)-6*\ln(c)*\text{Pi}^2*b^3*c\text{sgn}(I*x^n)^2*c\text{sgn}(I*c*x^n)^4+12*\ln(c)*\text{Pi}^2*b^3*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)^5+12*\ln(c)*\text{Pi}^2*b^3*c\text{sgn}(I*c*x^n)^5*c\text{sgn}(I*c)-12*\text{Pi}^2*b^3*n*c\text{sgn}(I*c*x^n)^5*c\text{sgn}(I*c)+6*\text{Pi}^2*b^3*n*c\text{sgn}(I*c*x^n)^4*c\text{sgn}(I*c)^2-24*I*\ln(c)*\text{Pi}*a*b^2*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)*c\text{sgn}(I*c)-48*b^3*n^3+24*I*\text{Pi}*a*b^2*n*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)*c\text{sgn}(I*c)+12*\text{Pi}^2*a*b^2*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)^5-24*I*\text{Pi}*b^3*n^2*c\text{sgn}(I*c*x^n)^3-12*I*\text{Pi}*a^2*b*c\text{sgn}(I*c*x^n)^3-I*\text{Pi}^3*b^3*c\text{sgn}(I*x^n)^3*c\text{sgn}(I*c*x^n)^6+3*I*\text{Pi}^3*b^3*c\text{sgn}(I*x^n)^2*c\text{sgn}(I*c*x^n)^7-24*\ln(c)^2*b^3*n+48*\ln(c)*b^3*n^2+24*\ln(c)*a^2*b+24*\ln(c)^2*a*b^2+I*\text{Pi}^3*b^3*c\text{sgn}(I*c*x^n)^9+12*\text{Pi}^2*a*b^2*c\text{sgn}(I*c*x^n)^5*c\text{sgn}(I*c)-6*\text{Pi}^2*a*b^2*c\text{sgn}(I*c*x^n)^4*c\text{sgn}(I*c)^2-6*\ln(c)*\text{Pi}^2*b^3*c\text{sgn}(I*c*x^n)^4*c\text{sgn}(I*c)^2+6*\text{Pi}^2*b^3*n*c\text{sgn}(I*x^n)^2*c\text{sgn}(I*c*x^n)^4-12*\text{Pi}^2*b^3*n*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)^5+6*\text{Pi}^2*b^3*n*c\text{sgn}(I*c*x^n)^6-6*\ln(c)*\text{Pi}^2*b^3*c\text{sgn}(I*c*x^n)^6-6*\text{Pi}^2*a*b^2*c\text{sgn}(I*c*x^n)^6+12*\ln(c)*\text{Pi}^2*b^3*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)^3*c\text{sgn}(I*c)^2+12*\text{Pi}^2*a*b^2*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)^3*c\text{sgn}(I*c)^2-6*\ln(c)*\text{Pi}^2*b^3*c\text{sgn}(I*x^n)^2*c\text{sgn}(I*c*x^n)^2*c\text{sgn}(I*c)^2-24*\ln(c)*\text{Pi}^2*b^3*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)^4*c\text{sgn}(I*c)-12*\text{Pi}^2*b^3*n*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)^3*c\text{sgn}(I*c)^2+12*\ln(c)*\text{Pi}^2*b^3*c\text{sgn}(I*x^n)^2*c\text{sgn}(I*c*x^n)^3*c\text{sgn}(I*c)-12*I*\ln(c)^2*\text{Pi}*b^3*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)*c\text{sgn}(I*c)+12*\text{Pi}^2*a*b^2*c\text{sgn}(I*x^n)^2*c\text{sgn}(I*c*x^n)^3*c\text{sgn}(I*c)-6*\text{Pi}^2*a*b^2*c\text{sgn}(I*x^n)^2*c\text{sgn}(I*c*x^n)^2*c\text{sgn}(I*c)^2-24*I*\text{Pi}*b^3*n^2*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)*c\text{sgn}(I*c)+12*I*\text{Pi}*a^2*b*c\text{sgn}(I*c*x^n)^2*c\text{sgn}(I*c)+I*\text{Pi}^3*b^3*c\text{sgn}(I*x^n)^3*c\text{sgn}(I*c*x^n)^3*c\text{sgn}(I*c)^3+3*I*\text{Pi}^3*b^3*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)^5*c\text{sgn}(I*c)^3+12*I*\ln(c)^2*\text{Pi}*b^3*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)^2+12*I*\ln(c)^2*\text{Pi}*b^3*c\text{sgn}(I*c*x^n)^2*c\text{sgn}(I*c)-24*I*\ln(c)*\text{Pi}*a*b^2*c\text{sgn}(I*c*x^n)^3+12*I*\text{Pi}*a^2*b*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)^2-3*I*\text{Pi}^3*b^3*c\text{sgn}(I*x^n)^2*c\text{sgn}(I*c*x^n)^4*c\text{sgn}(I*c)^3+9*I*\text{Pi}^3*b^3*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)^7*c\text{sgn}(I*c)-9*I*\text{Pi}^3*b^3*c\text{sgn}(I*x^n)*c\text{sgn}(I*c*x^n)^6*c\text{sgn}(I*c)^2+24*I*\text{Pi}*b^3*n^2*c\text{sgn}(I*x^n)*c\text{sgn}(I*c$

$$x^n)^2 + 3i\pi^3 b^3 \operatorname{csgn}(ix^n)^3 \operatorname{csgn}(icx^n)^5 \operatorname{csgn}(ic) + 24i\pi b^3 n^2 \operatorname{csgn}(icx^n)^2 \operatorname{csgn}(ic) - 3i\pi^3 b^3 \operatorname{csgn}(ix^n)^3 \operatorname{csgn}(icx^n)^4 \operatorname{csgn}(ic)^2 - 9i\pi^3 b^3 \operatorname{csgn}(ix^n)^2 \operatorname{csgn}(icx^n)^6 \operatorname{csgn}(ic) + 9i\pi^3 b^3 \operatorname{csgn}(ix^n)^2 \operatorname{csgn}(icx^n)^5 \operatorname{csgn}(ic)^2 - 48 \ln(c) a b^2 n + 24i \ln(c) \pi a b^2 \operatorname{csgn}(ix^n) \operatorname{csgn}(icx^n)^2 + 24i \ln(c) \pi a b^2 \operatorname{csgn}(icx^n)^2 \operatorname{csgn}(ic) - 24i \ln(c) \pi b^3 n \operatorname{csgn}(icx^n)^2 \operatorname{csgn}(ic) - 24i \pi a b^2 n \operatorname{csgn}(icx^n)^2 \operatorname{csgn}(ic) - 24i \pi a b^2 n \operatorname{csgn}(ix^n) \operatorname{csgn}(icx^n)^2 - 24i \ln(c) \pi b^3 n \operatorname{csgn}(ix^n) \operatorname{csgn}(icx^n)^2 - 12i \pi a^2 b \operatorname{csgn}(ix^n) \operatorname{csgn}(icx^n) \operatorname{csgn}(ic)) x$$

**Maxima [A]** time = 1.21959, size = 153, normalized size = 2.32

$$b^3 x \log(cx^n)^3 + 3ab^2 x \log(cx^n)^2 - 3a^2 b n x + 3a^2 b x \log(cx^n) + 6(n^2 x - n x \log(cx^n)) a b^2 - 3(n x \log(cx^n))^2 + 2(n^2 x - n$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out]  $b^3 x \log(cx^n)^3 + 3a b^2 x \log(cx^n)^2 - 3a^2 b n x + 3a^2 b x \log(cx^n) + 6(n^2 x - n x \log(cx^n)) a b^2 - 3(n x \log(cx^n))^2 + 2(n^2 x - n x \log(cx^n)) n b^3 + a^3 x$

**Fricas [B]** time = 0.899181, size = 435, normalized size = 6.59

$$b^3 n^3 x \log(x)^3 + b^3 x \log(c)^3 - 3(b^3 n - ab^2) x \log(c)^2 + 3(2b^3 n^2 - 2ab^2 n + a^2 b) x \log(c) + 3(b^3 n^2 x \log(c) - (b^3 n^3 - ab^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out]  $b^3 n^3 x \log(x)^3 + b^3 x \log(c)^3 - 3(b^3 n - ab^2) x \log(c)^2 + 3(2b^3 n^2 - 2ab^2 n + a^2 b) x \log(c) + 3(b^3 n^2 x \log(c) - (b^3 n^3 - ab^2) x \log(c)^2 - (6b^3 n^3 - 6ab^2 n^2 + 3a^2 b n - a^3) x + 3(b^3 n^3 x \log(c)^2 - 2(b^3 n^2 - ab^2) x \log(c) + (2b^3 n^3 - 2ab^2 n^2 + a^2 b n) x) \log(x)$

**Sympy [B]** time = 1.39415, size = 270, normalized size = 4.09

$$a^3 x + 3a^2 b n x \log(x) - 3a^2 b n x + 3a^2 b x \log(c) + 3ab^2 n^2 x \log(x)^2 - 6ab^2 n^2 x \log(x) + 6ab^2 n^2 x + 6ab^2 n x \log(c) \log(x) -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))\*\*3,x)

[Out]  $a^3 x + 3a^2 b n x \log(x) - 3a^2 b n x + 3a^2 b x \log(c) + 3a b^2 n^2 x \log(x)^2 - 6a b^2 n^2 x \log(x) + 6a b^2 n^2 x + 6a b^2 n x \log(c) \log(x) - 6a b^2 n^2 x \log(c) + 3a b^2 n^2 x \log(c)^2 + b^3 n^3 x \log(x)^3 - 3b^3 n^3 x \log(x)^2 + 6b^3 n^3 x \log(x) - 6b^3 n^3 x + 3b^3 n^2 x \log(c) \log(x)^2 - 6b^3 n^2 x \log(c) \log(x) + 6b^3 n^2 x \log(c) + 3b^3 n x \log(c)^2 \log(x) - 3b^3 n x \log(c)^2 + b^3 x \log(c)^3$

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**Giac [B]** time = 1.22994, size = 296, normalized size = 4.48

$$b^3 n^3 x \log(x)^3 - 3 b^3 n^3 x \log(x)^2 + 3 b^3 n^2 x \log(c) \log(x)^2 + 6 b^3 n^3 x \log(x) - 6 b^3 n^2 x \log(c) \log(x) + 3 b^3 n x \log(c)^2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out]  $b^3 n^3 x \log(x)^3 - 3 b^3 n^3 x \log(x)^2 + 3 b^3 n^2 x \log(c) \log(x)^2 + 6 b^3 n^3 x \log(x) - 6 b^3 n^2 x \log(c) \log(x) + 3 b^3 n x \log(c)^2 \log(x) + 3 a b^2 n^2 x \log(x)^2 - 6 b^3 n^3 x + 6 b^3 n^2 x \log(c) - 3 b^3 n x \log(c)^2 + b^3 x \log(c)^3 - 6 a b^2 n^2 x \log(x) + 6 a b^2 n x \log(c) \log(x) + 6 a b^2 n^2 x - 6 a b^2 n x \log(c) + 3 a b^2 x \log(c)^2 + 3 a^2 b n x \log(x) - 3 a^2 b n x + 3 a^2 b x \log(c) + a^3 x$

$$3.61 \quad \int \frac{(a+b \log(cx^n))^3}{x} dx$$

**Optimal.** Leaf size=22

$$\frac{(a + b \log(cx^n))^4}{4bn}$$

[Out] (a + b\*Log[c\*x^n])^4/(4\*b\*n)

**Rubi [A]** time = 0.0222569, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2302, 30}

$$\frac{(a + b \log(cx^n))^4}{4bn}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^3/x, x]

[Out] (a + b\*Log[c\*x^n])^4/(4\*b\*n)

#### Rule 2302

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] :> Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{(a + b \log(cx^n))^3}{x} dx &= \frac{\text{Subst}\left(\int x^3 dx, x, a + b \log(cx^n)\right)}{bn} \\ &= \frac{(a + b \log(cx^n))^4}{4bn} \end{aligned}$$

**Mathematica [A]** time = 0.003283, size = 22, normalized size = 1.

$$\frac{(a + b \log(cx^n))^4}{4bn}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^3/x, x]

[Out] (a + b\*Log[c\*x^n])^4/(4\*b\*n)



**Maple [B]** time = 0.036, size = 75, normalized size = 3.4

$$\frac{b^3 (\ln(cx^n))^4}{4n} + \frac{b^2 (\ln(cx^n))^3 a}{n} + \frac{3b (\ln(cx^n))^2 a^2}{2n} + \frac{\ln(cx^n) a^3}{n} + \frac{a^4}{4bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^n))^3/x,x)

[Out] 1/4\*b^3/n\*ln(c\*x^n)^4+b^2/n\*ln(c\*x^n)^3\*a+3/2\*b/n\*ln(c\*x^n)^2\*a^2+1/n\*ln(c\*x^n)\*a^3+1/4/b/n\*a^4

**Maxima [A]** time = 1.10159, size = 27, normalized size = 1.23

$$\frac{(b \log(cx^n) + a)^4}{4bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^3/x,x, algorithm="maxima")

[Out] 1/4\*(b\*log(c\*x^n) + a)^4/(b\*n)

**Fricas [B]** time = 0.810149, size = 255, normalized size = 11.59

$$\frac{1}{4} b^3 n^3 \log(x)^4 + (b^3 n^2 \log(c) + ab^2 n^2) \log(x)^3 + \frac{3}{2} (b^3 n \log(c)^2 + 2ab^2 n \log(c) + a^2 bn) \log(x)^2 + (b^3 \log(c)^3 + 3ab^2 \log(c)^2 + 3a^2 b \log(c) + a^3) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^3/x,x, algorithm="fricas")

[Out] 1/4\*b^3\*n^3\*log(x)^4 + (b^3\*n^2\*log(c) + a\*b^2\*n^2)\*log(x)^3 + 3/2\*(b^3\*n\*log(c)^2 + 2\*a\*b^2\*n\*log(c) + a^2\*b\*n)\*log(x)^2 + (b^3\*log(c)^3 + 3\*a\*b^2\*log(c)^2 + 3\*a^2\*b\*log(c) + a^3)\*log(x)

**Sympy [B]** time = 37.4416, size = 92, normalized size = 4.18

$$\begin{cases} \frac{a^3 \log(cx^n) + \frac{3a^2 b \log(cx^n)^2}{2} + ab^2 \log(cx^n)^3 + \frac{b^3 \log(cx^n)^4}{4}}{n} & \text{for } n \neq 0 \\ (a^3 + 3a^2 b \log(c) + 3ab^2 \log(c)^2 + b^3 \log(c)^3) \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))\*\*3/x,x)

[Out] Piecewise(((a\*\*3\*log(c\*x\*\*n) + 3\*a\*\*2\*b\*log(c\*x\*\*n)\*\*2/2 + a\*b\*\*2\*log(c\*x\*\*n))\*\*3 + b\*\*3\*log(c\*x\*\*n)\*\*4/4)/n, Ne(n, 0)), ((a\*\*3 + 3\*a\*\*2\*b\*log(c) + 3\*a\*b\*\*2\*log(c)\*\*2 + b\*\*3\*log(c)\*\*3)\*log(x), True))

**Giac [B]** time = 1.2242, size = 154, normalized size = 7.

$$\frac{1}{4}b^3n^3 \log(x)^4 + b^3n^2 \log(c) \log(x)^3 + \frac{3}{2}b^3n \log(c)^2 \log(x)^2 + ab^2n^2 \log(x)^3 + b^3 \log(c)^3 \log(x) + 3ab^2n \log(c) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^3/x,x, algorithm="giac")

[Out] 1/4\*b^3\*n^3\*log(x)^4 + b^3\*n^2\*log(c)\*log(x)^3 + 3/2\*b^3\*n\*log(c)^2\*log(x)^2 + a\*b^2\*n^2\*log(x)^3 + b^3\*log(c)^3\*log(x) + 3\*a\*b^2\*n\*log(c)\*log(x)^2 + 3\*a\*b^2\*log(c)^2\*log(x) + 3/2\*a^2\*b\*n\*log(x)^2 + 3\*a^2\*b\*log(c)\*log(x) + a^3\*log(x)

$$3.62 \quad \int \frac{(a+b \log(cx^n))^3}{x^2} dx$$

**Optimal.** Leaf size=69

$$-\frac{6b^2n^2(a+b \log(cx^n))}{x} - \frac{3bn(a+b \log(cx^n))^2}{x} - \frac{(a+b \log(cx^n))^3}{x} - \frac{6b^3n^3}{x}$$

[Out]  $(-6*b^3*n^3)/x - (6*b^2*n^2*(a + b*Log[c*x^n]))/x - (3*b*n*(a + b*Log[c*x^n])^2)/x - (a + b*Log[c*x^n])^3/x$

**Rubi [A]** time = 0.0595732, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2305, 2304}

$$-\frac{6b^2n^2(a+b \log(cx^n))}{x} - \frac{3bn(a+b \log(cx^n))^2}{x} - \frac{(a+b \log(cx^n))^3}{x} - \frac{6b^3n^3}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^3/x^2, x]

[Out]  $(-6*b^3*n^3)/x - (6*b^2*n^2*(a + b*Log[c*x^n]))/x - (3*b*n*(a + b*Log[c*x^n])^2)/x - (a + b*Log[c*x^n])^3/x$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m+1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m+1)), x] - Dist[(b\*n\*p)/(m+1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p-1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m+1)\*(a + b\*Log[c\*x^n]))/(d\*(m+1)), x] - Simp[(b\*n\*(d\*x)^(m+1))/(d\*(m+1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{(a+b \log(cx^n))^3}{x^2} dx &= -\frac{(a+b \log(cx^n))^3}{x} + (3bn) \int \frac{(a+b \log(cx^n))^2}{x^2} dx \\ &= -\frac{3bn(a+b \log(cx^n))^2}{x} - \frac{(a+b \log(cx^n))^3}{x} + (6b^2n^2) \int \frac{a+b \log(cx^n)}{x^2} dx \\ &= -\frac{6b^3n^3}{x} - \frac{6b^2n^2(a+b \log(cx^n))}{x} - \frac{3bn(a+b \log(cx^n))^2}{x} - \frac{(a+b \log(cx^n))^3}{x} \end{aligned}$$

**Mathematica [A]** time = 0.018329, size = 52, normalized size = 0.75

$$-\frac{(a+b \log(cx^n))^3 + 3bn((a+b \log(cx^n))^2 + 2bn(a+b \log(cx^n)) + bn)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])^3/x^2,x]
```

```
[Out] -(((a + b*Log[c*x^n])^3 + 3*b*n*((a + b*Log[c*x^n])^2 + 2*b*n*(a + b*n + b*Log[c*x^n]))) / x)
```

**Maple [C]** time = 0.24, size = 2674, normalized size = 38.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3/x^2,x)
```

```
[Out] -b^3/x*ln(x^n)^3-3/2*(I*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b^3*csgn(I*c*x^n)^3+I*Pi*b^3*csgn(I*c*x^n)^2*csgn(I*c)+2*ln(c)*b^3+2*b^3*n+2*a*b^2)/x*ln(x^n)^2-3/4*(4*a^2*b+8*b^3*n^2+8*ln(c)*a*b^2+8*n*ln(c)*b^3+2*Pi^2*b^3*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2-4*I*ln(c)*Pi*b^3*csgn(I*c*x^n)^3-4*I*Pi*a*b^2*csgn(I*c*x^n)^3-4*I*n*Pi*b^3*csgn(I*c*x^n)^3+8*a*b^2*n+2*Pi^2*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-Pi^2*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2-4*Pi^2*b^3*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)+4*ln(c)^2*b^3-Pi^2*b^3*csgn(I*c*x^n)^6-Pi^2*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^4+2*Pi^2*b^3*csgn(I*x^n)*csgn(I*c*x^n)^5+2*Pi^2*b^3*csgn(I*c*x^n)^5*csgn(I*c)-Pi^2*b^3*csgn(I*c*x^n)^4*csgn(I*c)^2-4*I*ln(c)*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-4*I*Pi*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-4*I*n*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+4*I*Pi*a*b^2*csgn(I*c*x^n)^2*csgn(I*c)+4*I*n*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)^2+4*I*n*Pi*b^3*csgn(I*c*x^n)^2*csgn(I*c)+4*I*ln(c)*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)^2+4*I*ln(c)*Pi*b^3*csgn(I*c*x^n)^2*csgn(I*c)+4*I*Pi*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2)/x*ln(x^n)-1/8*(8*a^3+48*a*b^2*n^2+24*a^2*b*n-6*Pi^2*a*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4+8*ln(c)^3*b^3-24*Pi^2*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)-3*I*Pi^3*b^3*csgn(I*x^n)*csgn(I*c*x^n)^8-3*I*Pi^3*b^3*csgn(I*c*x^n)^8*csgn(I*c)+3*I*Pi^3*b^3*csgn(I*c*x^n)^7*csgn(I*c)^2-I*Pi^3*b^3*csgn(I*c*x^n)^6*csgn(I*c)^3-12*I*ln(c)^2*Pi*b^3*csgn(I*c*x^n)^3+12*Pi^2*b^3*n*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-24*I*ln(c)*Pi*b^3*n*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-6*Pi^2*b^3*n*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2-24*Pi^2*b^3*n*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)-6*ln(c)*Pi^2*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^4+12*ln(c)*Pi^2*b^3*csgn(I*x^n)*csgn(I*c*x^n)^5+12*ln(c)*Pi^2*b^3*csgn(I*c*x^n)^5*csgn(I*c)+12*Pi^2*b^3*n*csgn(I*c*x^n)^5*csgn(I*c)-6*Pi^2*b^3*n*csgn(I*c*x^n)^4*csgn(I*c)^2-24*I*ln(c)*Pi*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-24*I*Pi*a*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+48*b^3*n^3+12*Pi^2*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)^5-24*I*Pi*b^3*n^2*csgn(I*c*x^n)^3-12*I*Pi*a^2*b*csgn(I*c*x^n)^3-I*Pi^3*b^3*csgn(I*x^n)^3*csgn(I*c*x^n)^6+3*I*Pi^3*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^7+24*ln(c)^2*b^3*n+48*ln(c)*b^3*n^2+24*ln(c)*a^2*b+24*ln(c)^2*a*b^2+I*Pi^3*b^3*csgn(I*c*x^n)^9+12*Pi^2*a*b^2*csgn(I*c*x^n)^5*csgn(I*c)-6*Pi^2*a*b^2*csgn(I*c*x^n)^4*csgn(I*c)^2-6*ln(c)*Pi^2*b^3*csgn(I*c*x^n)^4*csgn(I*c)^2-6*Pi^2*b^3*n*csgn(I*x^n)^2*csgn(I*c*x^n)^4+12*Pi^2*b^3*n*csgn(I*x^n)*csgn(I*c*x^n)^5-6*Pi^2*b^3*n*csgn(I*c*x^n)^6-6*ln(c)*Pi^2*b^3*csgn(I*c*x^n)^6-6*Pi^2*a*b^2*csgn(I*c*x^n)^6+12*ln(c)*Pi^2*b^3*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2+12*Pi^2*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2-6*ln(c)*Pi^2*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2-24*ln(c)*Pi^2*b^3*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)+12*Pi^2*b^3*n*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2+12*ln(c)*Pi^2*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-12*I*ln(c)^2*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+12*Pi^2*a*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-6*Pi^2*a*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2-24*I*Pi*b^3*n^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+12*I*Pi*a^2*b*csgn(I*c*x^n)^2*csgn(I*c)+I*Pi^3*b^3*csgn(I*x^n)^3*csgn(I*c*x^n)^3*csgn(I*c)^3+3*I*Pi^3*b^3*c
```

$$\begin{aligned} & \operatorname{sgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^5*\operatorname{csgn}(I*c)^3+12*I*\ln(c)^2*\Pi*b^3*\operatorname{csgn}(I*x^n)*\operatorname{csgn} \\ & (I*c*x^n)^2+12*I*\ln(c)^2*\Pi*b^3*\operatorname{csgn}(I*c*x^n)^2*\operatorname{csgn}(I*c)-24*I*\ln(c)*\Pi*a*b \\ & ^2*\operatorname{csgn}(I*c*x^n)^3+12*I*\Pi*a^2*b*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^2-3*I*\Pi^3*b^3*c \\ & \operatorname{sgn}(I*x^n)^2*\operatorname{csgn}(I*c*x^n)^4*\operatorname{csgn}(I*c)^3+9*I*\Pi^3*b^3*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c* \\ & x^n)^7*\operatorname{csgn}(I*c)-9*I*\Pi^3*b^3*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^6*\operatorname{csgn}(I*c)^2+24*I* \\ & \Pi*b^3*n^2*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^2+3*I*\Pi^3*b^3*\operatorname{csgn}(I*x^n)^3*\operatorname{csgn}(I*c* \\ & x^n)^5*\operatorname{csgn}(I*c)+24*I*\Pi*b^3*n^2*\operatorname{csgn}(I*c*x^n)^2*\operatorname{csgn}(I*c)-3*I*\Pi^3*b^3*\operatorname{csgn} \\ & (I*x^n)^3*\operatorname{csgn}(I*c*x^n)^4*\operatorname{csgn}(I*c)^2-9*I*\Pi^3*b^3*\operatorname{csgn}(I*x^n)^2*\operatorname{csgn}(I*c* \\ & x^n)^6*\operatorname{csgn}(I*c)+9*I*\Pi^3*b^3*\operatorname{csgn}(I*x^n)^2*\operatorname{csgn}(I*c*x^n)^5*\operatorname{csgn}(I*c)^2+48* \\ & \ln(c)*a*b^2*n+24*I*\ln(c)*\Pi*a*b^2*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^2+24*I*\ln(c)*\Pi \\ & *a*b^2*\operatorname{csgn}(I*c*x^n)^2*\operatorname{csgn}(I*c)-24*I*\ln(c)*\Pi*b^3*n*\operatorname{csgn}(I*c*x^n)^3+24*I*n \\ & *\Pi*a*b^2*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^2-24*I*\Pi*a*b^2*n*\operatorname{csgn}(I*c*x^n)^3+24*I* \\ & \Pi*a*b^2*n*\operatorname{csgn}(I*c*x^n)^2*\operatorname{csgn}(I*c)+24*I*\ln(c)*\Pi*b^3*n*\operatorname{csgn}(I*c*x^n)^2*\operatorname{csgn} \\ & (I*c)+24*I*\ln(c)*\Pi*b^3*n*\operatorname{csgn}(I*x^n)*\operatorname{csgn}(I*c*x^n)^2-12*I*\Pi*a^2*b*\operatorname{csgn}( \\ & I*x^n)*\operatorname{csgn}(I*c*x^n)*\operatorname{csgn}(I*c))/x \end{aligned}$$

**Maxima [A]** time = 1.15879, size = 180, normalized size = 2.61

$$-\frac{b^3 \log(cx^n)^3}{x} - 3 \left( 2n \left( \frac{n^2}{x} + \frac{n \log(cx^n)}{x} \right) + \frac{n \log(cx^n)^2}{x} \right) b^3 - 6ab^2 \left( \frac{n^2}{x} + \frac{n \log(cx^n)}{x} \right) - \frac{3ab^2 \log(cx^n)^2}{x} - \frac{3a^2bn}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^3/x^2,x, algorithm="maxima")

[Out]  $-b^3 \log(c*x^n)^3/x - 3*(2*n*(n^2/x + n*\log(c*x^n)/x) + n*\log(c*x^n)^2/x)*b^3 - 6*a*b^2*(n^2/x + n*\log(c*x^n)/x) - 3*a*b^2*\log(c*x^n)^2/x - 3*a^2*b*n/x - 3*a^2*b*\log(c*x^n)/x - a^3/x$

**Fricas [B]** time = 0.881548, size = 406, normalized size = 5.88

$$b^3 n^3 \log(x)^3 + 6 b^3 n^3 + b^3 \log(c)^3 + 6 a b^2 n^2 + 3 a^2 b n + a^3 + 3 (b^3 n + a b^2) \log(c)^2 + 3 (b^3 n^3 + b^3 n^2 \log(c) + a b^2 n^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^3/x^2,x, algorithm="fricas")

[Out]  $-(b^3*n^3*\log(x)^3 + 6*b^3*n^3 + b^3*\log(c)^3 + 6*a*b^2*n^2 + 3*a^2*b*n + a^3 + 3*(b^3*n + a*b^2)*\log(c)^2 + 3*(b^3*n^3 + b^3*n^2*\log(c) + a*b^2*n^2)*\log(x)^2 + 3*(2*b^3*n^2 + 2*a*b^2*n + a^2*b)*\log(c) + 3*(2*b^3*n^3 + b^3*n*\log(c)^2 + 2*a*b^2*n^2 + a^2*b*n + 2*(b^3*n^2 + a*b^2*n)*\log(c))*\log(x))/x$

**Sympy [B]** time = 1.601, size = 272, normalized size = 3.94

$$\frac{a^3}{x} - \frac{3a^2bn \log(x)}{x} - \frac{3a^2bn}{x} - \frac{3a^2b \log(c)}{x} - \frac{3ab^2n^2 \log(x)^2}{x} - \frac{6ab^2n^2 \log(x)}{x} - \frac{6ab^2n^2}{x} - \frac{6ab^2n \log(c) \log(x)}{x} - \frac{6a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))\*\*3/x\*\*2,x)

```
[Out] -a**3/x - 3*a**2*b*n*log(x)/x - 3*a**2*b*n/x - 3*a**2*b*log(c)/x - 3*a*b**2
*n**2*log(x)**2/x - 6*a*b**2*n**2*log(x)/x - 6*a*b**2*n**2/x - 6*a*b**2*n*l
og(c)*log(x)/x - 6*a*b**2*n*log(c)/x - 3*a*b**2*log(c)**2/x - b**3*n**3*log
(x)**3/x - 3*b**3*n**3*log(x)**2/x - 6*b**3*n**3*log(x)/x - 6*b**3*n**3/x -
3*b**3*n**2*log(c)*log(x)**2/x - 6*b**3*n**2*log(c)*log(x)/x - 6*b**3*n**2
*log(c)/x - 3*b**3*n*log(c)**2*log(x)/x - 3*b**3*n*log(c)**2/x - b**3*log(c)
)**3/x
```

**Giac [B]** time = 1.21943, size = 266, normalized size = 3.86

$$\frac{b^3 n^3 \log(x)^3}{x} - \frac{3(b^3 n^3 + b^3 n^2 \log(c) + ab^2 n^2) \log(x)^2}{x} - \frac{3(2b^3 n^3 + 2b^3 n^2 \log(c) + b^3 n \log(c)^2 + 2ab^2 n^2 + 2ab^2 n \log(c) + a^2 b^2 n)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3/x^2,x, algorithm="giac")
```

```
[Out] -b^3*n^3*log(x)^3/x - 3*(b^3*n^3 + b^3*n^2*log(c) + a*b^2*n^2)*log(x)^2/x -
3*(2*b^3*n^3 + 2*b^3*n^2*log(c) + b^3*n*log(c)^2 + 2*a*b^2*n^2 + 2*a*b^2*n
*log(c) + a^2*b*n)*log(x)/x - (6*b^3*n^3 + 6*b^3*n^2*log(c) + 3*b^3*n*log(c)
)^2 + b^3*log(c)^3 + 6*a*b^2*n^2 + 6*a*b^2*n*log(c) + 3*a*b^2*log(c)^2 + 3*
a^2*b*n + 3*a^2*b*log(c) + a^3)/x
```

$$3.63 \quad \int \frac{(a+b \log(cx^n))^3}{x^3} dx$$

**Optimal.** Leaf size=77

$$-\frac{3b^2n^2(a+b \log(cx^n))}{4x^2} - \frac{3bn(a+b \log(cx^n))^2}{4x^2} - \frac{(a+b \log(cx^n))^3}{2x^2} - \frac{3b^3n^3}{8x^2}$$

[Out]  $(-3*b^3*n^3)/(8*x^2) - (3*b^2*n^2*(a + b*Log[c*x^n]))/(4*x^2) - (3*b*n*(a + b*Log[c*x^n])^2)/(4*x^2) - (a + b*Log[c*x^n])^3/(2*x^2)$

**Rubi [A]** time = 0.0593631, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2305, 2304}

$$-\frac{3b^2n^2(a+b \log(cx^n))}{4x^2} - \frac{3bn(a+b \log(cx^n))^2}{4x^2} - \frac{(a+b \log(cx^n))^3}{2x^2} - \frac{3b^3n^3}{8x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^3/x^3, x]

[Out]  $(-3*b^3*n^3)/(8*x^2) - (3*b^2*n^2*(a + b*Log[c*x^n]))/(4*x^2) - (3*b*n*(a + b*Log[c*x^n])^2)/(4*x^2) - (a + b*Log[c*x^n])^3/(2*x^2)$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{(a+b \log(cx^n))^3}{x^3} dx &= -\frac{(a+b \log(cx^n))^3}{2x^2} + \frac{1}{2}(3bn) \int \frac{(a+b \log(cx^n))^2}{x^3} dx \\ &= -\frac{3bn(a+b \log(cx^n))^2}{4x^2} - \frac{(a+b \log(cx^n))^3}{2x^2} + \frac{1}{2}(3b^2n^2) \int \frac{a+b \log(cx^n)}{x^3} dx \\ &= -\frac{3b^3n^3}{8x^2} - \frac{3b^2n^2(a+b \log(cx^n))}{4x^2} - \frac{3bn(a+b \log(cx^n))^2}{4x^2} - \frac{(a+b \log(cx^n))^3}{2x^2} \end{aligned}$$

**Mathematica [A]** time = 0.0206204, size = 60, normalized size = 0.78

$$-\frac{4(a+b \log(cx^n))^3 + 3bn(2(a+b \log(cx^n))^2 + bn(2a + 2b \log(cx^n) + bn))}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])^3/x^3,x]
```

```
[Out] -(4*(a + b*Log[c*x^n])^3 + 3*b*n*(2*(a + b*Log[c*x^n])^2 + b*n*(2*a + b*n + 2*b*Log[c*x^n]))) / (8*x^2)
```

**Maple [C]** time = 0.237, size = 2673, normalized size = 34.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3/x^3,x)
```

```
[Out] -1/2*b^3/x^2*ln(x^n)^3-3/4*(I*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*b^3*csgn(I*c*x^n)^3+I*Pi*b^3*csgn(I*c*x^n)^2*csgn(I*c)+2*ln(c)*b^3+b^3*n+2*a*b^2)/x^2*ln(x^n)^2-3/8*(4*a^2*b+2*b^3*n^2-2*I*n*Pi*b^3*csgn(I*c*x^n)^3+8*ln(c)*a*b^2+4*n*ln(c)*b^3+2*Pi^2*b^3*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2-4*I*ln(c)*Pi*b^3*csgn(I*c*x^n)^3-4*I*Pi*a*b^2*csgn(I*c*x^n)^3+4*a*b^2*n+2*Pi^2*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-Pi^2*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2-4*Pi^2*b^3*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)+4*ln(c)^2*b^3-Pi^2*b^3*csgn(I*c*x^n)^6-Pi^2*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^4+2*Pi^2*b^3*csgn(I*x^n)*csgn(I*c*x^n)^5+2*Pi^2*b^3*csgn(I*c*x^n)^5*csgn(I*c)-Pi^2*b^3*csgn(I*c*x^n)^4*csgn(I*c)^2-2*I*n*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-4*I*ln(c)*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-4*I*Pi*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+4*I*Pi*a*b^2*csgn(I*c*x^n)^2*csgn(I*c)+4*I*ln(c)*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)^2+4*I*ln(c)*Pi*b^3*csgn(I*c*x^n)^2*csgn(I*c)+4*I*Pi*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2+2*I*n*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)^2+2*I*n*Pi*b^3*csgn(I*c*x^n)^2*csgn(I*c))/x^2*ln(x^n)-1/16*(8*a^3+12*a*b^2*n^2+12*a^2*b*n-6*I*Pi*b^3*n^2*csgn(I*c*x^n)^3-6*Pi^2*a*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4+8*ln(c)^3*b^3-24*Pi^2*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)-3*I*Pi^3*b^3*csgn(I*x^n)*csgn(I*c*x^n)^8-3*I*Pi^3*b^3*csgn(I*c*x^n)^8*csgn(I*c)+3*I*Pi^3*b^3*csgn(I*c*x^n)^7*csgn(I*c)^2-I*Pi^3*b^3*csgn(I*c*x^n)^6*csgn(I*c)^3-12*I*ln(c)^2*Pi*b^3*csgn(I*c*x^n)^3+6*I*Pi*b^3*n^2*csgn(I*x^n)*csgn(I*c*x^n)^2+6*I*Pi*b^3*n^2*csgn(I*c*x^n)^2*csgn(I*c)+6*Pi^2*b^3*n*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-12*I*Pi*a*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-12*I*ln(c)*Pi*b^3*n*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-3*Pi^2*b^3*n*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2-12*Pi^2*b^3*n*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)-6*ln(c)*Pi^2*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^4+12*ln(c)*Pi^2*b^3*csgn(I*x^n)*csgn(I*c*x^n)^5+12*ln(c)*Pi^2*b^3*csgn(I*c*x^n)^5*csgn(I*c)+6*Pi^2*b^3*n*csgn(I*c*x^n)^5*csgn(I*c)-3*Pi^2*b^3*n*csgn(I*c*x^n)^4*csgn(I*c)^2-24*I*ln(c)*Pi*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+6*b^3*n^3+12*Pi^2*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)^5-12*I*Pi*a^2*b*csgn(I*c*x^n)^3-I*Pi^3*b^3*csgn(I*x^n)^3*csgn(I*c*x^n)^6+3*I*Pi^3*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^7+12*ln(c)^2*b^3*n+12*ln(c)*b^3*n^2+24*ln(c)*a^2*b+24*ln(c)^2*a*b^2+I*Pi^3*b^3*csgn(I*c*x^n)^9+12*Pi^2*a*b^2*csgn(I*c*x^n)^5*csgn(I*c)-6*Pi^2*a*b^2*csgn(I*c*x^n)^4*csgn(I*c)^2-6*ln(c)*Pi^2*b^3*csgn(I*c*x^n)^4*csgn(I*c)^2-3*Pi^2*b^3*n*csgn(I*x^n)^2*csgn(I*c*x^n)^4+6*Pi^2*b^3*n*csgn(I*x^n)*csgn(I*c*x^n)^5-3*Pi^2*b^3*n*csgn(I*c*x^n)^6-6*ln(c)*Pi^2*b^3*csgn(I*c*x^n)^6-6*Pi^2*a*b^2*csgn(I*c*x^n)^6+12*ln(c)*Pi^2*b^3*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2+12*Pi^2*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2-6*ln(c)*Pi^2*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2-24*ln(c)*Pi^2*b^3*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)+6*Pi^2*b^3*n*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2+12*ln(c)*Pi^2*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-12*I*ln(c)^2*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+12*Pi^2*a*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-6*Pi^2*a*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2+12*I*Pi*a^2*b*csgn(I*c*x^n)^2*csgn(I*c)+I*Pi^3*b^3*csgn(I*x^n)^3*csgn(I*
```



$$c*x^n)^3*\text{csgn}(I*c)^3+3*I*\text{Pi}^3*b^3*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^5*\text{csgn}(I*c)^3+12*I*\ln(c)^2*\text{Pi}*b^3*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^2+12*I*\ln(c)^2*\text{Pi}*b^3*\text{csgn}(I*c*x^n)^2*\text{csgn}(I*c)-24*I*\ln(c)*\text{Pi}*a*b^2*\text{csgn}(I*c*x^n)^3+12*I*\text{Pi}*a^2*b*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^2-3*I*\text{Pi}^3*b^3*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^4*\text{csgn}(I*c)^3+9*I*\text{Pi}^3*b^3*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^7*\text{csgn}(I*c)-9*I*\text{Pi}^3*b^3*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^6*\text{csgn}(I*c)^2+3*I*\text{Pi}^3*b^3*\text{csgn}(I*x^n)^3*\text{csgn}(I*c*x^n)^5*\text{csgn}(I*c)-3*I*\text{Pi}^3*b^3*\text{csgn}(I*x^n)^3*\text{csgn}(I*c*x^n)^4*\text{csgn}(I*c)^2-9*I*\text{Pi}^3*b^3*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^6*\text{csgn}(I*c)+9*I*\text{Pi}^3*b^3*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^5*\text{csgn}(I*c)^2+24*\ln(c)*a*b^2*n+24*I*\ln(c)*\text{Pi}*a*b^2*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^2+24*I*\ln(c)*\text{Pi}*a*b^2*\text{csgn}(I*c*x^n)^2*\text{csgn}(I*c)+12*I*\ln(c)*\text{Pi}*b^3*n*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^2+12*I*\text{Pi}*a*b^2*n*\text{csgn}(I*c*x^n)^2*\text{csgn}(I*c)-6*I*\text{Pi}*b^3*n^2*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)*\text{csgn}(I*c)+12*I*n*\text{Pi}*a*b^2*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^2+12*I*\ln(c)*\text{Pi}*b^3*n*\text{csgn}(I*c*x^n)^2*\text{csgn}(I*c)-12*I*\ln(c)*\text{Pi}*b^3*n*\text{csgn}(I*c*x^n)^3-12*I*\text{Pi}*a*b^2*n*\text{csgn}(I*c*x^n)^3-12*I*\text{Pi}*a^2*b*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)*\text{csgn}(I*c))/x^2$$

**Maxima [A]** time = 1.17584, size = 182, normalized size = 2.36

$$-\frac{3}{8}\left(n\left(\frac{n^2}{x^2} + \frac{2n \log(cx^n)}{x^2}\right) + \frac{2n \log(cx^n)^2}{x^2}\right)b^3 - \frac{3}{4}ab^2\left(\frac{n^2}{x^2} + \frac{2n \log(cx^n)}{x^2}\right) - \frac{b^3 \log(cx^n)^3}{2x^2} - \frac{3ab^2 \log(cx^n)^2}{2x^2} - \frac{3a^2b \log(cx^n)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^3/x^3,x, algorithm="maxima")

[Out]  $-3/8*(n*(n^2/x^2 + 2*n*log(c*x^n)/x^2) + 2*n*log(c*x^n)^2/x^2)*b^3 - 3/4*a*b^2*(n^2/x^2 + 2*n*log(c*x^n)/x^2) - 1/2*b^3*log(c*x^n)^3/x^2 - 3/2*a*b^2*log(c*x^n)^2/x^2 - 3/4*a^2*b*n/x^2 - 3/2*a^2*b*log(c*x^n)/x^2 - 1/2*a^3/x^2$

**Fricas [B]** time = 0.938325, size = 436, normalized size = 5.66

$$4b^3n^3 \log(x)^3 + 3b^3n^3 + 4b^3 \log(c)^3 + 6ab^2n^2 + 6a^2bn + 4a^3 + 6(b^3n + 2ab^2) \log(c)^2 + 6(b^3n^3 + 2b^3n^2 \log(c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^3/x^3,x, algorithm="fricas")

[Out]  $-1/8*(4*b^3*n^3*log(x)^3 + 3*b^3*n^3 + 4*b^3*log(c)^3 + 6*a*b^2*n^2 + 6*a^2*b*n + 4*a^3 + 6*(b^3*n + 2*a*b^2)*log(c)^2 + 6*(b^3*n^3 + 2*b^3*n^2*log(c) + 2*a*b^2*n^2)*log(x)^2 + 6*(b^3*n^2 + 2*a*b^2*n + 2*a^2*b)*log(c) + 6*(b^3*n^3 + 2*b^3*n*log(c)^2 + 2*a*b^2*n^2 + 2*a^2*b*n + 2*(b^3*n^2 + 2*a*b^2*n)*log(c))*log(x))/x^2$

**Sympy [B]** time = 1.69011, size = 338, normalized size = 4.39

$$\frac{a^3}{2x^2} - \frac{3a^2bn \log(x)}{2x^2} - \frac{3a^2bn}{4x^2} - \frac{3a^2b \log(c)}{2x^2} - \frac{3ab^2n^2 \log(x)^2}{2x^2} - \frac{3ab^2n^2 \log(x)}{2x^2} - \frac{3ab^2n^2}{4x^2} - \frac{3ab^2n \log(c) \log(x)}{x^2} - \frac{3a^3}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))\*\*3/x\*\*3,x)

```
[Out] -a**3/(2*x**2) - 3*a**2*b*n*log(x)/(2*x**2) - 3*a**2*b*n/(4*x**2) - 3*a**2*
b*log(c)/(2*x**2) - 3*a*b**2*n**2*log(x)**2/(2*x**2) - 3*a*b**2*n**2*log(x)
/(2*x**2) - 3*a*b**2*n**2/(4*x**2) - 3*a*b**2*n*log(c)*log(x)/x**2 - 3*a*b*
**2*n*log(c)/(2*x**2) - 3*a*b**2*log(c)**2/(2*x**2) - b**3*n**3*log(x)**3/(2
*x**2) - 3*b**3*n**3*log(x)**2/(4*x**2) - 3*b**3*n**3*log(x)/(4*x**2) - 3*b
**3*n**3/(8*x**2) - 3*b**3*n**2*log(c)*log(x)**2/(2*x**2) - 3*b**3*n**2*log
(c)*log(x)/(2*x**2) - 3*b**3*n**2*log(c)/(4*x**2) - 3*b**3*n*log(c)**2*log(
x)/(2*x**2) - 3*b**3*n*log(c)**2/(4*x**2) - b**3*log(c)**3/(2*x**2)
```

**Giac [B]** time = 1.22237, size = 274, normalized size = 3.56

$$\frac{b^3 n^3 \log(x)^3}{2x^2} - \frac{3(b^3 n^3 + 2b^3 n^2 \log(c) + 2ab^2 n^2) \log(x)^2}{4x^2} - \frac{3(b^3 n^3 + 2b^3 n^2 \log(c) + 2b^3 n \log(c)^2 + 2ab^2 n^2 + 4ab^2 n \log(c) + 4a^2 b^2) \log(x)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^3/x^3,x, algorithm="giac")
```

```
[Out] -1/2*b^3*n^3*log(x)^3/x^2 - 3/4*(b^3*n^3 + 2*b^3*n^2*log(c) + 2*a*b^2*n^2)*
log(x)^2/x^2 - 3/4*(b^3*n^3 + 2*b^3*n^2*log(c) + 2*b^3*n*log(c)^2 + 2*a*b^2
*n^2 + 4*a*b^2*n*log(c) + 2*a^2*b*n)*log(x)/x^2 - 1/8*(3*b^3*n^3 + 6*b^3*n^
2*log(c) + 6*b^3*n*log(c)^2 + 4*b^3*log(c)^3 + 6*a*b^2*n^2 + 12*a*b^2*n*log
(c) + 12*a*b^2*log(c)^2 + 6*a^2*b*n + 12*a^2*b*log(c) + 4*a^3)/x^2
```

$$3.64 \quad \int \frac{(a+b \log(cx^n))^3}{x^4} dx$$

**Optimal.** Leaf size=77

$$-\frac{2b^2n^2(a+b \log(cx^n))}{9x^3} - \frac{bn(a+b \log(cx^n))^2}{3x^3} - \frac{(a+b \log(cx^n))^3}{3x^3} - \frac{2b^3n^3}{27x^3}$$

[Out]  $(-2*b^3*n^3)/(27*x^3) - (2*b^2*n^2*(a + b*Log[c*x^n]))/(9*x^3) - (b*n*(a + b*Log[c*x^n])^2)/(3*x^3) - (a + b*Log[c*x^n])^3/(3*x^3)$

**Rubi [A]** time = 0.0606733, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2305, 2304}

$$-\frac{2b^2n^2(a+b \log(cx^n))}{9x^3} - \frac{bn(a+b \log(cx^n))^2}{3x^3} - \frac{(a+b \log(cx^n))^3}{3x^3} - \frac{2b^3n^3}{27x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^3/x^4, x]

[Out]  $(-2*b^3*n^3)/(27*x^3) - (2*b^2*n^2*(a + b*Log[c*x^n]))/(9*x^3) - (b*n*(a + b*Log[c*x^n])^2)/(3*x^3) - (a + b*Log[c*x^n])^3/(3*x^3)$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{(a+b \log(cx^n))^3}{x^4} dx &= -\frac{(a+b \log(cx^n))^3}{3x^3} + (bn) \int \frac{(a+b \log(cx^n))^2}{x^4} dx \\ &= -\frac{bn(a+b \log(cx^n))^2}{3x^3} - \frac{(a+b \log(cx^n))^3}{3x^3} + \frac{1}{3}(2b^2n^2) \int \frac{a+b \log(cx^n)}{x^4} dx \\ &= -\frac{2b^3n^3}{27x^3} - \frac{2b^2n^2(a+b \log(cx^n))}{9x^3} - \frac{bn(a+b \log(cx^n))^2}{3x^3} - \frac{(a+b \log(cx^n))^3}{3x^3} \end{aligned}$$

**Mathematica [A]** time = 0.0242901, size = 60, normalized size = 0.78

$$-\frac{9(a+b \log(cx^n))^3 + bn(9(a+b \log(cx^n))^2 + 2bn(3a + 3b \log(cx^n) + bn))}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[c*x^n])^3/x^4,x]
```

```
[Out] -(9*(a + b*Log[c*x^n])^3 + b*n*(9*(a + b*Log[c*x^n])^2 + 2*b*n*(3*a + b*n +
3*b*Log[c*x^n]))) / (27*x^3)
```

**Maple [C]** time = 0.249, size = 2674, normalized size = 34.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(c*x^n))^3/x^4,x)
```

```
[Out] -1/3*b^3/x^3*ln(x^n)^3-1/6*(3*I*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)^2-3*I*Pi*b^
^3*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-3*I*Pi*b^3*csgn(I*c*x^n)^3+3*I*Pi*b^
^3*csgn(I*c*x^n)^2*csgn(I*c)+6*ln(c)*b^3+2*b^3*n+6*a*b^2)/x^3*ln(x^n)^2-1/36
*(36*a^2*b+8*b^3*n^2-36*I*ln(c)*Pi*b^3*csgn(I*c*x^n)^3-36*I*Pi*a*b^2*csgn(I
*c*x^n)^3-12*I*n*Pi*b^3*csgn(I*c*x^n)^3+72*ln(c)*a*b^2+24*n*ln(c)*b^3+18*Pi
^2*b^3*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2+24*a*b^2*n+18*Pi^2*b^3*csgn(
I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-9*Pi^2*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^2
*csgn(I*c)^2-36*Pi^2*b^3*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)+36*ln(c)^2*b
^3-9*Pi^2*b^3*csgn(I*c*x^n)^6+36*I*ln(c)*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)^2
+36*I*ln(c)*Pi*b^3*csgn(I*c*x^n)^2*csgn(I*c)+36*I*Pi*a*b^2*csgn(I*x^n)*csgn
(I*c*x^n)^2-9*Pi^2*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^4+18*Pi^2*b^3*csgn(I*x^n
)*csgn(I*c*x^n)^5+18*Pi^2*b^3*csgn(I*c*x^n)^5*csgn(I*c)-9*Pi^2*b^3*csgn(I*c
*x^n)^4*csgn(I*c)^2-36*I*Pi*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-12*I
*n*Pi*b^3*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-36*I*ln(c)*Pi*b^3*csgn(I*x^n)*
csgn(I*c*x^n)*csgn(I*c)+36*I*Pi*a*b^2*csgn(I*c*x^n)^2*csgn(I*c)+12*I*n*Pi*b
^3*csgn(I*x^n)*csgn(I*c*x^n)^2+12*I*n*Pi*b^3*csgn(I*c*x^n)^2*csgn(I*c))/x^3
*ln(x^n)-1/216*(72*a^3+48*a*b^2*n^2+72*a^2*b*n-54*Pi^2*a*b^2*csgn(I*x^n)^2*
csgn(I*c*x^n)^4+72*ln(c)^3*b^3-216*Pi^2*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)^4*c
sgn(I*c)+36*Pi^2*b^3*n*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-18*Pi^2*b^3*
n*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2-72*Pi^2*b^3*n*csgn(I*x^n)*csgn(
I*c*x^n)^4*csgn(I*c)-54*ln(c)*Pi^2*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^4+108*ln
(c)*Pi^2*b^3*csgn(I*x^n)*csgn(I*c*x^n)^5+108*ln(c)*Pi^2*b^3*csgn(I*c*x^n)^5
*csgn(I*c)+36*Pi^2*b^3*n*csgn(I*c*x^n)^5*csgn(I*c)-18*Pi^2*b^3*n*csgn(I*c*x
^n)^4*csgn(I*c)^2-216*I*ln(c)*Pi*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+
16*b^3*n^3-72*I*ln(c)*Pi*b^3*n*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+9*I*Pi^3
*b^3*csgn(I*c*x^n)^9+108*Pi^2*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)^5-108*I*ln(c)
^2*Pi*b^3*csgn(I*c*x^n)^3-108*I*Pi*a^2*b*csgn(I*c*x^n)^3-24*I*Pi*b^3*n^2*cs
gn(I*c*x^n)^3-9*I*Pi^3*b^3*csgn(I*x^n)^3*csgn(I*c*x^n)^6-27*I*Pi^3*b^3*csgn
(I*x^n)*csgn(I*c*x^n)^8-27*I*Pi^3*b^3*csgn(I*c*x^n)^8*csgn(I*c)+27*I*Pi^3*b
^3*csgn(I*c*x^n)^7*csgn(I*c)^2-9*I*Pi^3*b^3*csgn(I*c*x^n)^6*csgn(I*c)^3+72*
ln(c)^2*b^3*n+48*ln(c)*b^3*n^2+216*ln(c)*a^2*b+216*ln(c)^2*a*b^2+108*Pi^2*a
*b^2*csgn(I*c*x^n)^5*csgn(I*c)-54*Pi^2*a*b^2*csgn(I*c*x^n)^4*csgn(I*c)^2-54
*ln(c)*Pi^2*b^3*csgn(I*c*x^n)^4*csgn(I*c)^2-18*Pi^2*b^3*n*csgn(I*x^n)^2*csg
n(I*c*x^n)^4+36*Pi^2*b^3*n*csgn(I*x^n)*csgn(I*c*x^n)^5-18*Pi^2*b^3*n*csgn(I
*c*x^n)^6-54*ln(c)*Pi^2*b^3*csgn(I*c*x^n)^6-54*Pi^2*a*b^2*csgn(I*c*x^n)^6+2
7*I*Pi^3*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^7-72*I*Pi*a*b^2*n*csgn(I*x^n)*csgn
(I*c*x^n)*csgn(I*c)+27*I*Pi^3*b^3*csgn(I*x^n)^3*csgn(I*c*x^n)^5*csgn(I*c)-2
7*I*Pi^3*b^3*csgn(I*x^n)^3*csgn(I*c*x^n)^4*csgn(I*c)^2+9*I*Pi^3*b^3*csgn(I
*x^n)^3*csgn(I*c*x^n)^3*csgn(I*c)^3-81*I*Pi^3*b^3*csgn(I*x^n)^2*csgn(I*c*x^n
)^6*csgn(I*c)+108*ln(c)*Pi^2*b^3*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2+10
8*Pi^2*a*b^2*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2-54*ln(c)*Pi^2*b^3*csgn
(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2-216*ln(c)*Pi^2*b^3*csgn(I*x^n)*csgn(I
*c*x^n)^4*csgn(I*c)+36*Pi^2*b^3*n*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2+1
08*ln(c)*Pi^2*b^3*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)+108*Pi^2*a*b^2*cs
```

$$\begin{aligned} & \operatorname{sgn}(I*x^n)^2 * \operatorname{csgn}(I*c*x^n)^3 * \operatorname{csgn}(I*c) - 54 * \pi^2 * a * b^2 * \operatorname{csgn}(I*x^n)^2 * \operatorname{csgn}(I*c * \\ & x^n)^2 * \operatorname{csgn}(I*c)^2 - 24 * I * \pi * b^3 * n^2 * \operatorname{csgn}(I*x^n) * \operatorname{csgn}(I*c*x^n) * \operatorname{csgn}(I*c) + 72 * I \\ & * n * \pi * a * b^2 * \operatorname{csgn}(I*x^n) * \operatorname{csgn}(I*c*x^n)^2 + 72 * I * \pi * a * b^2 * n * \operatorname{csgn}(I*c*x^n)^2 * \operatorname{csgn} \\ & n(I*c) + 24 * I * \pi * b^3 * n^2 * \operatorname{csgn}(I*x^n) * \operatorname{csgn}(I*c*x^n)^2 + 24 * I * \pi * b^3 * n^2 * \operatorname{csgn}(I*c \\ & * x^n)^2 * \operatorname{csgn}(I*c) - 216 * I * \ln(c) * \pi * a * b^2 * \operatorname{csgn}(I*c*x^n)^3 + 108 * I * \pi * a^2 * b * \operatorname{csgn}( \\ & I*x^n) * \operatorname{csgn}(I*c*x^n)^2 + 108 * I * \pi * a^2 * b * \operatorname{csgn}(I*c*x^n)^2 * \operatorname{csgn}(I*c) - 81 * I * \pi^3 * b \\ & ^3 * \operatorname{csgn}(I*x^n) * \operatorname{csgn}(I*c*x^n)^6 * \operatorname{csgn}(I*c)^2 + 27 * I * \pi^3 * b^3 * \operatorname{csgn}(I*x^n) * \operatorname{csgn}(I \\ & * c*x^n)^5 * \operatorname{csgn}(I*c)^3 + 108 * I * \ln(c)^2 * \pi * b^3 * \operatorname{csgn}(I*x^n) * \operatorname{csgn}(I*c*x^n)^2 + 108 * \\ & I * \ln(c)^2 * \pi * b^3 * \operatorname{csgn}(I*c*x^n)^2 * \operatorname{csgn}(I*c) + 144 * \ln(c) * a * b^2 * n + 81 * I * \pi^3 * b^3 * \\ & \operatorname{csgn}(I*x^n)^2 * \operatorname{csgn}(I*c*x^n)^5 * \operatorname{csgn}(I*c)^2 - 27 * I * \pi^3 * b^3 * \operatorname{csgn}(I*x^n)^2 * \operatorname{csgn}( \\ & I*c*x^n)^4 * \operatorname{csgn}(I*c)^3 + 81 * I * \pi^3 * b^3 * \operatorname{csgn}(I*x^n) * \operatorname{csgn}(I*c*x^n)^7 * \operatorname{csgn}(I*c) - \\ & 108 * I * \ln(c)^2 * \pi * b^3 * \operatorname{csgn}(I*x^n) * \operatorname{csgn}(I*c*x^n) * \operatorname{csgn}(I*c) - 72 * I * \ln(c) * \pi * b^3 * \\ & n * \operatorname{csgn}(I*c*x^n)^3 - 72 * I * \pi * a * b^2 * n * \operatorname{csgn}(I*c*x^n)^3 - 108 * I * \pi * a^2 * b * \operatorname{csgn}(I*x^n) \\ & ) * \operatorname{csgn}(I*c*x^n) * \operatorname{csgn}(I*c) + 72 * I * \ln(c) * \pi * b^3 * n * \operatorname{csgn}(I*c*x^n)^2 * \operatorname{csgn}(I*c) + 72 * \\ & I * \ln(c) * \pi * b^3 * n * \operatorname{csgn}(I*x^n) * \operatorname{csgn}(I*c*x^n)^2 + 216 * I * \ln(c) * \pi * a * b^2 * \operatorname{csgn}(I*x^n) \\ & ) * \operatorname{csgn}(I*c*x^n)^2 + 216 * I * \ln(c) * \pi * a * b^2 * \operatorname{csgn}(I*c*x^n)^2 * \operatorname{csgn}(I*c) \Big) / x^3 \end{aligned}$$

**Maxima [A]** time = 1.10039, size = 184, normalized size = 2.39

$$-\frac{1}{27} \left( 2n \left( \frac{n^2}{x^3} + \frac{3n \log(cx^n)}{x^3} \right) + \frac{9n \log(cx^n)^2}{x^3} \right) b^3 - \frac{2}{9} ab^2 \left( \frac{n^2}{x^3} + \frac{3n \log(cx^n)}{x^3} \right) - \frac{b^3 \log(cx^n)^3}{3x^3} - \frac{ab^2 \log(cx^n)^2}{x^3} - \frac{a^2bn}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^3/x^4,x, algorithm="maxima")

[Out]  $-1/27 * (2 * n * (n^2/x^3 + 3 * n * \log(c * x^n)/x^3) + 9 * n * \log(c * x^n)^2/x^3) * b^3 - 2/9 * a * b^2 * (n^2/x^3 + 3 * n * \log(c * x^n)/x^3) - 1/3 * b^3 * \log(c * x^n)^3/x^3 - a * b^2 * \log(c * x^n)^2/x^3 - 1/3 * a^2 * b * n/x^3 - a^2 * b * \log(c * x^n)/x^3 - 1/3 * a^3/x^3$

**Fricas [B]** time = 0.788247, size = 443, normalized size = 5.75

$$\frac{9b^3n^3 \log(x)^3 + 2b^3n^3 + 9b^3 \log(c)^3 + 6ab^2n^2 + 9a^2bn + 9a^3 + 9(b^3n + 3ab^2) \log(c)^2 + 9(b^3n^3 + 3b^3n^2 \log(c))}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^3/x^4,x, algorithm="fricas")

[Out]  $-1/27 * (9 * b^3 * n^3 * \log(x)^3 + 2 * b^3 * n^3 + 9 * b^3 * \log(c)^3 + 6 * a * b^2 * n^2 + 9 * a^2 * b * n + 9 * a^3 + 9 * (b^3 * n + 3 * a * b^2) * \log(c)^2 + 9 * (b^3 * n^3 + 3 * b^3 * n^2 * \log(c) + 3 * a * b^2 * n^2) * \log(x)^2 + 3 * (2 * b^3 * n^2 + 6 * a * b^2 * n + 9 * a^2 * b) * \log(c) + 3 * (2 * b^3 * n^3 + 9 * b^3 * n * \log(c)^2 + 6 * a * b^2 * n^2 + 9 * a^2 * b * n + 6 * (b^3 * n^2 + 3 * a * b^2 * n) * \log(c)) * \log(x)) / x^3$

**Sympy [B]** time = 3.78093, size = 313, normalized size = 4.06

$$\frac{a^3}{3x^3} - \frac{a^2bn \log(x)}{x^3} - \frac{a^2bn}{3x^3} - \frac{a^2b \log(c)}{x^3} - \frac{ab^2n^2 \log(x)^2}{x^3} - \frac{2ab^2n^2 \log(x)}{3x^3} - \frac{2ab^2n^2}{9x^3} - \frac{2ab^2n \log(c) \log(x)}{x^3} - \frac{2ab^2n}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))\*\*3/x\*\*4,x)

[Out]  $-a^{**3}/(3*x^{**3}) - a^{**2}*b*n*\log(x)/x^{**3} - a^{**2}*b*n/(3*x^{**3}) - a^{**2}*b*\log(c)/x^{**3} - a*b^{**2}*n^{**2}*\log(x)^{**2}/x^{**3} - 2*a*b^{**2}*n^{**2}*\log(x)/(3*x^{**3}) - 2*a*b^{**2}*n^{**2}/(9*x^{**3}) - 2*a*b^{**2}*n*\log(c)*\log(x)/x^{**3} - 2*a*b^{**2}*n*\log(c)/(3*x^{**3}) - a*b^{**2}*\log(c)^{**2}/x^{**3} - b^{**3}*n^{**3}*\log(x)^{**3}/(3*x^{**3}) - b^{**3}*n^{**3}*\log(x)*2/(3*x^{**3}) - 2*b^{**3}*n^{**3}*\log(x)/(9*x^{**3}) - 2*b^{**3}*n^{**3}/(27*x^{**3}) - b^{**3}*n^{**2}*\log(c)*\log(x)^{**2}/x^{**3} - 2*b^{**3}*n^{**2}*\log(c)*\log(x)/(3*x^{**3}) - 2*b^{**3}*n^{**2}*\log(c)/(9*x^{**3}) - b^{**3}*n*\log(c)^{**2}*\log(x)/x^{**3} - b^{**3}*n*\log(c)^{**2}/(3*x^{**3}) - b^{**3}*\log(c)^{**3}/(3*x^{**3})$

**Giac [B]** time = 1.17249, size = 275, normalized size = 3.57

$$\frac{b^3 n^3 \log(x)^3}{3x^3} - \frac{(b^3 n^3 + 3b^3 n^2 \log(c) + 3ab^2 n^2) \log(x)^2}{3x^3} - \frac{(2b^3 n^3 + 6b^3 n^2 \log(c) + 9b^3 n \log(c)^2 + 6ab^2 n^2 + 18ab^2 n \log(c) + 9a^2 b^2 n) \log(x)}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^3/x^4,x, algorithm="giac")

[Out]  $-1/3*b^3*n^3*\log(x)^3/x^3 - 1/3*(b^3*n^3 + 3*b^3*n^2*\log(c) + 3*a*b^2*n^2)*\log(x)^2/x^3 - 1/9*(2*b^3*n^3 + 6*b^3*n^2*\log(c) + 9*b^3*n*\log(c)^2 + 6*a*b^2*n^2 + 18*a*b^2*n*\log(c) + 9*a^2*b*n)*\log(x)/x^3 - 1/27*(2*b^3*n^3 + 6*b^3*n^2*\log(c) + 9*b^3*n*\log(c)^2 + 9*b^3*\log(c)^3 + 6*a*b^2*n^2 + 18*a*b^2*n*\log(c) + 27*a*b^2*\log(c)^2 + 9*a^2*b*n + 27*a^2*b*\log(c) + 9*a^3)/x^3$

$$3.65 \quad \int \frac{x^3}{a+b \log(cx^n)} dx$$

**Optimal.** Leaf size=51

$$\frac{x^4 e^{-\frac{4a}{bn}} (cx^n)^{-4/n} \operatorname{Ei}\left(\frac{4(a+b \log(cx^n))}{bn}\right)}{bn}$$

[Out] (x^4\*ExpIntegralEi[(4\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b\*E^((4\*a)/(b\*n))\*n\*(c\*x^n)^(4/n))

**Rubi [A]** time = 0.0559981, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2310, 2178}

$$\frac{x^4 e^{-\frac{4a}{bn}} (cx^n)^{-4/n} \operatorname{Ei}\left(\frac{4(a+b \log(cx^n))}{bn}\right)}{bn}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b\*Log[c\*x^n]),x]

[Out] (x^4\*ExpIntegralEi[(4\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b\*E^((4\*a)/(b\*n))\*n\*(c\*x^n)^(4/n))

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x)/n]\*(a + b\*x)^p, x], x, Log[c\*x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

#### Rubi steps

$$\begin{aligned} \int \frac{x^3}{a + b \log(cx^n)} dx &= \frac{(x^4 (cx^n)^{-4/n}) \operatorname{Subst}\left(\int \frac{e^{\frac{4x}{a+bx}}}{a+bx} dx, x, \log(cx^n)\right)}{n} \\ &= \frac{e^{-\frac{4a}{bn}} x^4 (cx^n)^{-4/n} \operatorname{Ei}\left(\frac{4(a+b \log(cx^n))}{bn}\right)}{bn} \end{aligned}$$

**Mathematica [A]** time = 0.0577181, size = 51, normalized size = 1.

$$\frac{x^4 e^{-\frac{4a}{bn}} (cx^n)^{-4/n} \operatorname{Ei}\left(\frac{4(a+b \log(cx^n))}{bn}\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b\*Log[c\*x^n]),x]

[Out] (x^4\*ExpIntegralEi[(4\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b\*E^((4\*a)/(b\*n))\*n\*(c\*x^n)^(4/n))

**Maple [F]** time = 0.167, size = 0, normalized size = 0.

$$\int \frac{x^3}{a + b \ln(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a+b\*ln(c\*x^n)),x)

[Out] int(x^3/(a+b\*ln(c\*x^n)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{b \log(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out] integrate(x^3/(b\*log(c\*x^n) + a), x)

**Fricas [A]** time = 0.904, size = 108, normalized size = 2.12

$$\frac{e^{\left(-\frac{4(b \log(c)+a)}{bn}\right)} \log\_integral\left(x^4 e^{\left(\frac{4(b \log(c)+a)}{bn}\right)}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] e^(-4\*(b\*log(c) + a)/(b\*n))\*log\_integral(x^4\*e^(4\*(b\*log(c) + a)/(b\*n)))/(b\*n)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{a + b \log(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(a+b\*ln(c\*x\*\*n)),x)



[Out] Integral( $x^3/(a + b \cdot \log(c \cdot x^n))$ ), x)

---

**Giac [A]** time = 1.19458, size = 65, normalized size = 1.27

$$\frac{\operatorname{Ei}\left(\frac{4 \log(c)}{n} + \frac{4a}{bn} + 4 \log(x)\right) e^{\left(-\frac{4a}{bn}\right)}}{bc^{\frac{4}{n}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate( $x^3/(a+b \cdot \log(c \cdot x^n))$ ),x, algorithm="giac")

[Out] Ei( $4 \cdot \log(c)/n + 4 \cdot a/(b \cdot n) + 4 \cdot \log(x)$ )\*e<sup>(-4\*a/(b\*n))</sup>/(b\*c<sup>(4/n)</sup>\*n)

$$3.66 \quad \int \frac{x^2}{a+b \log(cx^n)} dx$$

**Optimal.** Leaf size=51

$$\frac{x^3 e^{-\frac{3a}{bn}} (cx^n)^{-3/n} \operatorname{Ei}\left(\frac{3(a+b \log(cx^n))}{bn}\right)}{bn}$$

[Out] (x^3\*ExpIntegralEi[(3\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b\*E^(((3\*a)/(b\*n))\*n\*(c\*x^n)^(3/n)))

**Rubi [A]** time = 0.0553589, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2310, 2178}

$$\frac{x^3 e^{-\frac{3a}{bn}} (cx^n)^{-3/n} \operatorname{Ei}\left(\frac{3(a+b \log(cx^n))}{bn}\right)}{bn}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b\*Log[c\*x^n]), x]

[Out] (x^3\*ExpIntegralEi[(3\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b\*E^(((3\*a)/(b\*n))\*n\*(c\*x^n)^(3/n)))

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{x^2}{a+b \log(cx^n)} dx &= \frac{(x^3 (cx^n)^{-3/n}) \operatorname{Subst}\left(\int \frac{e^{\frac{3x}{a+bx}}}{a+bx} dx, x, \log(cx^n)\right)}{n} \\ &= \frac{e^{-\frac{3a}{bn}} x^3 (cx^n)^{-3/n} \operatorname{Ei}\left(\frac{3(a+b \log(cx^n))}{bn}\right)}{bn} \end{aligned}$$

**Mathematica [A]** time = 0.0545041, size = 51, normalized size = 1.

$$\frac{x^3 e^{-\frac{3a}{bn}} (cx^n)^{-3/n} \operatorname{Ei}\left(\frac{3(a+b \log(cx^n))}{bn}\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b\*Log[c\*x^n]),x]

[Out] (x^3\*ExpIntegralEi[(3\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b\*E^((3\*a)/(b\*n))\*n\*(c\*x^n)^(3/n))

**Maple [F]** time = 0.168, size = 0, normalized size = 0.

$$\int \frac{x^2}{a + b \ln(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b\*ln(c\*x^n)),x)

[Out] int(x^2/(a+b\*ln(c\*x^n)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{b \log(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out] integrate(x^2/(b\*log(c\*x^n) + a), x)

**Fricas [A]** time = 0.85628, size = 108, normalized size = 2.12

$$\frac{e^{\left(-\frac{3(b \log(c) + a)}{bn}\right)} \log\_integral\left(x^3 e^{\left(\frac{3(b \log(c) + a)}{bn}\right)}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] e^(-3\*(b\*log(c) + a)/(b\*n))\*log\_integral(x^3\*e^(3\*(b\*log(c) + a)/(b\*n)))/(b\*n)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{a + b \log(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(a+b\*ln(c\*x\*\*n)),x)

[Out] Integral(x\*\*2/(a + b\*log(c\*x\*\*n)), x)

---

**Giac [A]** time = 1.18288, size = 65, normalized size = 1.27

$$\frac{\operatorname{Ei}\left(\frac{3 \log(c)}{n} + \frac{3a}{bn} + 3 \log(x)\right) e^{\left(-\frac{3a}{bn}\right)}}{bc^{\frac{3}{n}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] Ei(3\*log(c)/n + 3\*a/(b\*n) + 3\*log(x))\*e^(-3\*a/(b\*n))/(b\*c^(3/n)\*n)

$$3.67 \quad \int \frac{x}{a+b \log(cx^n)} dx$$

**Optimal.** Leaf size=51

$$\frac{x^2 e^{-\frac{2a}{bn}} (cx^n)^{-2/n} \operatorname{Ei}\left(\frac{2(a+b \log(cx^n))}{bn}\right)}{bn}$$

[Out] (x^2\*ExpIntegralEi[(2\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b\*E^((2\*a)/(b\*n))\*n\*(c\*x^n)^(2/n))

**Rubi [A]** time = 0.0453849, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2310, 2178}

$$\frac{x^2 e^{-\frac{2a}{bn}} (cx^n)^{-2/n} \operatorname{Ei}\left(\frac{2(a+b \log(cx^n))}{bn}\right)}{bn}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b\*Log[c\*x^n]),x]

[Out] (x^2\*ExpIntegralEi[(2\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b\*E^((2\*a)/(b\*n))\*n\*(c\*x^n)^(2/n))

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{x}{a+b \log(cx^n)} dx &= \frac{(x^2 (cx^n)^{-2/n}) \operatorname{Subst}\left(\int \frac{e^{\frac{2x}{a+bx}}}{a+bx} dx, x, \log(cx^n)\right)}{n} \\ &= \frac{e^{-\frac{2a}{bn}} x^2 (cx^n)^{-2/n} \operatorname{Ei}\left(\frac{2(a+b \log(cx^n))}{bn}\right)}{bn} \end{aligned}$$

**Mathematica [A]** time = 0.0512214, size = 51, normalized size = 1.

$$\frac{x^2 e^{-\frac{2a}{bn}} (cx^n)^{-2/n} \operatorname{Ei}\left(\frac{2(a+b \log(cx^n))}{bn}\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b\*Log[c\*x^n]),x]

[Out] (x^2\*ExpIntegralEi[(2\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b\*E^((2\*a)/(b\*n))\*n\*(c\*x^n)^(2/n))

**Maple [F]** time = 0.155, size = 0, normalized size = 0.

$$\int \frac{x}{a + b \ln(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b\*ln(c\*x^n)),x)

[Out] int(x/(a+b\*ln(c\*x^n)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b \log(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out] integrate(x/(b\*log(c\*x^n) + a), x)

**Fricas [A]** time = 0.818718, size = 108, normalized size = 2.12

$$\frac{e^{\left(-\frac{2(b \log(c)+a)}{bn}\right)} \log\_integral\left(x^2 e^{\left(\frac{2(b \log(c)+a)}{bn}\right)}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] e^(-2\*(b\*log(c) + a)/(b\*n))\*log\_integral(x^2\*e^(2\*(b\*log(c) + a)/(b\*n)))/(b\*n)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{a + b \log(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*ln(c\*x\*\*n)),x)

[Out] Integral(x/(a + b\*log(c\*x\*\*n)), x)

---

**Giac [A]** time = 1.20011, size = 65, normalized size = 1.27

$$\frac{\operatorname{Ei}\left(\frac{2 \log(c)}{n} + \frac{2a}{bn} + 2 \log(x)\right) e^{\left(-\frac{2a}{bn}\right)}}{bc^{\frac{2}{n}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] Ei(2\*log(c)/n + 2\*a/(b\*n) + 2\*log(x))\*e^(-2\*a/(b\*n))/(b\*c^(2/n)\*n)

$$3.68 \quad \int \frac{1}{a+b \log(cx^n)} dx$$

**Optimal.** Leaf size=48

$$\frac{x e^{-\frac{a}{bn}} (cx^n)^{-1/n} \operatorname{Ei}\left(\frac{a+b \log(cx^n)}{bn}\right)}{bn}$$

[Out] (x\*ExpIntegralEi[(a + b\*Log[c\*x^n])/(b\*n)])/(b\*E^(a/(b\*n))\*n\*(c\*x^n)^n^(-1))

**Rubi [A]** time = 0.0358402, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2300, 2178}

$$\frac{x e^{-\frac{a}{bn}} (cx^n)^{-1/n} \operatorname{Ei}\left(\frac{a+b \log(cx^n)}{bn}\right)}{bn}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^(-1), x]

[Out] (x\*ExpIntegralEi[(a + b\*Log[c\*x^n])/(b\*n)])/(b\*E^(a/(b\*n))\*n\*(c\*x^n)^n^(-1))

#### Rule 2300

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^p, x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[E^(x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{1}{a+b \log(cx^n)} dx &= \frac{(x (cx^n)^{-1/n}) \operatorname{Subst}\left(\int \frac{e^{\frac{x}{n}}}{a+bx} dx, x, \log(cx^n)\right)}{n} \\ &= \frac{e^{-\frac{a}{bn}} x (cx^n)^{-1/n} \operatorname{Ei}\left(\frac{a+b \log(cx^n)}{bn}\right)}{bn} \end{aligned}$$

**Mathematica [A]** time = 0.0410216, size = 48, normalized size = 1.

$$\frac{x e^{-\frac{a}{bn}} (cx^n)^{-1/n} \operatorname{Ei}\left(\frac{a+b \log(cx^n)}{bn}\right)}{bn}$$

Antiderivative was successfully verified.



[In] Integrate[(a + b\*Log[c\*x^n])^(-1), x]

[Out] (x\*ExpIntegralEi[(a + b\*Log[c\*x^n])/(b\*n)])/(b\*E^(a/(b\*n))\*n\*(c\*x^n)^n^(-1))

**Maple [C]** time = 0.267, size = 241, normalized size = 5.

$$-\frac{1}{bn} e^{-\frac{ib\pi \operatorname{csgn}(ix^n)(\operatorname{csgn}(icx^n))^2 - ib\pi \operatorname{csgn}(ix^n)\operatorname{csgn}(icx^n)\operatorname{csgn}(ic) - ib\pi (\operatorname{csgn}(icx^n))^3 + ib\pi (\operatorname{csgn}(icx^n))^2 \operatorname{csgn}(ic) - 2 \ln(x)bn + 2b \ln(c) + 2b \ln(x^n) + 2a}{2bn}} \operatorname{Ei}\left(1, -\ln(x) - \frac{a}{bn}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*ln(c\*x^n)), x)

[Out] -1/b/n\*exp(-1/2\*(I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2 - I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c) - I\*b\*Pi\*csgn(I\*c\*x^n)^3 + I\*b\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*c) - 2\*ln(x)\*b\*n + 2\*b\*ln(c) + 2\*b\*ln(x^n) + 2\*a)/b/n)\*Ei(1, -ln(x) - 1/2\*(I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2 - I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c) - I\*b\*Pi\*csgn(I\*c\*x^n)^3 + I\*b\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*c) + 2\*b\*ln(c) + 2\*b\*(ln(x^n) - n\*ln(x)) + 2\*a)/b/n)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b \log(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*log(c\*x^n)), x, algorithm="maxima")

[Out] integrate(1/(b\*log(c\*x^n) + a), x)

**Fricas [A]** time = 0.836507, size = 100, normalized size = 2.08

$$\frac{e^{\left(-\frac{b \log(c)+a}{bn}\right)} \log\_integral\left(x e^{\left(\frac{b \log(c)+a}{bn}\right)}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*log(c\*x^n)), x, algorithm="fricas")

[Out] e^(-(b\*log(c) + a)/(b\*n))\*log\_integral(x\*e^((b\*log(c) + a)/(b\*n)))/(b\*n)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a + b \log(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*ln(c\*x\*\*n)),x)

[Out] Integral(1/(a + b\*log(c\*x\*\*n)), x)

**Giac [A]** time = 1.21367, size = 57, normalized size = 1.19

$$\frac{\operatorname{Ei}\left(\frac{\log(c)}{n} + \frac{a}{bn} + \log(x)\right) e^{\left(-\frac{a}{bn}\right)}}{bc^{\left(\frac{1}{n}\right)}n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] Ei(log(c)/n + a/(b\*n) + log(x))\*e^(-a/(b\*n))/(b\*c^(1/n)\*n)

$$3.69 \quad \int \frac{1}{x(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=18

$$\frac{\log(a + b \log(cx^n))}{bn}$$

[Out] Log[a + b\*Log[c\*x^n]]/(b\*n)

**Rubi [A]** time = 0.0246855, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2302, 29}

$$\frac{\log(a + b \log(cx^n))}{bn}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(a + b\*Log[c\*x^n])),x]

[Out] Log[a + b\*Log[c\*x^n]]/(b\*n)

**Rule 2302**

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] := Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

**Rule 29**

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

**Rubi steps**

$$\begin{aligned} \int \frac{1}{x(a + b \log(cx^n))} dx &= \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, a + b \log(cx^n)\right)}{bn} \\ &= \frac{\log(a + b \log(cx^n))}{bn} \end{aligned}$$

**Mathematica [A]** time = 0.0158856, size = 18, normalized size = 1.

$$\frac{\log(a + b \log(cx^n))}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(a + b\*Log[c\*x^n])),x]

[Out] Log[a + b\*Log[c\*x^n]]/(b\*n)

**Maple [A]** time = 0.038, size = 19, normalized size = 1.1

$$\frac{\ln(a + b \ln(cx^n))}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b\*ln(c\*x^n)),x)

[Out] ln(a+b\*ln(c\*x^n))/b/n

**Maxima [A]** time = 1.05798, size = 24, normalized size = 1.33

$$\frac{\log(b \log(cx^n) + a)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out] log(b\*log(c\*x^n) + a)/(b\*n)

**Fricas [A]** time = 0.827532, size = 51, normalized size = 2.83

$$\frac{\log(bn \log(x) + b \log(c) + a)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] log(b\*n\*log(x) + b\*log(c) + a)/(b\*n)

**Sympy [A]** time = 1.98887, size = 32, normalized size = 1.78

$$\begin{cases} \frac{\log(x)}{\log(x)} & \text{for } b = 0 \wedge (b = 0 \vee n = 0) \\ \frac{a + b \log(c)}{\log(\frac{a}{b} + n \log(x) + \log(c))} & \text{for } n = 0 \\ \frac{\log(\frac{a}{b} + n \log(x) + \log(c))}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*ln(c\*x\*\*n)),x)

[Out] Piecewise((log(x)/a, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)/(a + b\*log(c)), Eq(n, 0)), (log(a/b + n\*log(x) + log(c))/(b\*n), True))

**Giac [B]** time = 1.23544, size = 61, normalized size = 3.39

$$\frac{\log\left(\frac{1}{4}(\pi bn(\operatorname{sgn}(x) - 1) + \pi b(\operatorname{sgn}(c) - 1))^2 + (bn \log(|x|) + b \log(|c|) + a)^2\right)}{2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/2*log(1/4*(pi*b*n*(sgn(x) - 1) + pi*b*(sgn(c) - 1))^2 + (b*n*log(abs(x)) + b*log(abs(c)) + a)^2)/(b*n)
```

$$3.70 \quad \int \frac{1}{x^2(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=48

$$\frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} \operatorname{Ei}\left(-\frac{a+b \log(cx^n)}{bn}\right)}{bnx}$$

[Out] (E^(a/(b\*n))\*(c\*x^n)^n^(-1)\*ExpIntegralEi[-((a + b\*Log[c\*x^n])/(b\*n))])/(b\*n\*x)

**Rubi [A]** time = 0.0513248, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2310, 2178}

$$\frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} \operatorname{Ei}\left(-\frac{a+b \log(cx^n)}{bn}\right)}{bnx}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(a + b\*Log[c\*x^n])),x]

[Out] (E^(a/(b\*n))\*(c\*x^n)^n^(-1)\*ExpIntegralEi[-((a + b\*Log[c\*x^n])/(b\*n))])/(b\*n\*x)

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{1}{x^2(a+b \log(cx^n))} dx &= \frac{(cx^n)^{\frac{1}{n}} \operatorname{Subst}\left(\int \frac{e^{-\frac{x}{n}}}{a+bx} dx, x, \log(cx^n)\right)}{nx} \\ &= \frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} \operatorname{Ei}\left(-\frac{a+b \log(cx^n)}{bn}\right)}{bnx} \end{aligned}$$

**Mathematica [A]** time = 0.0485528, size = 48, normalized size = 1.

$$\frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} \operatorname{Ei}\left(-\frac{a+b \log(cx^n)}{bn}\right)}{bnx}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(a + b\*Log[c\*x^n])),x]

[Out] (E^(a/(b\*n))\*(c\*x^n)^n^(-1)\*ExpIntegralEi[-((a + b\*Log[c\*x^n])/(b\*n))])/(b\*n\*x)

**Maple [F]** time = 0.151, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a + b \ln(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+b\*ln(c\*x^n)),x)

[Out] int(1/x^2/(a+b\*ln(c\*x^n)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \log(cx^n) + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out] integrate(1/((b\*log(c\*x^n) + a)\*x^2), x)

**Fricas [A]** time = 0.736541, size = 100, normalized size = 2.08

$$\frac{e^{\left(\frac{b \log(c)+a}{bn}\right)} \log\_integral\left(\frac{e^{\left(-\frac{b \log(c)+a}{bn}\right)}}{x}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] e^((b\*log(c) + a)/(b\*n))\*log\_integral(e^(-(b\*log(c) + a)/(b\*n))/x)/(b\*n)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a + b \log(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(a+b\*ln(c\*x\*\*n)),x)

[Out] Integral(1/(x\*\*2\*(a + b\*log(c\*x\*\*n))), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \log(cx^n) + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] integrate(1/((b\*log(c\*x^n) + a)\*x^2), x)



$$3.71 \quad \int \frac{1}{x^3(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=51

$$\frac{e^{\frac{2a}{bn}} (cx^n)^{2/n} \operatorname{Ei}\left(-\frac{2(a+b \log(cx^n))}{bn}\right)}{bnx^2}$$

[Out] (E^((2\*a)/(b\*n))\*(c\*x^n)^(2/n)\*ExpIntegralEi[(-2\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b\*n\*x^2)

**Rubi [A]** time = 0.0513079, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2310, 2178}

$$\frac{e^{\frac{2a}{bn}} (cx^n)^{2/n} \operatorname{Ei}\left(-\frac{2(a+b \log(cx^n))}{bn}\right)}{bnx^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*(a + b\*Log[c\*x^n])), x]

[Out] (E^((2\*a)/(b\*n))\*(c\*x^n)^(2/n)\*ExpIntegralEi[(-2\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b\*n\*x^2)

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x)/n]\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{1}{x^3(a+b \log(cx^n))} dx &= \frac{(cx^n)^{2/n} \operatorname{Subst}\left(\int \frac{e^{-\frac{2x}{a+bx}}}{a+bx} dx, x, \log(cx^n)\right)}{nx^2} \\ &= \frac{e^{\frac{2a}{bn}} (cx^n)^{2/n} \operatorname{Ei}\left(-\frac{2(a+b \log(cx^n))}{bn}\right)}{bnx^2} \end{aligned}$$

**Mathematica [A]** time = 0.0500862, size = 51, normalized size = 1.

$$\frac{e^{\frac{2a}{bn}} (cx^n)^{2/n} \operatorname{Ei}\left(-\frac{2(a+b \log(cx^n))}{bn}\right)}{bnx^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(a + b\*Log[c\*x^n])),x]

[Out] (E^((2\*a)/(b\*n))\*(c\*x^n)^(2/n)\*ExpIntegralEi[(-2\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b\*n\*x^2)

**Maple [F]** time = 0.165, size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (a + b \ln(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(a+b\*ln(c\*x^n)),x)

[Out] int(1/x^3/(a+b\*ln(c\*x^n)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \log(cx^n) + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out] integrate(1/((b\*log(c\*x^n) + a)\*x^3), x)

**Fricas [A]** time = 0.785426, size = 108, normalized size = 2.12

$$\frac{e^{\left(\frac{2(b \log(c)+a)}{bn}\right)} \log\_integral\left(\frac{e^{\left(\frac{-2(b \log(c)+a)}{bn}\right)}}{x^2}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] e^(2\*(b\*log(c) + a)/(b\*n))\*log\_integral(e^(-2\*(b\*log(c) + a)/(b\*n))/x^2)/(b\*n)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (a + b \log(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/(a+b\*ln(c\*x\*\*n)),x)

[Out] Integral(1/(x\*\*3\*(a + b\*log(c\*x\*\*n))), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \log(cx^n) + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] integrate(1/((b\*log(c\*x^n) + a)\*x^3), x)

$$3.72 \quad \int \frac{1}{x^4(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=51

$$\frac{e^{\frac{3a}{bn}} (cx^n)^{3/n} \operatorname{Ei}\left(-\frac{3(a+b \log(cx^n))}{bn}\right)}{bnx^3}$$

[Out] (E^((3\*a)/(b\*n))\*(c\*x^n)^(3/n)\*ExpIntegralEi[(-3\*(a + b\*Log[c\*x^n]))]/(b\*n)]/(b\*n\*x^3)

**Rubi [A]** time = 0.0517936, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2310, 2178}

$$\frac{e^{\frac{3a}{bn}} (cx^n)^{3/n} \operatorname{Ei}\left(-\frac{3(a+b \log(cx^n))}{bn}\right)}{bnx^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a + b\*Log[c\*x^n])),x]

[Out] (E^((3\*a)/(b\*n))\*(c\*x^n)^(3/n)\*ExpIntegralEi[(-3\*(a + b\*Log[c\*x^n]))]/(b\*n)]/(b\*n\*x^3)

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{1}{x^4(a+b \log(cx^n))} dx &= \frac{(cx^n)^{3/n} \operatorname{Subst}\left(\int \frac{e^{-\frac{3x}{a+bx}}}{a+bx} dx, x, \log(cx^n)\right)}{nx^3} \\ &= \frac{e^{\frac{3a}{bn}} (cx^n)^{3/n} \operatorname{Ei}\left(-\frac{3(a+b \log(cx^n))}{bn}\right)}{bnx^3} \end{aligned}$$

**Mathematica [A]** time = 0.052378, size = 51, normalized size = 1.

$$\frac{e^{\frac{3a}{bn}} (cx^n)^{3/n} \operatorname{Ei}\left(-\frac{3(a+b \log(cx^n))}{bn}\right)}{bnx^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(a + b\*Log[c\*x^n])),x]

[Out] (E^((3\*a)/(b\*n))\*(c\*x^n)^(3/n)\*ExpIntegralEi[(-3\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b\*n\*x^3)

**Maple [F]** time = 0.184, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 (a + b \ln(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(a+b\*ln(c\*x^n)),x)

[Out] int(1/x^4/(a+b\*ln(c\*x^n)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \log(cx^n) + a)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out] integrate(1/((b\*log(c\*x^n) + a)\*x^4), x)

**Fricas [A]** time = 0.799937, size = 108, normalized size = 2.12

$$\frac{e^{\left(\frac{3(b \log(c)+a)}{bn}\right)} \log\_integral\left(\frac{e^{\left(-\frac{3(b \log(c)+a)}{bn}\right)}}{x^3}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] e^(3\*(b\*log(c) + a)/(b\*n))\*log\_integral(e^(-3\*(b\*log(c) + a)/(b\*n))/x^3)/(b\*n)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 (a + b \log(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*4/(a+b\*ln(c\*x\*\*n)),x)

[Out] Integral(1/(x\*\*4\*(a + b\*log(c\*x\*\*n))), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \log(cx^n) + a)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] integrate(1/((b\*log(c\*x^n) + a)\*x^4), x)

### 3.73 $\int \frac{x^3}{(a+b \log(cx^n))^2} dx$

**Optimal.** Leaf size=76

$$\frac{4x^4 e^{-\frac{4a}{bn}} (cx^n)^{-4/n} \operatorname{Ei}\left(\frac{4(a+b \log(cx^n))}{bn}\right)}{b^2 n^2} - \frac{x^4}{bn(a+b \log(cx^n))}$$

[Out]  $(4*x^4*ExpIntegralEi[(4*(a + b*Log[c*x^n]))/(b*n)])/(b^2*E^((4*a)/(b*n))*n^2*(c*x^n)^(4/n)) - x^4/(b*n*(a + b*Log[c*x^n]))$

**Rubi [A]** time = 0.0785481, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {2306, 2310, 2178}

$$\frac{4x^4 e^{-\frac{4a}{bn}} (cx^n)^{-4/n} \operatorname{Ei}\left(\frac{4(a+b \log(cx^n))}{bn}\right)}{b^2 n^2} - \frac{x^4}{bn(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3/(a + b*\text{Log}[c*x^n])^2, x]$

[Out]  $(4*x^4*ExpIntegralEi[(4*(a + b*Log[c*x^n]))/(b*n)])/(b^2*E^((4*a)/(b*n))*n^2*(c*x^n)^(4/n)) - x^4/(b*n*(a + b*Log[c*x^n]))$

#### Rule 2306

$\text{Int}[(a_. + \text{Log}[(c_.)*(x_.)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_.))^(m_.), x\_Symbol] \rightarrow \text{Simp}[(d*x)^(m+1)*(a + b*\text{Log}[c*x^n])^(p+1)/(b*d*n*(p+1)), x] - \text{Dist}[(m+1)/(b*n*(p+1)), \text{Int}[(d*x)^(m*(a + b*\text{Log}[c*x^n])^(p+1), x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{NeQ}[m, -1] \&\& \text{LtQ}[p, -1]$

#### Rule 2310

$\text{Int}[(a_. + \text{Log}[(c_.)*(x_.)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_.))^(m_.), x\_Symbol] \rightarrow \text{Dist}[(d*x)^(m+1)/(d*n*(c*x^n)^((m+1)/n)), \text{Subst}[\text{Int}[E^((m+1)*x/n)*(a + b*x)^p, x], x, \text{Log}[c*x^n]], x] /; \text{FreeQ}\{a, b, c, d, m, n, p\}, x]$

#### Rule 2178

$\text{Int}[(F_)^((g_.)*((e_.) + (f_.)*(x_.)))/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \text{Simp}[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*\text{Log}[F])/d])/d, x] /; \text{FreeQ}\{F, c, d, e, f, g\}, x] \&\& !\$UseGamma === \text{True}$

#### Rubi steps

$$\begin{aligned} \int \frac{x^3}{(a + b \log(cx^n))^2} dx &= -\frac{x^4}{bn(a + b \log(cx^n))} + \frac{4 \int \frac{x^3}{a + b \log(cx^n)} dx}{bn} \\ &= -\frac{x^4}{bn(a + b \log(cx^n))} + \frac{(4x^4 (cx^n)^{-4/n}) \text{Subst} \left( \int \frac{e^{\frac{4x}{a+bx}}}{a+bx} dx, x, \log(cx^n) \right)}{bn^2} \\ &= \frac{4e^{-\frac{4a}{bn}} x^4 (cx^n)^{-4/n} \text{Ei} \left( \frac{4(a+b \log(cx^n))}{bn} \right)}{b^2 n^2} - \frac{x^4}{bn(a + b \log(cx^n))} \end{aligned}$$

**Mathematica [A]** time = 0.118405, size = 70, normalized size = 0.92

$$\frac{x^4 \left( 4e^{-\frac{4a}{bn}} (cx^n)^{-4/n} \text{Ei} \left( \frac{4(a+b \log(cx^n))}{bn} \right) - \frac{bn}{a+b \log(cx^n)} \right)}{b^2 n^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b\*Log[c\*x^n])^2,x]

[Out] (x^4\*((4\*ExpIntegralEi[(4\*(a + b\*Log[c\*x^n]))/(b\*n)])/(E^((4\*a)/(b\*n))\*(c\*x^n)^(4/n)) - (b\*n)/(a + b\*Log[c\*x^n])))/(b^2\*n^2)

**Maple [F]** time = 0.747, size = 0, normalized size = 0.

$$\int \frac{x^3}{(a + b \ln(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a+b\*ln(c\*x^n))^2,x)

[Out] int(x^3/(a+b\*ln(c\*x^n))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{x^4}{b^2 n \log(c) + b^2 n \log(x^n) + abn} + 4 \int \frac{x^3}{b^2 n \log(c) + b^2 n \log(x^n) + abn} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] -x^4/(b^2\*n\*log(c) + b^2\*n\*log(x^n) + a\*b\*n) + 4\*integrate(x^3/(b^2\*n\*log(c) + b^2\*n\*log(x^n) + a\*b\*n), x)

**Fricas [A]** time = 0.833669, size = 257, normalized size = 3.38

$$\frac{\left( bnx^4 e^{\left( \frac{4(b \log(c)+a)}{bn} \right)} - 4(bn \log(x) + b \log(c) + a) \log\_integral \left( x^4 e^{\left( \frac{4(b \log(c)+a)}{bn} \right)} \right) \right) e^{-\frac{4(b \log(c)+a)}{bn}}}{b^3 n^3 \log(x) + b^3 n^2 \log(c) + ab^2 n^2}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out]  $-(b*n*x^4*e^{(4*(b*\log(c) + a)/(b*n))} - 4*(b*n*\log(x) + b*\log(c) + a)*\log\_integral(x^4*e^{(4*(b*\log(c) + a)/(b*n))}))*e^{(-4*(b*\log(c) + a)/(b*n))}/(b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(a + b \log(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Integral(x\*\*3/(a + b\*log(c\*x\*\*n))\*\*2, x)

**Giac [B]** time = 1.28218, size = 352, normalized size = 4.63

$$\frac{bnx^4}{b^3n^3 \log(x) + b^3n^2 \log(c) + ab^2n^2} + \frac{4bn\text{Ei}\left(\frac{4\log(c)}{n} + \frac{4a}{bn} + 4\log(x)\right)e^{\left(-\frac{4a}{bn}\right)\log(x)}}{(b^3n^3 \log(x) + b^3n^2 \log(c) + ab^2n^2)c^{\frac{4}{n}}} + \frac{4b\text{Ei}\left(\frac{4\log(c)}{n} + \frac{4a}{bn} + 4\log(x)\right)}{(b^3n^3 \log(x) + b^3n^2 \log(c) + ab^2n^2)c^{\frac{4}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out]  $-b*n*x^4/(b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2) + 4*b*n*\text{Ei}(4*\log(c)/n + 4*a/(b*n) + 4*\log(x))*e^{(-4*a/(b*n))*\log(x)}/((b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2)*c^{(4/n)}) + 4*b*\text{Ei}(4*\log(c)/n + 4*a/(b*n) + 4*\log(x))*e^{(-4*a/(b*n))*\log(c)}/((b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2)*c^{(4/n)}) + 4*a*\text{Ei}(4*\log(c)/n + 4*a/(b*n) + 4*\log(x))*e^{(-4*a/(b*n))}/((b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2)*c^{(4/n)})$

$$3.74 \quad \int \frac{x^2}{(a+b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=76

$$\frac{3x^3 e^{-\frac{3a}{bn}} (cx^n)^{-3/n} \operatorname{Ei}\left(\frac{3(a+b \log(cx^n))}{bn}\right)}{b^2 n^2} - \frac{x^3}{bn(a+b \log(cx^n))}$$

[Out] (3\*x^3\*ExpIntegralEi[(3\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b^2\*E^((3\*a)/(b\*n))\*n^2\*(c\*x^n)^(3/n)) - x^3/(b\*n\*(a + b\*Log[c\*x^n]))

**Rubi [A]** time = 0.0793871, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {2306, 2310, 2178}

$$\frac{3x^3 e^{-\frac{3a}{bn}} (cx^n)^{-3/n} \operatorname{Ei}\left(\frac{3(a+b \log(cx^n))}{bn}\right)}{b^2 n^2} - \frac{x^3}{bn(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b\*Log[c\*x^n])^2, x]

[Out] (3\*x^3\*ExpIntegralEi[(3\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b^2\*E^((3\*a)/(b\*n))\*n^2\*(c\*x^n)^(3/n)) - x^3/(b\*n\*(a + b\*Log[c\*x^n]))

#### Rule 2306

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^(p + 1))/(b*d*n*(p + 1)), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

#### Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol]
:> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

#### Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

#### Rubi steps

$$\begin{aligned} \int \frac{x^2}{(a + b \log(cx^n))^2} dx &= -\frac{x^3}{bn(a + b \log(cx^n))} + \frac{3 \int \frac{x^2}{a + b \log(cx^n)} dx}{bn} \\ &= -\frac{x^3}{bn(a + b \log(cx^n))} + \frac{(3x^3 (cx^n)^{-3/n}) \text{Subst}\left(\int \frac{e^{\frac{3x}{a+bx}}}{a+bx} dx, x, \log(cx^n)\right)}{bn^2} \\ &= \frac{3e^{-\frac{3a}{bn}} x^3 (cx^n)^{-3/n} \text{Ei}\left(\frac{3(a+b \log(cx^n))}{bn}\right)}{b^2 n^2} - \frac{x^3}{bn(a + b \log(cx^n))} \end{aligned}$$

**Mathematica [A]** time = 0.120686, size = 70, normalized size = 0.92

$$\frac{x^3 \left( 3e^{-\frac{3a}{bn}} (cx^n)^{-3/n} \text{Ei}\left(\frac{3(a+b \log(cx^n))}{bn}\right) - \frac{bn}{a+b \log(cx^n)} \right)}{b^2 n^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b\*Log[c\*x^n])^2,x]

[Out] (x^3\*((3\*ExpIntegralEi[(3\*(a + b\*Log[c\*x^n]))]/(b\*n)))/(E^((3\*a)/(b\*n))\*(c\*x^n)^(3/n)) - (b\*n)/(a + b\*Log[c\*x^n]))/(b^2\*n^2)

**Maple [F]** time = 0.691, size = 0, normalized size = 0.

$$\int \frac{x^2}{(a + b \ln(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b\*ln(c\*x^n))^2,x)

[Out] int(x^2/(a+b\*ln(c\*x^n))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{x^3}{b^2 n \log(c) + b^2 n \log(x^n) + abn} + 3 \int \frac{x^2}{b^2 n \log(c) + b^2 n \log(x^n) + abn} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] -x^3/(b^2\*n\*log(c) + b^2\*n\*log(x^n) + a\*b\*n) + 3\*integrate(x^2/(b^2\*n\*log(c) + b^2\*n\*log(x^n) + a\*b\*n), x)

**Fricas [A]** time = 0.888405, size = 257, normalized size = 3.38

$$\frac{\left( bnx^3 e^{\left( \frac{3(b \log(c)+a)}{bn} \right)} - 3(bn \log(x) + b \log(c) + a) \log\_integral \left( x^3 e^{\left( \frac{3(b \log(c)+a)}{bn} \right)} \right) \right) e^{\left( -\frac{3(b \log(c)+a)}{bn} \right)}}{b^3 n^3 \log(x) + b^3 n^2 \log(c) + ab^2 n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out]  $-(b*n*x^3*e^{(3*(b*\log(c) + a)/(b*n))} - 3*(b*n*\log(x) + b*\log(c) + a)*\log\_integral(x^3*e^{(3*(b*\log(c) + a)/(b*n))})*e^{(-3*(b*\log(c) + a)/(b*n))}/(b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a + b \log(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Integral(x\*\*2/(a + b\*log(c\*x\*\*n))\*\*2, x)

**Giac [B]** time = 1.33935, size = 352, normalized size = 4.63

$$-\frac{bnx^3}{b^3n^3 \log(x) + b^3n^2 \log(c) + ab^2n^2} + \frac{3bnEi\left(\frac{3 \log(c)}{n} + \frac{3a}{bn} + 3 \log(x)\right) e^{\left(-\frac{3a}{bn}\right) \log(x)}}{(b^3n^3 \log(x) + b^3n^2 \log(c) + ab^2n^2)c^{\frac{3}{n}}} + \frac{3bEi\left(\frac{3 \log(c)}{n} + \frac{3a}{bn} + 3 \log(x)\right)}{(b^3n^3 \log(x) + b^3n^2 \log(c) + ab^2n^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out]  $-b*n*x^3/(b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2) + 3*b*n*Ei(3*\log(c)/n + 3*a/(b*n) + 3*\log(x))*e^{(-3*a/(b*n))*\log(x)}/((b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2)*c^{(3/n)}) + 3*b*Ei(3*\log(c)/n + 3*a/(b*n) + 3*\log(x))*e^{(-3*a/(b*n))*\log(c)}/((b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2)*c^{(3/n)}) + 3*a*Ei(3*\log(c)/n + 3*a/(b*n) + 3*\log(x))*e^{(-3*a/(b*n))}/((b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2)*c^{(3/n)})$

$$3.75 \quad \int \frac{x}{(a+b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=76

$$\frac{2x^2 e^{-\frac{2a}{bn}} (cx^n)^{-2/n} \operatorname{Ei}\left(\frac{2(a+b \log(cx^n))}{bn}\right)}{b^2 n^2} - \frac{x^2}{bn(a+b \log(cx^n))}$$

[Out] (2\*x^2\*ExpIntegralEi[(2\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b^2\*E^((2\*a)/(b\*n))\*n^2\*(c\*x^n)^(2/n)) - x^2/(b\*n\*(a + b\*Log[c\*x^n]))

**Rubi [A]** time = 0.0591005, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {2306, 2310, 2178}

$$\frac{2x^2 e^{-\frac{2a}{bn}} (cx^n)^{-2/n} \operatorname{Ei}\left(\frac{2(a+b \log(cx^n))}{bn}\right)}{b^2 n^2} - \frac{x^2}{bn(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b\*Log[c\*x^n])^2,x]

[Out] (2\*x^2\*ExpIntegralEi[(2\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b^2\*E^((2\*a)/(b\*n))\*n^2\*(c\*x^n)^(2/n)) - x^2/(b\*n\*(a + b\*Log[c\*x^n]))

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{x}{(a+b \log(cx^n))^2} dx &= -\frac{x^2}{bn(a+b \log(cx^n))} + \frac{2 \int \frac{x}{a+b \log(cx^n)} dx}{bn} \\ &= -\frac{x^2}{bn(a+b \log(cx^n))} + \frac{(2x^2 (cx^n)^{-2/n}) \operatorname{Subst}\left(\int \frac{e^{\frac{2x}{a+bx}}}{a+bx} dx, x, \log(cx^n)\right)}{bn^2} \\ &= \frac{2e^{-\frac{2a}{bn}} x^2 (cx^n)^{-2/n} \operatorname{Ei}\left(\frac{2(a+b \log(cx^n))}{bn}\right)}{b^2 n^2} - \frac{x^2}{bn(a+b \log(cx^n))} \end{aligned}$$

**Mathematica [A]** time = 0.117886, size = 70, normalized size = 0.92

$$\frac{x^2 \left( 2e^{-\frac{2a}{bn}} (cx^n)^{-2/n} \operatorname{Ei} \left( \frac{2(a+b \log(cx^n))}{bn} \right) - \frac{bn}{a+b \log(cx^n)} \right)}{b^2 n^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b\*Log[c\*x^n])^2,x]

[Out] (x^2\*((2\*ExpIntegralEi[(2\*(a + b\*Log[c\*x^n]))/(b\*n)])/(E^((2\*a)/(b\*n))\*(c\*x^n)^(2/n)) - (b\*n)/(a + b\*Log[c\*x^n])))/(b^2\*n^2)

**Maple [F]** time = 0.65, size = 0, normalized size = 0.

$$\int \frac{x}{(a + b \ln(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b\*ln(c\*x^n))^2,x)

[Out] int(x/(a+b\*ln(c\*x^n))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{x^2}{b^2 n \log(c) + b^2 n \log(x^n) + abn} + 2 \int \frac{x}{b^2 n \log(c) + b^2 n \log(x^n) + abn} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] -x^2/(b^2\*n\*log(c) + b^2\*n\*log(x^n) + a\*b\*n) + 2\*integrate(x/(b^2\*n\*log(c) + b^2\*n\*log(x^n) + a\*b\*n), x)

**Fricas [A]** time = 0.899177, size = 257, normalized size = 3.38

$$\frac{\left( bnx^2 e^{\left( \frac{2(b \log(c)+a)}{bn} \right)} - 2(bn \log(x) + b \log(c) + a) \log\_integral \left( x^2 e^{\left( \frac{2(b \log(c)+a)}{bn} \right)} \right) \right) e^{\left( -\frac{2(b \log(c)+a)}{bn} \right)}}{b^3 n^3 \log(x) + b^3 n^2 \log(c) + ab^2 n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] -(b\*n\*x^2\*e^(2\*(b\*log(c) + a)/(b\*n)) - 2\*(b\*n\*log(x) + b\*log(c) + a)\*log\_integral(x^2\*e^(2\*(b\*log(c) + a)/(b\*n))))\*e^(-2\*(b\*log(c) + a)/(b\*n))/(b^3\*n^3\*log(x) + b^3\*n^2\*log(c) + a\*b^2\*n^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a + b \log(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Integral(x/(a + b\*log(c\*x\*\*n))\*\*2, x)

**Giac [B]** time = 1.32602, size = 352, normalized size = 4.63

$$\frac{bnx^2}{b^3n^3 \log(x) + b^3n^2 \log(c) + ab^2n^2} + \frac{2bn\text{Ei}\left(\frac{2\log(c)}{n} + \frac{2a}{bn} + 2\log(x)\right)e^{\left(-\frac{2a}{bn}\right)\log(x)} \log(x)}{\left(b^3n^3 \log(x) + b^3n^2 \log(c) + ab^2n^2\right)c^{\frac{2}{n}}} + \frac{2b\text{Ei}\left(\frac{2\log(c)}{n} + \frac{2a}{bn} + 2\log(x)\right)}{\left(b^3n^3 \log(x) + b^3n^2 \log(c) + ab^2n^2\right)c^{\frac{2}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] 
$$-b*n*x^2/(b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2) + 2*b*n*\text{Ei}(2*\log(c)/n + 2*a/(b*n) + 2*\log(x))*e^{(-2*a/(b*n))*\log(x)}/((b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2)*c^{(2/n)}) + 2*b*\text{Ei}(2*\log(c)/n + 2*a/(b*n) + 2*\log(x))*e^{(-2*a/(b*n))*\log(c)}/((b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2)*c^{(2/n)}) + 2*a*\text{Ei}(2*\log(c)/n + 2*a/(b*n) + 2*\log(x))*e^{(-2*a/(b*n))}/((b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2)*c^{(2/n)})$$

$$3.76 \quad \int \frac{1}{(a+b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=70

$$\frac{x e^{-\frac{a}{bn}} (cx^n)^{-1/n} \operatorname{Ei}\left(\frac{a+b \log(cx^n)}{bn}\right)}{b^2 n^2} - \frac{x}{bn(a+b \log(cx^n))}$$

[Out] (x\*ExpIntegralEi[(a + b\*Log[c\*x^n])/(b\*n)])/(b^2\*E^(a/(b\*n))\*n^2\*(c\*x^n)^n^(-1)) - x/(b\*n\*(a + b\*Log[c\*x^n]))

**Rubi [A]** time = 0.039918, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {2297, 2300, 2178}

$$\frac{x e^{-\frac{a}{bn}} (cx^n)^{-1/n} \operatorname{Ei}\left(\frac{a+b \log(cx^n)}{bn}\right)}{b^2 n^2} - \frac{x}{bn(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^(-2), x]

[Out] (x\*ExpIntegralEi[(a + b\*Log[c\*x^n])/(b\*n)])/(b^2\*E^(a/(b\*n))\*n^2\*(c\*x^n)^n^(-1)) - x/(b\*n\*(a + b\*Log[c\*x^n]))

#### Rule 2297

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*n\*(p + 1)), x] - Dist[1/(b\*n\*(p + 1)), Int[(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && IntegerQ[2\*p]

#### Rule 2300

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[E^(x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{1}{(a+b \log(cx^n))^2} dx &= -\frac{x}{bn(a+b \log(cx^n))} + \frac{\int \frac{1}{a+b \log(cx^n)} dx}{bn} \\ &= -\frac{x}{bn(a+b \log(cx^n))} + \frac{(x(cx^n)^{-1/n}) \operatorname{Subst}\left(\int \frac{e^{\frac{x}{a+bx}}}{a+bx} dx, x, \log(cx^n)\right)}{bn^2} \\ &= \frac{e^{-\frac{a}{bn}} x (cx^n)^{-1/n} \operatorname{Ei}\left(\frac{a+b \log(cx^n)}{bn}\right)}{b^2 n^2} - \frac{x}{bn(a+b \log(cx^n))} \end{aligned}$$



**Mathematica [A]** time = 0.0975662, size = 66, normalized size = 0.94

$$\frac{x \left( e^{-\frac{a}{bn}} (cx^n)^{-1/n} \operatorname{Ei} \left( \frac{a+b \log(cx^n)}{bn} \right) - \frac{bn}{a+b \log(cx^n)} \right)}{b^2 n^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^(-2), x]

[Out] (x\*(ExpIntegralEi[(a + b\*Log[c\*x^n])/(b\*n)]/(E^(a/(b\*n))\*(c\*x^n)^n^(-1)) - (b\*n)/(a + b\*Log[c\*x^n]))) / (b^2\*n^2)

**Maple [C]** time = 0.265, size = 351, normalized size = 5.

$$-2 \frac{x}{bn(2a + 2b \ln(c) + 2b \ln(x^n) + ib\pi \operatorname{csgn}(ix^n) (\operatorname{csgn}(icx^n))^2 - ib\pi \operatorname{csgn}(ix^n) \operatorname{csgn}(icx^n) \operatorname{csgn}(ic) - ib\pi (\operatorname{csgn}(ic) \operatorname{csgn}(ix^n) \operatorname{csgn}(icx^n)))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*ln(c\*x^n))^2, x)

[Out] -2/b/n\*x/(2\*a+2\*b\*ln(c)+2\*b\*ln(x^n)+I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2-I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c)-I\*b\*Pi\*csgn(I\*c\*x^n)^3+I\*b\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*c))-1/b^2/n^2\*Ei(1,-ln(x)-1/2\*(I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2-I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c)-I\*b\*Pi\*csgn(I\*c\*x^n)^3+I\*b\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*c)+2\*b\*ln(c)+2\*b\*(ln(x^n)-n\*ln(x))+2\*a)/b/n)\*exp(1/2\*(-I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2+I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c)+I\*b\*Pi\*csgn(I\*c\*x^n)^3-I\*b\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*c)+2\*ln(x)\*b\*n-2\*b\*ln(x^n)-2\*b\*ln(c)-2\*a)/b/n)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{x}{b^2 n \log(c) + b^2 n \log(x^n) + abn} + \int \frac{1}{b^2 n \log(c) + b^2 n \log(x^n) + abn} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*log(c\*x^n))^2, x, algorithm="maxima")

[Out] -x/(b^2\*n\*log(c) + b^2\*n\*log(x^n) + a\*b\*n) + integrate(1/(b^2\*n\*log(c) + b^2\*n\*log(x^n) + a\*b\*n), x)

**Fricas [A]** time = 0.951567, size = 240, normalized size = 3.43

$$\frac{\left( bnx e^{\left( \frac{b \log(c)+a}{bn} \right)} - (bn \log(x) + b \log(c) + a) \log\_integral \left( x e^{\left( \frac{b \log(c)+a}{bn} \right)} \right) \right) e^{\left( -\frac{b \log(c)+a}{bn} \right)}}{b^3 n^3 \log(x) + b^3 n^2 \log(c) + ab^2 n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*log(c\*x^n))^2, x, algorithm="fricas")

[Out]  $-(b*n*x*e^{((b*\log(c) + a)/(b*n))} - (b*n*\log(x) + b*\log(c) + a)*\log\_integral(x*e^{((b*\log(c) + a)/(b*n))}))*e^{-((b*\log(c) + a)/(b*n))}/(b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \log(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Integral((a + b\*log(c\*x\*\*n))\*\*(-2), x)

**Giac [B]** time = 1.22164, size = 321, normalized size = 4.59

$$\frac{bn\text{Ei}\left(\frac{\log(c)}{n} + \frac{a}{bn} + \log(x)\right)e^{\left(-\frac{a}{bn}\right)\log(x)}}{\left(b^3n^3\log(x) + b^3n^2\log(c) + ab^2n^2\right)c^{\left(\frac{1}{n}\right)}} - \frac{bnx}{b^3n^3\log(x) + b^3n^2\log(c) + ab^2n^2} + \frac{b\text{Ei}\left(\frac{\log(c)}{n} + \frac{a}{bn} + \log(x)\right)e^{\left(-\frac{a}{bn}\right)\log(x)}}{\left(b^3n^3\log(x) + b^3n^2\log(c) + ab^2n^2\right)c^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out]  $b*n*\text{Ei}(\log(c)/n + a/(b*n) + \log(x))*e^{-a/(b*n)}*\log(x)/((b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2)*c^{(1/n)}) - b*n*x/(b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2) + b*\text{Ei}(\log(c)/n + a/(b*n) + \log(x))*e^{-a/(b*n)}*\log(c)/((b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2)*c^{(1/n)}) + a*\text{Ei}(\log(c)/n + a/(b*n) + \log(x))*e^{-a/(b*n)}/((b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2)*c^{(1/n)})$

$$3.77 \quad \int \frac{1}{x(a+b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=20

$$-\frac{1}{bn(a+b \log(cx^n))}$$

[Out] -(1/(b\*n\*(a + b\*Log[c\*x^n])))

**Rubi [A]** time = 0.024049, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2302, 30}

$$-\frac{1}{bn(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(a + b\*Log[c\*x^n])^2), x]

[Out] -(1/(b\*n\*(a + b\*Log[c\*x^n])))

**Rule 2302**

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^p.]/(x\_), x\_Symbol] := Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

**Rule 30**

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

**Rubi steps**

$$\begin{aligned} \int \frac{1}{x(a+b \log(cx^n))^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2} dx, x, a+b \log(cx^n)\right)}{bn} \\ &= -\frac{1}{bn(a+b \log(cx^n))} \end{aligned}$$

**Mathematica [A]** time = 0.0045306, size = 20, normalized size = 1.

$$-\frac{1}{bn(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(a + b\*Log[c\*x^n])^2), x]

[Out] -(1/(b\*n\*(a + b\*Log[c\*x^n])))

**Maple [A]** time = 0.034, size = 21, normalized size = 1.1

$$-\frac{1}{bn(a + b \ln(cx^n))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b\*ln(c\*x^n))^2,x)

[Out] -1/b/n/(a+b\*ln(c\*x^n))

**Maxima [A]** time = 1.13194, size = 27, normalized size = 1.35

$$-\frac{1}{(b \log(cx^n) + a)bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] -1/((b\*log(c\*x^n) + a)\*b\*n)

**Fricas [A]** time = 0.804778, size = 59, normalized size = 2.95

$$-\frac{1}{b^2n^2 \log(x) + b^2n \log(c) + abn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] -1/(b^2\*n^2\*log(x) + b^2\*n\*log(c) + a\*b\*n)

**Sympy [A]** time = 49.4998, size = 70, normalized size = 3.5

$$\begin{cases} \frac{\infty \log(x)}{\log(c)^2} & \text{for } a = 0 \wedge b = 0 \wedge n = 0 \\ \infty n \log(x) & \text{for } a = -b(n \log(x) + \log(c)) \\ \frac{\log(x)}{a^2} & \text{for } b = 0 \\ \frac{\log(x)}{(a+b \log(c))^2} & \text{for } n = 0 \\ -\frac{1}{abn+b^2n^2 \log(x)+b^2n \log(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Piecewise((zoo\*log(x)/log(c)\*\*2, Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (zoo\*n\*log(x), Eq(a, -b\*(n\*log(x) + log(c)))), (log(x)/a\*\*2, Eq(b, 0)), (log(x)/(a + b\*log(c))\*\*2, Eq(n, 0)), (-1/(a\*b\*n + b\*\*2\*n\*\*2\*log(x) + b\*\*2\*n\*log(c)), True))

---

**Giac [A]** time = 1.18794, size = 28, normalized size = 1.4

$$-\frac{1}{(bn \log(x) + b \log(c) + a)bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] -1/((b\*n\*log(x) + b\*log(c) + a)\*b\*n)

$$3.78 \quad \int \frac{1}{x^2(a+b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=73

$$-\frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} \operatorname{Ei}\left(-\frac{a+b \log(cx^n)}{bn}\right)}{b^2 n^2 x} - \frac{1}{bnx(a+b \log(cx^n))}$$

[Out] -((E^(a/(b\*n))\*(c\*x^n)^n^(-1)\*ExpIntegralEi[-((a + b\*Log[c\*x^n])/(b\*n))]))/(b^2\*n^2\*x) - 1/(b\*n\*x\*(a + b\*Log[c\*x^n]))

**Rubi [A]** time = 0.0745142, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {2306, 2310, 2178}

$$-\frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} \operatorname{Ei}\left(-\frac{a+b \log(cx^n)}{bn}\right)}{b^2 n^2 x} - \frac{1}{bnx(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(a + b\*Log[c\*x^n])^2), x]

[Out] -((E^(a/(b\*n))\*(c\*x^n)^n^(-1)\*ExpIntegralEi[-((a + b\*Log[c\*x^n])/(b\*n))]))/(b^2\*n^2\*x) - 1/(b\*n\*x\*(a + b\*Log[c\*x^n]))

#### Rule 2306

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol]
  := Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^(p + 1))/(b*d*n*(p + 1)), x] -
  Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
  /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

#### Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol]
  := Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
  /n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

#### Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Si
  mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
  reeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

#### Rubi steps

$$\begin{aligned} \int \frac{1}{x^2 (a + b \log(cx^n))^2} dx &= -\frac{1}{bnx (a + b \log(cx^n))} - \frac{\int \frac{1}{x^2(a+b \log(cx^n))} dx}{bn} \\ &= -\frac{1}{bnx (a + b \log(cx^n))} - \frac{(cx^n)^{\frac{1}{n}} \text{Subst}\left(\int \frac{e^{-\frac{x}{a+bx}}}{a+bx} dx, x, \log(cx^n)\right)}{bn^2x} \\ &= -\frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} \text{Ei}\left(-\frac{a+b \log(cx^n)}{bn}\right)}{b^2n^2x} - \frac{1}{bnx (a + b \log(cx^n))} \end{aligned}$$

**Mathematica [A]** time = 0.0889523, size = 76, normalized size = 1.04

$$\frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} (a + b \log(cx^n)) \text{Ei}\left(-\frac{a+b \log(cx^n)}{bn}\right) + bn}{b^2n^2x (a + b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(a + b\*Log[c\*x^n])^2),x]

[Out] -((b\*n + E^(a/(b\*n))\*(c\*x^n)^n^(-1)\*ExpIntegralEi[-((a + b\*Log[c\*x^n])/(b\*n))])\*(a + b\*Log[c\*x^n]))/(b^2\*n^2\*x\*(a + b\*Log[c\*x^n]))

**Maple [F]** time = 0.659, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a + b \ln(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+b\*ln(c\*x^n))^2,x)

[Out] int(1/x^2/(a+b\*ln(c\*x^n))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{b^2nx \log(x^n) + (b^2n \log(c) + abn)x} - \int \frac{1}{b^2nx^2 \log(x^n) + (b^2n \log(c) + abn)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] -1/(b^2\*n\*x\*log(x^n) + (b^2\*n\*log(c) + a\*b\*n)\*x) - integrate(1/(b^2\*n\*x^2\*log(x^n) + (b^2\*n\*log(c) + a\*b\*n)\*x^2), x)

**Fricas [A]** time = 0.772961, size = 220, normalized size = 3.01

$$\frac{(bnx \log(x) + bx \log(c) + ax)e^{\left(\frac{b \log(c)+a}{bn}\right)} \log\_integral\left(\frac{e^{\left(\frac{-b \log(c)+a}{bn}\right)}}{x}\right) + bn}{b^3n^3x \log(x) + b^3n^2x \log(c) + ab^2n^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] -((b\*n\*x\*log(x) + b\*x\*log(c) + a\*x)\*e^((b\*log(c) + a)/(b\*n))\*log\_integral(e^(-(b\*log(c) + a)/(b\*n))/x) + b\*n)/(b^3\*n^3\*x\*log(x) + b^3\*n^2\*x\*log(c) + a\*b^2\*n^2\*x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a + b \log(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Integral(1/(x\*\*2\*(a + b\*log(c\*x\*\*n))\*\*2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \log(cx^n) + a)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] integrate(1/((b\*log(c\*x^n) + a)^2\*x^2), x)



$$3.79 \quad \int \frac{1}{x^3(a+b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=76

$$-\frac{2e^{\frac{2a}{bn}}(cx^n)^{2/n} \operatorname{Ei}\left(-\frac{2(a+b \log(cx^n))}{bn}\right)}{b^2 n^2 x^2} - \frac{1}{bnx^2(a+b \log(cx^n))}$$

[Out]  $(-2 * E^{((2 * a) / (b * n))} * (c * x^n)^{(2 / n)} * \operatorname{ExpIntegralEi}[(-2 * (a + b * \operatorname{Log}[c * x^n])) / (b * n)]) / (b^2 * n^2 * x^2) - 1 / (b * n * x^2 * (a + b * \operatorname{Log}[c * x^n]))$

**Rubi [A]** time = 0.0777883, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {2306, 2310, 2178}

$$-\frac{2e^{\frac{2a}{bn}}(cx^n)^{2/n} \operatorname{Ei}\left(-\frac{2(a+b \log(cx^n))}{bn}\right)}{b^2 n^2 x^2} - \frac{1}{bnx^2(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*(a + b\*Log[c\*x^n])^2), x]

[Out]  $(-2 * E^{((2 * a) / (b * n))} * (c * x^n)^{(2 / n)} * \operatorname{ExpIntegralEi}[(-2 * (a + b * \operatorname{Log}[c * x^n])) / (b * n)]) / (b^2 * n^2 * x^2) - 1 / (b * n * x^2 * (a + b * \operatorname{Log}[c * x^n]))$

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\int \frac{1}{x^3 (a + b \log(cx^n))^2} dx = -\frac{1}{bnx^2 (a + b \log(cx^n))} - \frac{2 \int \frac{1}{x^3 (a + b \log(cx^n))} dx}{bn}$$

$$= -\frac{1}{bnx^2 (a + b \log(cx^n))} - \frac{(2 (cx^n)^{2/n}) \text{Subst} \left( \int \frac{e^{-\frac{2x}{a+bx}}}{a+bx} dx, x, \log(cx^n) \right)}{bn^2 x^2}$$

$$= -\frac{2e^{\frac{2a}{bn}} (cx^n)^{2/n} \text{Ei} \left( -\frac{2(a+b \log(cx^n))}{bn} \right)}{b^2 n^2 x^2} - \frac{1}{bnx^2 (a + b \log(cx^n))}$$

**Mathematica [A]** time = 0.0910069, size = 80, normalized size = 1.05

$$\frac{2e^{\frac{2a}{bn}} (cx^n)^{2/n} (a + b \log(cx^n)) \text{Ei} \left( -\frac{2(a+b \log(cx^n))}{bn} \right) + bn}{b^2 n^2 x^2 (a + b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(a + b\*Log[c\*x^n])^2),x]

[Out] -((b\*n + 2\*E^((2\*a)/(b\*n))\*(c\*x^n)^(2/n)\*ExpIntegralEi[(-2\*(a + b\*Log[c\*x^n]))]/(b\*n)]\*(a + b\*Log[c\*x^n]))/(b^2\*n^2\*x^2\*(a + b\*Log[c\*x^n]))

**Maple [F]** time = 0.725, size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (a + b \ln(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(a+b\*ln(c\*x^n))^2,x)

[Out] int(1/x^3/(a+b\*ln(c\*x^n))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{b^2 n x^2 \log(x^n) + (b^2 n \log(c) + a b n) x^2} - 2 \int \frac{1}{b^2 n x^3 \log(x^n) + (b^2 n \log(c) + a b n) x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] -1/(b^2\*n\*x^2\*log(x^n) + (b^2\*n\*log(c) + a\*b\*n)\*x^2) - 2\*integrate(1/(b^2\*n\*x^3\*log(x^n) + (b^2\*n\*log(c) + a\*b\*n)\*x^3), x)

**Fricas [A]** time = 0.952235, size = 247, normalized size = 3.25

$$\frac{2 (bnx^2 \log(x) + bx^2 \log(c) + ax^2) e^{\left( \frac{2(b \log(c)+a)}{bn} \right)} \log\_integral \left( \frac{e^{\left( \frac{-2(b \log(c)+a)}{bn} \right)}}{x^2} \right) + bn}{b^3 n^3 x^2 \log(x) + b^3 n^2 x^2 \log(c) + ab^2 n^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] -(2*(b*n*x^2*log(x) + b*x^2*log(c) + a*x^2)*e^(2*(b*log(c) + a)/(b*n))*log_
integral(e^(-2*(b*log(c) + a)/(b*n))/x^2) + b*n)/(b^3*n^3*x^2*log(x) + b^3*
n^2*x^2*log(c) + a*b^2*n^2*x^2)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (a + b \log(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*ln(c*x**n))**2,x)
```

```
[Out] Integral(1/(x**3*(a + b*log(c*x**n))**2), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \log(cx^n) + a)^2 x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*log(c*x^n) + a)^2*x^3), x)
```

$$3.80 \quad \int \frac{1}{x^4(a+b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=76

$$-\frac{3e^{\frac{3a}{bn}}(cx^n)^{3/n} \operatorname{Ei}\left(-\frac{3(a+b \log(cx^n))}{bn}\right)}{b^2 n^2 x^3} - \frac{1}{bnx^3(a+b \log(cx^n))}$$

[Out]  $(-3E^{((3*a)/(b*n))}*(c*x^n)^{(3/n)}*ExpIntegralEi[(-3*(a + b*Log[c*x^n]))/(b*n)])/(b^2*n^2*x^3) - 1/(b*n*x^3*(a + b*Log[c*x^n]))$

**Rubi [A]** time = 0.0751389, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {2306, 2310, 2178}

$$-\frac{3e^{\frac{3a}{bn}}(cx^n)^{3/n} \operatorname{Ei}\left(-\frac{3(a+b \log(cx^n))}{bn}\right)}{b^2 n^2 x^3} - \frac{1}{bnx^3(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a + b\*Log[c\*x^n])^2), x]

[Out]  $(-3E^{((3*a)/(b*n))}*(c*x^n)^{(3/n)}*ExpIntegralEi[(-3*(a + b*Log[c*x^n]))/(b*n)])/(b^2*n^2*x^3) - 1/(b*n*x^3*(a + b*Log[c*x^n]))$

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{1}{x^4 (a + b \log(cx^n))^2} dx &= -\frac{1}{bnx^3 (a + b \log(cx^n))} - \frac{3 \int \frac{1}{x^4 (a + b \log(cx^n))} dx}{bn} \\ &= -\frac{1}{bnx^3 (a + b \log(cx^n))} - \frac{(3 (cx^n)^{3/n}) \text{Subst} \left( \int \frac{e^{-\frac{3x}{a+bx}}}{a+bx} dx, x, \log(cx^n) \right)}{bn^2 x^3} \\ &= -\frac{3e^{\frac{3a}{bn}} (cx^n)^{3/n} \text{Ei} \left( -\frac{3(a+b \log(cx^n))}{bn} \right)}{b^2 n^2 x^3} - \frac{1}{bnx^3 (a + b \log(cx^n))} \end{aligned}$$

**Mathematica [A]** time = 0.0942537, size = 80, normalized size = 1.05

$$-\frac{3e^{\frac{3a}{bn}} (cx^n)^{3/n} (a + b \log(cx^n)) \text{Ei} \left( -\frac{3(a+b \log(cx^n))}{bn} \right) + bn}{b^2 n^2 x^3 (a + b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(a + b\*Log[c\*x^n])^2), x]

[Out] -((b\*n + 3\*E^((3\*a)/(b\*n))\*(c\*x^n)^(3/n)\*ExpIntegralEi[(-3\*(a + b\*Log[c\*x^n]))/(b\*n)]\*(a + b\*Log[c\*x^n]))/(b^2\*n^2\*x^3\*(a + b\*Log[c\*x^n]))

**Maple [F]** time = 0.756, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 (a + b \ln(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(a+b\*ln(c\*x^n))^2,x)

[Out] int(1/x^4/(a+b\*ln(c\*x^n))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{b^2 n x^3 \log(x^n) + (b^2 n \log(c) + a b n) x^3} - 3 \int \frac{1}{b^2 n x^4 \log(x^n) + (b^2 n \log(c) + a b n) x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] -1/(b^2\*n\*x^3\*log(x^n) + (b^2\*n\*log(c) + a\*b\*n)\*x^3) - 3\*integrate(1/(b^2\*n\*x^4\*log(x^n) + (b^2\*n\*log(c) + a\*b\*n)\*x^4), x)

**Fricas [A]** time = 0.856755, size = 247, normalized size = 3.25

$$\frac{3 (bnx^3 \log(x) + bx^3 \log(c) + ax^3) e^{\left( \frac{3(b \log(c) + a)}{bn} \right)} \log\_integral \left( \frac{e^{\left( -\frac{3(b \log(c) + a)}{bn} \right)}}{x^3} \right) + bn}{b^3 n^3 x^3 \log(x) + b^3 n^2 x^3 \log(c) + a b^2 n^2 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out]  $-(3*(b*n*x^3*\log(x) + b*x^3*\log(c) + a*x^3)*e^{(3*(b*\log(c) + a)/(b*n))*\log\_integral(e^{(-3*(b*\log(c) + a)/(b*n))/x^3) + b*n)/(b^3*n^3*x^3*\log(x) + b^3*n^2*x^3*\log(c) + a*b^2*n^2*x^3)}$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 (a + b \log(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*4/(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Integral(1/(x\*\*4\*(a + b\*log(c\*x\*\*n))\*\*2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \log(cx^n) + a)^2 x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] integrate(1/((b\*log(c\*x^n) + a)^2\*x^4), x)

$$3.81 \quad \int \frac{x^3}{(a+b \log(cx^n))^3} dx$$

**Optimal.** Leaf size=101

$$\frac{8x^4 e^{-\frac{4a}{bn}} (cx^n)^{-4/n} \operatorname{Ei}\left(\frac{4(a+b \log(cx^n))}{bn}\right)}{b^3 n^3} - \frac{2x^4}{b^2 n^2 (a+b \log(cx^n))} - \frac{x^4}{2bn (a+b \log(cx^n))^2}$$

[Out] (8\*x^4\*ExpIntegralEi[(4\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b^3\*E^((4\*a)/(b\*n))\*n^3\*(c\*x^n)^(4/n)) - x^4/(2\*b\*n\*(a + b\*Log[c\*x^n])^2) - (2\*x^4)/(b^2\*n^2\*(a + b\*Log[c\*x^n]))

**Rubi [A]** time = 0.10761, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {2306, 2310, 2178}

$$\frac{8x^4 e^{-\frac{4a}{bn}} (cx^n)^{-4/n} \operatorname{Ei}\left(\frac{4(a+b \log(cx^n))}{bn}\right)}{b^3 n^3} - \frac{2x^4}{b^2 n^2 (a+b \log(cx^n))} - \frac{x^4}{2bn (a+b \log(cx^n))^2}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b\*Log[c\*x^n])^3,x]

[Out] (8\*x^4\*ExpIntegralEi[(4\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b^3\*E^((4\*a)/(b\*n))\*n^3\*(c\*x^n)^(4/n)) - x^4/(2\*b\*n\*(a + b\*Log[c\*x^n])^2) - (2\*x^4)/(b^2\*n^2\*(a + b\*Log[c\*x^n]))

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a + b \log(cx^n))^3} dx &= -\frac{x^4}{2bn(a + b \log(cx^n))^2} + \frac{2 \int \frac{x^3}{(a+b \log(cx^n))^2} dx}{bn} \\
&= -\frac{x^4}{2bn(a + b \log(cx^n))^2} - \frac{2x^4}{b^2n^2(a + b \log(cx^n))} + \frac{8 \int \frac{x^3}{a+b \log(cx^n)} dx}{b^2n^2} \\
&= -\frac{x^4}{2bn(a + b \log(cx^n))^2} - \frac{2x^4}{b^2n^2(a + b \log(cx^n))} + \frac{(8x^4 (cx^n)^{-4/n}) \text{Subst} \left( \int \frac{e^{\frac{4x}{a+bx}}}{a+bx} dx, x, \log(cx^n) \right)}{b^2n^3} \\
&= \frac{8e^{-\frac{4a}{bn}} x^4 (cx^n)^{-4/n} \text{Ei} \left( \frac{4(a+b \log(cx^n))}{bn} \right)}{b^3n^3} - \frac{x^4}{2bn(a + b \log(cx^n))^2} - \frac{2x^4}{b^2n^2(a + b \log(cx^n))}
\end{aligned}$$

**Mathematica [A]** time = 0.135277, size = 89, normalized size = 0.88

$$\frac{x^4 \left( 16e^{-\frac{4a}{bn}} (cx^n)^{-4/n} \text{Ei} \left( \frac{4(a+b \log(cx^n))}{bn} \right) - \frac{bn(4a+4b \log(cx^n)+bn)}{(a+b \log(cx^n))^2} \right)}{2b^3n^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b\*Log[c\*x^n])^3,x]

[Out] (x^4\*((16\*ExpIntegralEi[(4\*(a + b\*Log[c\*x^n]))/(b\*n)])/(E^((4\*a)/(b\*n))\*(c\*x^n)^(4/n)) - (b\*n\*(4\*a + b\*n + 4\*b\*Log[c\*x^n]))/(a + b\*Log[c\*x^n])^2))/(2\*b^3\*n^3)

**Maple [F]** time = 0.705, size = 0, normalized size = 0.

$$\int \frac{x^3}{(a + b \ln(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a+b\*ln(c\*x^n))^3,x)

[Out] int(x^3/(a+b\*ln(c\*x^n))^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{4bx^4 \log(x^n) + (b(n + 4 \log(c)) + 4a)x^4}{2(b^4n^2 \log(c)^2 + b^4n^2 \log(x^n)^2 + 2ab^3n^2 \log(c) + a^2b^2n^2 + 2(b^4n^2 \log(c) + ab^3n^2) \log(x^n))} + 8 \int \frac{x^3}{b^3n^2 \log(c) + b^3n^2 \log(x^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out] -1/2\*(4\*b\*x^4\*log(x^n) + (b\*(n + 4\*log(c)) + 4\*a)\*x^4)/(b^4\*n^2\*log(c)^2 + b^4\*n^2\*log(x^n)^2 + 2\*a\*b^3\*n^2\*log(c) + a^2\*b^2\*n^2 + 2\*(b^4\*n^2\*log(c) + a\*b^3\*n^2)\*log(x^n)) + 8\*integrate(x^3/(b^3\*n^2\*log(c) + b^3\*n^2\*log(x^n)),x)



+ a\*b^2\*n^2), x)

**Fricas [B]** time = 0.827485, size = 520, normalized size = 5.15

$$\frac{\left(4b^2n^2x^4 \log(x) + 4b^2nx^4 \log(c) + (b^2n^2 + 4abn)x^4\right)e^{\frac{4(b \log(c)+a)}{bn}} - 16\left(b^2n^2 \log(x)^2 + b^2 \log(c)^2 + 2ab \log(c) + a^2\right)}{2\left(b^5n^5 \log(x)^2 + b^5n^3 \log(c)^2 + 2ab^4n^3 \log(c) + a^2b^3n^3 + 2\left(b^5n^5 \log(x) + b^5n^3 \log(c) + 2ab^4n^3 \log(c) + a^2b^3n^3\right)\log(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out] -1/2\*((4\*b^2\*n^2\*x^4\*log(x) + 4\*b^2\*n\*x^4\*log(c) + (b^2\*n^2 + 4\*a\*b\*n)\*x^4)\*e^(4\*(b\*log(c) + a)/(b\*n)) - 16\*(b^2\*n^2\*log(x)^2 + b^2\*log(c)^2 + 2\*a\*b\*log(c) + a^2 + 2\*(b^2\*n\*log(c) + a\*b\*n)\*log(x))\*log\_integral(x^4\*e^(4\*(b\*log(c) + a)/(b\*n))))\*e^(-4\*(b\*log(c) + a)/(b\*n))/(b^5\*n^5\*log(x)^2 + b^5\*n^3\*log(c)^2 + 2\*a\*b^4\*n^3\*log(c) + a^2\*b^3\*n^3 + 2\*(b^5\*n^4\*log(c) + a\*b^4\*n^4)\*log(x))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(a + b \log(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(a+b\*ln(c\*x\*\*n))\*\*3,x)

[Out] Integral(x\*\*3/(a + b\*log(c\*x\*\*n))\*\*3, x)

**Giac [B]** time = 1.44135, size = 1389, normalized size = 13.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out] -2\*b^2\*n^2\*x^4\*log(x)/(b^5\*n^5\*log(x)^2 + 2\*b^5\*n^4\*log(c)\*log(x) + b^5\*n^3\*log(c)^2 + 2\*a\*b^4\*n^4\*log(x) + 2\*a\*b^4\*n^3\*log(c) + a^2\*b^3\*n^3) - 1/2\*b^2\*n^2\*x^4/(b^5\*n^5\*log(x)^2 + 2\*b^5\*n^4\*log(c)\*log(x) + b^5\*n^3\*log(c)^2 + 2\*a\*b^4\*n^4\*log(x) + 2\*a\*b^4\*n^3\*log(c) + a^2\*b^3\*n^3) - 2\*b^2\*n\*x^4\*log(c)/(b^5\*n^5\*log(x)^2 + 2\*b^5\*n^4\*log(c)\*log(x) + b^5\*n^3\*log(c)^2 + 2\*a\*b^4\*n^4\*log(x) + 2\*a\*b^4\*n^3\*log(c) + a^2\*b^3\*n^3) - 2\*a\*b\*n\*x^4/(b^5\*n^5\*log(x)^2 + 2\*b^5\*n^4\*log(c)\*log(x) + b^5\*n^3\*log(c)^2 + 2\*a\*b^4\*n^4\*log(x) + 2\*a\*b^4\*n^3\*log(c) + a^2\*b^3\*n^3) + 8\*b^2\*n^2\*Ei(4\*log(c)/n + 4\*a/(b\*n) + 4\*log(x))\*e^(-4\*a/(b\*n))\*log(x)^2/((b^5\*n^5\*log(x)^2 + 2\*b^5\*n^4\*log(c)\*log(x) + b^5\*n^3\*log(c)^2 + 2\*a\*b^4\*n^4\*log(x) + 2\*a\*b^4\*n^3\*log(c) + a^2\*b^3\*n^3)\*c^(4/n)) + 16\*b^2\*n\*Ei(4\*log(c)/n + 4\*a/(b\*n) + 4\*log(x))\*e^(-4\*a/(b\*n))\*log(c)\*log(x)/((b^5\*n^5\*log(x)^2 + 2\*b^5\*n^4\*log(c)\*log(x) + b^5\*n^3\*log(c)^2 + 2\*a\*b^4\*n^4\*log(x) + 2\*a\*b^4\*n^3\*log(c) + a^2\*b^3\*n^3)\*c^(4/n)) + 8\*b^2\*

$$\begin{aligned}
& \text{Ei}(4\log(c)/n + 4a/(b*n) + 4\log(x)) * e^{(-4a/(b*n))} * \log(c)^2 / ((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(4/n)}) + 16*a*b*n*\text{Ei}(4\log(c)/n + 4a/(b*n) + 4\log(x)) * e^{(-4a/(b*n))} * \log(x) / ((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(4/n)}) + 16*a*b*\text{Ei}(4\log(c)/n + 4a/(b*n) + 4\log(x)) * e^{(-4a/(b*n))} * \log(c) / ((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(4/n)}) + 8*a^2*\text{Ei}(4\log(c)/n + 4a/(b*n) + 4\log(x)) * e^{(-4a/(b*n))} / ((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(4/n)})
\end{aligned}$$

$$3.82 \quad \int \frac{x^2}{(a+b \log(cx^n))^3} dx$$

**Optimal.** Leaf size=105

$$\frac{9x^3 e^{-\frac{3a}{bn}} (cx^n)^{-3/n} \operatorname{Ei}\left(\frac{3(a+b \log(cx^n))}{bn}\right)}{2b^3 n^3} - \frac{3x^3}{2b^2 n^2 (a+b \log(cx^n))} - \frac{x^3}{2bn (a+b \log(cx^n))^2}$$

[Out] (9\*x^3\*ExpIntegralEi[(3\*(a + b\*Log[c\*x^n]))/(b\*n)])/(2\*b^3\*E^((3\*a)/(b\*n))\*n^3\*(c\*x^n)^(3/n)) - x^3/(2\*b\*n\*(a + b\*Log[c\*x^n])^2) - (3\*x^3)/(2\*b^2\*n^2\*(a + b\*Log[c\*x^n]))

**Rubi [A]** time = 0.107642, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {2306, 2310, 2178}

$$\frac{9x^3 e^{-\frac{3a}{bn}} (cx^n)^{-3/n} \operatorname{Ei}\left(\frac{3(a+b \log(cx^n))}{bn}\right)}{2b^3 n^3} - \frac{3x^3}{2b^2 n^2 (a+b \log(cx^n))} - \frac{x^3}{2bn (a+b \log(cx^n))^2}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b\*Log[c\*x^n])^3,x]

[Out] (9\*x^3\*ExpIntegralEi[(3\*(a + b\*Log[c\*x^n]))/(b\*n)])/(2\*b^3\*E^((3\*a)/(b\*n))\*n^3\*(c\*x^n)^(3/n)) - x^3/(2\*b\*n\*(a + b\*Log[c\*x^n])^2) - (3\*x^3)/(2\*b^2\*n^2\*(a + b\*Log[c\*x^n]))

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x)/n\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(a+b \log (c x^n))^3} d x &= -\frac{x^3}{2 b n(a+b \log (c x^n))^2} + \frac{3 \int \frac{x^2}{(a+b \log (c x^n))^2} d x}{2 b n} \\
&= -\frac{x^3}{2 b n(a+b \log (c x^n))^2} - \frac{3 x^3}{2 b^2 n^2(a+b \log (c x^n))} + \frac{9 \int \frac{x^2}{a+b \log (c x^n)} d x}{2 b^2 n^2} \\
&= -\frac{x^3}{2 b n(a+b \log (c x^n))^2} - \frac{3 x^3}{2 b^2 n^2(a+b \log (c x^n))} + \frac{(9 x^3(c x^n)^{-3 / n}) \operatorname{Subst}\left(\int \frac{e^{\frac{3 x}{a+b x}}}{a+b x} d x, x, \log (c x^n)\right)}{2 b^2 n^3} \\
&= \frac{9 e^{-\frac{3 a}{b n}} x^3(c x^n)^{-3 / n} \operatorname{Ei}\left(\frac{3(a+b \log (c x^n))}{b n}\right)}{2 b^3 n^3} - \frac{x^3}{2 b n(a+b \log (c x^n))^2} - \frac{3 x^3}{2 b^2 n^2(a+b \log (c x^n))}
\end{aligned}$$

**Mathematica [A]** time = 0.130902, size = 89, normalized size = 0.85

$$\frac{x^3 \left( 9 e^{-\frac{3 a}{b n}} (c x^n)^{-3 / n} \operatorname{Ei}\left(\frac{3(a+b \log (c x^n))}{b n}\right) - \frac{b n(3 a+3 b \log (c x^n)+b n)}{(a+b \log (c x^n))^2} \right)}{2 b^3 n^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b\*Log[c\*x^n])^3,x]

[Out] (x^3\*((9\*ExpIntegralEi[(3\*(a + b\*Log[c\*x^n]))/(b\*n)])/(E^((3\*a)/(b\*n))\*(c\*x^n)^(3/n)) - (b\*n\*(3\*a + b\*n + 3\*b\*Log[c\*x^n]))/(a + b\*Log[c\*x^n])^2))/(2\*b^3\*n^3)

**Maple [F]** time = 0.662, size = 0, normalized size = 0.

$$\int \frac{x^2}{(a+b \ln (c x^n))^3} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+b\*ln(c\*x^n))^3,x)

[Out] int(x^2/(a+b\*ln(c\*x^n))^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{3 b x^3 \log (x^n) + (b(n+3 \log (c)) + 3 a) x^3}{2\left(b^4 n^2 \log (c)^2 + b^4 n^2 \log (x^n)^2 + 2 a b^3 n^2 \log (c) + a^2 b^2 n^2 + 2\left(b^4 n^2 \log (c) + a b^3 n^2\right) \log (x^n)\right)} + 9 \int \frac{1}{2\left(b^3 n^2 \log (c) + \dots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out] -1/2\*(3\*b\*x^3\*log(x^n) + (b\*(n + 3\*log(c)) + 3\*a)\*x^3)/(b^4\*n^2\*log(c)^2 + b^4\*n^2\*log(x^n)^2 + 2\*a\*b^3\*n^2\*log(c) + a^2\*b^2\*n^2 + 2\*(b^4\*n^2\*log(c) + a\*b^3\*n^2)\*log(x^n)) + 9\*integrate(1/2\*x^2/(b^3\*n^2\*log(c) + b^3\*n^2\*log(x^n)),x)



$$\begin{aligned}
& 9/2*b^2*Ei(3*log(c)/n + 3*a/(b*n) + 3*log(x))*e^(-3*a/(b*n))*log(c)^2/((b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*log(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3)*c^(3/n)) + 9*a*b*n*Ei(3*log(c)/n + 3*a/(b*n) + 3*log(x))*e^(-3*a/(b*n))*log(x)/((b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*log(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3)*c^(3/n)) + 9*a*b*Ei(3*log(c)/n + 3*a/(b*n) + 3*log(x))*e^(-3*a/(b*n))*log(c)/((b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*log(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3)*c^(3/n)) + 9/2*a^2*Ei(3*log(c)/n + 3*a/(b*n) + 3*log(x))*e^(-3*a/(b*n))/((b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*log(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3)*c^(3/n))
\end{aligned}$$

$$3.83 \quad \int \frac{x}{(a+b \log(cx^n))^3} dx$$

**Optimal.** Leaf size=101

$$\frac{2x^2 e^{-\frac{2a}{bn}} (cx^n)^{-2/n} \operatorname{Ei}\left(\frac{2(a+b \log(cx^n))}{bn}\right)}{b^3 n^3} - \frac{x^2}{b^2 n^2 (a+b \log(cx^n))} - \frac{x^2}{2bn (a+b \log(cx^n))^2}$$

[Out] (2\*x^2\*ExpIntegralEi[(2\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b^3\*E^((2\*a)/(b\*n))\*n^3\*(c\*x^n)^(2/n)) - x^2/(2\*b\*n\*(a + b\*Log[c\*x^n])^2) - x^2/(b^2\*n^2\*(a + b\*Log[c\*x^n]))

**Rubi [A]** time = 0.0798244, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {2306, 2310, 2178}

$$\frac{2x^2 e^{-\frac{2a}{bn}} (cx^n)^{-2/n} \operatorname{Ei}\left(\frac{2(a+b \log(cx^n))}{bn}\right)}{b^3 n^3} - \frac{x^2}{b^2 n^2 (a+b \log(cx^n))} - \frac{x^2}{2bn (a+b \log(cx^n))^2}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b\*Log[c\*x^n])^3, x]

[Out] (2\*x^2\*ExpIntegralEi[(2\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b^3\*E^((2\*a)/(b\*n))\*n^3\*(c\*x^n)^(2/n)) - x^2/(2\*b\*n\*(a + b\*Log[c\*x^n])^2) - x^2/(b^2\*n^2\*(a + b\*Log[c\*x^n]))

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

#### Rubi steps

$$\begin{aligned}
\int \frac{x}{(a + b \log(cx^n))^3} dx &= -\frac{x^2}{2bn(a + b \log(cx^n))^2} + \frac{\int \frac{x}{(a+b \log(cx^n))^2} dx}{bn} \\
&= -\frac{x^2}{2bn(a + b \log(cx^n))^2} - \frac{x^2}{b^2n^2(a + b \log(cx^n))} + \frac{2 \int \frac{x}{a+b \log(cx^n)} dx}{b^2n^2} \\
&= -\frac{x^2}{2bn(a + b \log(cx^n))^2} - \frac{x^2}{b^2n^2(a + b \log(cx^n))} + \frac{(2x^2 (cx^n)^{-2/n}) \text{Subst} \left( \int \frac{\frac{2x}{a+bx}}{a+bx} dx, x, \log(cx^n) \right)}{b^2n^3} \\
&= \frac{2e^{-\frac{2a}{bn}} x^2 (cx^n)^{-2/n} \text{Ei} \left( \frac{2(a+b \log(cx^n))}{bn} \right)}{b^3n^3} - \frac{x^2}{2bn(a + b \log(cx^n))^2} - \frac{x^2}{b^2n^2(a + b \log(cx^n))}
\end{aligned}$$

**Mathematica [A]** time = 0.129612, size = 89, normalized size = 0.88

$$\frac{x^2 \left( 4e^{-\frac{2a}{bn}} (cx^n)^{-2/n} \text{Ei} \left( \frac{2(a+b \log(cx^n))}{bn} \right) - \frac{bn(2a+2b \log(cx^n)+bn)}{(a+b \log(cx^n))^2} \right)}{2b^3n^3}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b\*Log[c\*x^n])^3,x]

[Out] (x^2\*((4\*ExpIntegralEi[(2\*(a + b\*Log[c\*x^n]))/(b\*n)])/(E^((2\*a)/(b\*n))\*(c\*x^n)^(2/n)) - (b\*n\*(2\*a + b\*n + 2\*b\*Log[c\*x^n]))/(a + b\*Log[c\*x^n]^2)))/(2\*b^3\*n^3)

**Maple [F]** time = 0.652, size = 0, normalized size = 0.

$$\int \frac{x}{(a + b \ln(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b\*ln(c\*x^n))^3,x)

[Out] int(x/(a+b\*ln(c\*x^n))^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2bx^2 \log(x^n) + (b(n + 2 \log(c)) + 2a)x^2}{2(b^4n^2 \log(c)^2 + b^4n^2 \log(x^n)^2 + 2ab^3n^2 \log(c) + a^2b^2n^2 + 2(b^4n^2 \log(c) + ab^3n^2) \log(x^n))} + 2 \int \frac{1}{b^3n^2 \log(c) + b^3n^2 \log(x^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out] -1/2\*(2\*b\*x^2\*log(x^n) + (b\*(n + 2\*log(c)) + 2\*a)\*x^2)/(b^4\*n^2\*log(c)^2 + b^4\*n^2\*log(x^n)^2 + 2\*a\*b^3\*n^2\*log(c) + a^2\*b^2\*n^2 + 2\*(b^4\*n^2\*log(c) + a\*b^3\*n^2)\*log(x^n)) + 2\*integrate(x/(b^3\*n^2\*log(c) + b^3\*n^2\*log(x^n) +



$a*b^{2*n^2}), x)$

**Fricas [B]** time = 0.782581, size = 518, normalized size = 5.13

$$\frac{\left(2b^2n^2x^2 \log(x) + 2b^2nx^2 \log(c) + (b^2n^2 + 2abn)x^2\right)e^{\left(\frac{2(b\log(c)+a)}{bn}\right)} - 4\left(b^2n^2 \log(x)^2 + b^2 \log(c)^2 + 2ab \log(c) + a^2\right)}{2\left(b^5n^5 \log(x)^2 + b^5n^3 \log(c)^2 + 2ab^4n^3 \log(c) + a^2b^3n^3 + 2\left(b^5n^5 \log(x) + b^5n^3 \log(c)\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out]  $-1/2*((2*b^{2*n^2}*x^2*\log(x) + 2*b^{2*n}*x^2*\log(c) + (b^{2*n^2} + 2*a*b*n)*x^2)*e^{(2*(b*\log(c) + a)/(b*n))} - 4*(b^{2*n^2}*\log(x)^2 + b^2*\log(c)^2 + 2*a*b*\log(c) + a^2 + 2*(b^{2*n}*\log(c) + a*b*n)*\log(x))*\log\_integral(x^2*e^{(2*(b*\log(c) + a)/(b*n))})*e^{(-2*(b*\log(c) + a)/(b*n))}/(b^5*n^5*\log(x)^2 + b^5*n^3*\log(c)^2 + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3 + 2*(b^5*n^4*\log(c) + a*b^4*n^4)*\log(x)))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a + b \log(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*ln(c\*x\*\*n))\*\*3,x)

[Out] Integral(x/(a + b\*log(c\*x\*\*n))\*\*3, x)

**Giac [B]** time = 1.80405, size = 1389, normalized size = 13.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out]  $-b^{2*n^2}*x^2*\log(x)/(b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3) + 2*b^{2*n^2}*Ei(2*\log(c)/n + 2*a/(b*n) + 2*\log(x))*e^{(-2*a/(b*n))*\log(x)^2}/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(2/n)}) - 1/2*b^{2*n^2}*x^2/(b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3) - b^{2*n}*x^2*\log(c)/(b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3) + 4*b^{2*n}*Ei(2*\log(c)/n + 2*a/(b*n) + 2*\log(x))*e^{(-2*a/(b*n))*\log(c)*\log(x)}/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(2/n)}) - a*b*n*x^2/(b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3) + 2*b^{2*n}*Ei(2*\log(c)/n + 2*a/(b*n) + 2*\log(x))*e^{(-2*a/(b*n))*\log(c)^2}/((b^5*n^5*\log(x)^2 +$

$$\begin{aligned}
& 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4* \\
& n^3*\log(c) + a^2*b^3*n^3*c^{(2/n)} + 4*a*b*n*Ei(2*\log(c)/n + 2*a/(b*n) + 2* \\
& \log(x))*e^{(-2*a/(b*n))*\log(x)/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) \\
& + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3) \\
& *c^{(2/n)}) + 4*a*b*Ei(2*\log(c)/n + 2*a/(b*n) + 2*\log(x))*e^{(-2*a/(b*n))*\log( \\
& c)/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^ \\
& 4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(2/n)}) + 2*a^2*Ei(2*\log( \\
& c)/n + 2*a/(b*n) + 2*\log(x))*e^{(-2*a/(b*n))}/((b^5*n^5*\log(x)^2 + 2*b^5*n^4* \\
& \log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) \\
& + a^2*b^3*n^3)*c^{(2/n)})
\end{aligned}$$

$$3.84 \quad \int \frac{1}{(a+b \log(cx^n))^3} dx$$

**Optimal.** Leaf size=98

$$\frac{x e^{-\frac{a}{bn}} (cx^n)^{-1/n} \operatorname{Ei}\left(\frac{a+b \log(cx^n)}{bn}\right)}{2b^3 n^3} - \frac{x}{2b^2 n^2 (a+b \log(cx^n))} - \frac{x}{2bn (a+b \log(cx^n))^2}$$

[Out] (x\*ExpIntegralEi[(a + b\*Log[c\*x^n])/(b\*n)])/(2\*b^3\*E^(a/(b\*n))\*n^3\*(c\*x^n)^n^(-1)) - x/(2\*b\*n\*(a + b\*Log[c\*x^n])^2) - x/(2\*b^2\*n^2\*(a + b\*Log[c\*x^n]))

**Rubi [A]** time = 0.0524806, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {2297, 2300, 2178}

$$\frac{x e^{-\frac{a}{bn}} (cx^n)^{-1/n} \operatorname{Ei}\left(\frac{a+b \log(cx^n)}{bn}\right)}{2b^3 n^3} - \frac{x}{2b^2 n^2 (a+b \log(cx^n))} - \frac{x}{2bn (a+b \log(cx^n))^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^(-3), x]

[Out] (x\*ExpIntegralEi[(a + b\*Log[c\*x^n])/(b\*n)])/(2\*b^3\*E^(a/(b\*n))\*n^3\*(c\*x^n)^n^(-1)) - x/(2\*b\*n\*(a + b\*Log[c\*x^n])^2) - x/(2\*b^2\*n^2\*(a + b\*Log[c\*x^n]))

#### Rule 2297

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^p], x\_Symbol] := Simp[(x\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*n\*(p + 1)), x] - Dist[1/(b\*n\*(p + 1)), Int[(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && IntegerQ[2\*p]

#### Rule 2300

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^p], x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[E^(x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + b \log(cx^n))^3} dx &= -\frac{x}{2bn(a + b \log(cx^n))^2} + \frac{\int \frac{1}{(a+b \log(cx^n))^2} dx}{2bn} \\
&= -\frac{x}{2bn(a + b \log(cx^n))^2} - \frac{x}{2b^2n^2(a + b \log(cx^n))} + \frac{\int \frac{1}{a+b \log(cx^n)} dx}{2b^2n^2} \\
&= -\frac{x}{2bn(a + b \log(cx^n))^2} - \frac{x}{2b^2n^2(a + b \log(cx^n))} + \frac{(x(cx^n)^{-1/n}) \text{Subst}\left(\int \frac{e^{\frac{x}{a+bx}}}{a+bx} dx, x, \log(cx^n)\right)}{2b^2n^3} \\
&= \frac{e^{-\frac{a}{bn}} x (cx^n)^{-1/n} \text{Ei}\left(\frac{a+b \log(cx^n)}{bn}\right)}{2b^3n^3} - \frac{x}{2bn(a + b \log(cx^n))^2} - \frac{x}{2b^2n^2(a + b \log(cx^n))}
\end{aligned}$$

**Mathematica [A]** time = 0.113674, size = 82, normalized size = 0.84

$$\frac{x \left( e^{-\frac{a}{bn}} (cx^n)^{-1/n} \text{Ei}\left(\frac{a+b \log(cx^n)}{bn}\right) - \frac{bn(a+b \log(cx^n)+bn)}{(a+b \log(cx^n))^2} \right)}{2b^3n^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^(-3), x]

[Out] (x\*(ExpIntegralEi[(a + b\*Log[c\*x^n])]/(b\*n)]/(E^(a/(b\*n))\*(c\*x^n)^n^(-1)) - (b\*n\*(a + b\*n + b\*Log[c\*x^n]))/(a + b\*Log[c\*x^n])^2))/(2\*b^3\*n^3)

**Maple [C]** time = 0.268, size = 460, normalized size = 4.7

$$\frac{2bnx + i\pi bxcsgn(ix^n)(csgn(icx^n))^2 - i\pi bxcsgn(ix^n)csgn(icx^n)csgn(ic) - i\pi bx(csgn(icx^n))^3 + i\pi bx(csgn(icx^n))}{(2a + 2b \ln(c) + 2b \ln(x^n) + ib\pi csgn(ix^n)(csgn(icx^n))^2 - ib\pi csgn(ix^n)csgn(icx^n)csgn(ic) - ib\pi(csgn(icx^n))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*ln(c\*x^n))^3, x)

[Out] -(2\*b\*n\*x+I\*Pi\*b\*x\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2-I\*Pi\*b\*x\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c)-I\*Pi\*b\*x\*csgn(I\*c\*x^n)^3+I\*Pi\*b\*x\*csgn(I\*c\*x^n)^2\*csgn(I\*c)+2\*ln(c)\*b\*x+2\*b\*x\*ln(x^n)+2\*a\*x)/(2\*a+2\*b\*ln(c)+2\*b\*ln(x^n)+I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2-I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c)-I\*b\*Pi\*csgn(I\*c\*x^n)^3+I\*b\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*c))^2/b^2/n^2-1/2/b^3/n^3\*exp(-1/2\*(I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2-I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c)-I\*b\*Pi\*csgn(I\*c\*x^n)^3+I\*b\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*c))^2/b^2/n^2-1/2\*(I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2-I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c)-I\*b\*Pi\*csgn(I\*c\*x^n)^3+I\*b\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*c)-2\*ln(x)\*b\*n+2\*b\*ln(c)+2\*b\*ln(x^n)+2\*a)/b/n)\*Ei(1,-ln(x)-1/2\*(I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)^2-I\*b\*Pi\*csgn(I\*x^n)\*csgn(I\*c\*x^n)\*csgn(I\*c)-I\*b\*Pi\*csgn(I\*c\*x^n)^3+I\*b\*Pi\*csgn(I\*c\*x^n)^2\*csgn(I\*c)+2\*b\*ln(c)+2\*b\*(ln(x^n)-n\*ln(x))+2\*a)/b/n)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{bx \log(x^n) + (b(n + \log(c)) + a)x}{2(b^4n^2 \log(c)^2 + b^4n^2 \log(x^n)^2 + 2ab^3n^2 \log(c) + a^2b^2n^2 + 2(b^4n^2 \log(c) + ab^3n^2) \log(x^n))} + \int \frac{1}{2(b^3n^2 \log(c) + b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out]  $-1/2*(b*x*\log(x^n) + (b*(n + \log(c)) + a)*x)/(b^4*n^2*\log(c)^2 + b^4*n^2*\log(x^n)^2 + 2*a*b^3*n^2*\log(c) + a^2*b^2*n^2 + 2*(b^4*n^2*\log(c) + a*b^3*n^2)*\log(x^n)) + \text{integrate}(1/2/(b^3*n^2*\log(c) + b^3*n^2*\log(x^n) + a*b^2*n^2), x)$

**Fricas [B]** time = 0.761523, size = 489, normalized size = 4.99

$$\frac{\left( (b^2 n^2 x \log(x) + b^2 n x \log(c) + (b^2 n^2 + a b n) x) e^{\left( \frac{b \log(c) + a}{b n} \right)} - (b^2 n^2 \log(x)^2 + b^2 \log(c)^2 + 2 a b \log(c) + a^2 + 2 (b^2 n \log(x) + b \log(c)) \log(c)) \right)}{2 (b^5 n^5 \log(x)^2 + b^5 n^3 \log(c)^2 + 2 a b^4 n^3 \log(c) + a^2 b^3 n^3 + 2 (b^5 n^4 \log(c) + a b^4 n^3) \log(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out]  $-1/2*((b^2*n^2*x*\log(x) + b^2*n*x*\log(c) + (b^2*n^2 + a*b*n)*x)*e^{((b*\log(c) + a)/(b*n))} - (b^2*n^2*\log(x)^2 + b^2*\log(c)^2 + 2*a*b*\log(c) + a^2 + 2*(b^2*n*\log(c) + a*b*n)*\log(x))*\log\_integral(x*e^{((b*\log(c) + a)/(b*n))})*e^{(-(b*\log(c) + a)/(b*n))}/(b^5*n^5*\log(x)^2 + b^5*n^3*\log(c)^2 + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3 + 2*(b^5*n^4*\log(c) + a*b^4*n^4)*\log(x)))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \log(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*ln(c\*x\*\*n))\*\*3,x)

[Out] Integral((a + b\*log(c\*x\*\*n))\*\*(-3), x)

**Giac [B]** time = 1.32889, size = 1326, normalized size = 13.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out]  $1/2*b^2*n^2*Ei(\log(c)/n + a/(b*n) + \log(x))*e^{-a/(b*n)}*\log(x)^2/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)*c^{(1/n)}) - 1/2*b^2*n^2*x*\log(x)/(b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3) + b^2*n*Ei(\log(c)/n + a/(b*n) + \log(x))*e^{-a/(b*n)}*\log(c)*\log(x)/((b^5*n^5*\log(x)^2 + 2*b^5*n^4*\log(c)*\log(x) + b^5*n^3*\log(c)^2 + 2*a*b^4*n^4*\log(x) + 2*a*b^4*n^3*\log(c) + a^2*b^3*n^3)$

$$\begin{aligned}
& ) * c^{(1/n)} - 1/2 * b^2 * n^2 * x / (b^5 * n^5 * \log(x)^2 + 2 * b^5 * n^4 * \log(c) * \log(x) + b^5 * n^3 * \log(c)^2 + 2 * a * b^4 * n^4 * \log(x) + 2 * a * b^4 * n^3 * \log(c) + a^2 * b^3 * n^3) - 1 \\
& / 2 * b^2 * n * x * \log(c) / (b^5 * n^5 * \log(x)^2 + 2 * b^5 * n^4 * \log(c) * \log(x) + b^5 * n^3 * \log(c)^2 + 2 * a * b^4 * n^4 * \log(x) + 2 * a * b^4 * n^3 * \log(c) + a^2 * b^3 * n^3) + 1/2 * b^2 * \text{Ei} \\
& (\log(c)/n + a/(b * n) + \log(x)) * e^{(-a/(b * n))} * \log(c)^2 / ((b^5 * n^5 * \log(x)^2 + 2 * b^5 * n^4 * \log(c) * \log(x) + b^5 * n^3 * \log(c)^2 + 2 * a * b^4 * n^4 * \log(x) + 2 * a * b^4 * n^3 * \log(c) + a^2 * b^3 * n^3) * c^{(1/n)}) + a * b * n * \text{Ei}(\log(c)/n + a/(b * n) + \log(x)) * e^{(-a/(b * n))} * \log(x) / ((b^5 * n^5 * \log(x)^2 + 2 * b^5 * n^4 * \log(c) * \log(x) + b^5 * n^3 * \log(c)^2 + 2 * a * b^4 * n^4 * \log(x) + 2 * a * b^4 * n^3 * \log(c) + a^2 * b^3 * n^3) * c^{(1/n)}) - 1 \\
& / 2 * a * b * n * x / (b^5 * n^5 * \log(x)^2 + 2 * b^5 * n^4 * \log(c) * \log(x) + b^5 * n^3 * \log(c)^2 + 2 * a * b^4 * n^4 * \log(x) + 2 * a * b^4 * n^3 * \log(c) + a^2 * b^3 * n^3) + a * b * \text{Ei}(\log(c)/n + a/(b * n) + \log(x)) * e^{(-a/(b * n))} * \log(c) / ((b^5 * n^5 * \log(x)^2 + 2 * b^5 * n^4 * \log(c) * \log(x) + b^5 * n^3 * \log(c)^2 + 2 * a * b^4 * n^4 * \log(x) + 2 * a * b^4 * n^3 * \log(c) + a^2 * b^3 * n^3) * c^{(1/n)}) + 1/2 * a^2 * \text{Ei}(\log(c)/n + a/(b * n) + \log(x)) * e^{(-a/(b * n))} / ((b^5 * n^5 * \log(x)^2 + 2 * b^5 * n^4 * \log(c) * \log(x) + b^5 * n^3 * \log(c)^2 + 2 * a * b^4 * n^4 * \log(x) + 2 * a * b^4 * n^3 * \log(c) + a^2 * b^3 * n^3) * c^{(1/n)})
\end{aligned}$$

$$3.85 \quad \int \frac{1}{x(a+b \log(cx^n))^3} dx$$

**Optimal.** Leaf size=22

$$-\frac{1}{2bn(a+b \log(cx^n))^2}$$

[Out] -1/(2\*b\*n\*(a + b\*Log[c\*x^n])^2)

**Rubi [A]** time = 0.0235673, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2302, 30}

$$-\frac{1}{2bn(a+b \log(cx^n))^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(a + b\*Log[c\*x^n])^3), x]

[Out] -1/(2\*b\*n\*(a + b\*Log[c\*x^n])^2)

Rule 2302

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] := Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{1}{x(a+b \log(cx^n))^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3} dx, x, a + b \log(cx^n)\right)}{bn} \\ &= -\frac{1}{2bn(a+b \log(cx^n))^2} \end{aligned}$$

**Mathematica [A]** time = 0.0038617, size = 22, normalized size = 1.

$$-\frac{1}{2bn(a+b \log(cx^n))^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(a + b\*Log[c\*x^n])^3), x]

[Out] -1/(2\*b\*n\*(a + b\*Log[c\*x^n])^2)

---

**Maple [A]** time = 0.035, size = 21, normalized size = 1.

$$\frac{1}{2bn(a + b \ln(cx^n))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b\*ln(c\*x^n))^3,x)

[Out] -1/2/b/n/(a+b\*ln(c\*x^n))^2

---

**Maxima [A]** time = 1.09555, size = 27, normalized size = 1.23

$$\frac{1}{2(b \log(cx^n) + a)^2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out] -1/2/((b\*log(c\*x^n) + a)^2\*b\*n)

---

**Fricas [B]** time = 0.792293, size = 150, normalized size = 6.82

$$\frac{1}{2(b^3n^3 \log(x)^2 + b^3n \log(c)^2 + 2ab^2n \log(c) + a^2bn + 2(b^3n^2 \log(c) + ab^2n^2) \log(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out] -1/2/(b^3\*n^3\*log(x)^2 + b^3\*n\*log(c)^2 + 2\*a\*b^2\*n\*log(c) + a^2\*b\*n + 2\*(b^3\*n^2\*log(c) + a\*b^2\*n^2)\*log(x))

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*ln(c\*x\*\*n))\*\*3,x)

[Out] Timed out

---

**Giac [A]** time = 1.28818, size = 28, normalized size = 1.27

$$\frac{1}{2(bn \log(x) + b \log(c) + a)^2bn}$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] -1/2/((b*n*log(x) + b*log(c) + a)^2*b*n)
```

$$3.86 \quad \int \frac{1}{x^2(a+b \log(cx^n))^3} dx$$

**Optimal.** Leaf size=102

$$\frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} \operatorname{Ei}\left(-\frac{a+b \log(cx^n)}{bn}\right)}{2b^3n^3x} + \frac{1}{2b^2n^2x(a+b \log(cx^n))} - \frac{1}{2bnx(a+b \log(cx^n))^2}$$

[Out] (E^(a/(b\*n))\*(c\*x^n)^n^(-1)\*ExpIntegralEi[-((a + b\*Log[c\*x^n])/(b\*n))])/(2\*b^3\*n^3\*x) - 1/(2\*b\*n\*x\*(a + b\*Log[c\*x^n])^2) + 1/(2\*b^2\*n^2\*x\*(a + b\*Log[c\*x^n]))

**Rubi [A]** time = 0.106101, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {2306, 2310, 2178}

$$\frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} \operatorname{Ei}\left(-\frac{a+b \log(cx^n)}{bn}\right)}{2b^3n^3x} + \frac{1}{2b^2n^2x(a+b \log(cx^n))} - \frac{1}{2bnx(a+b \log(cx^n))^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(a + b\*Log[c\*x^n])^3), x]

[Out] (E^(a/(b\*n))\*(c\*x^n)^n^(-1)\*ExpIntegralEi[-((a + b\*Log[c\*x^n])/(b\*n))])/(2\*b^3\*n^3\*x) - 1/(2\*b\*n\*x\*(a + b\*Log[c\*x^n])^2) + 1/(2\*b^2\*n^2\*x\*(a + b\*Log[c\*x^n]))

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 (a + b \log(cx^n))^3} dx &= -\frac{1}{2bnx (a + b \log(cx^n))^2} - \frac{\int \frac{1}{x^2 (a + b \log(cx^n))^2} dx}{2bn} \\
&= -\frac{1}{2bnx (a + b \log(cx^n))^2} + \frac{1}{2b^2 n^2 x (a + b \log(cx^n))} + \frac{\int \frac{1}{x^2 (a + b \log(cx^n))} dx}{2b^2 n^2} \\
&= -\frac{1}{2bnx (a + b \log(cx^n))^2} + \frac{1}{2b^2 n^2 x (a + b \log(cx^n))} + \frac{(cx^n)^{\frac{1}{n}} \text{Subst}\left(\int \frac{e^{-\frac{x}{a+bx}}}{a+bx} dx, x, \log(cx^n)\right)}{2b^2 n^3 x} \\
&= \frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} \text{Ei}\left(-\frac{a+b \log(cx^n)}{bn}\right)}{2b^3 n^3 x} - \frac{1}{2bnx (a + b \log(cx^n))^2} + \frac{1}{2b^2 n^2 x (a + b \log(cx^n))}
\end{aligned}$$

**Mathematica [A]** time = 0.0940234, size = 94, normalized size = 0.92

$$\frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} (a + b \log(cx^n))^2 \text{Ei}\left(-\frac{a+b \log(cx^n)}{bn}\right) + bn (a + b \log(cx^n) - bn)}{2b^3 n^3 x (a + b \log(cx^n))^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(a + b\*Log[c\*x^n])^3), x]

[Out] (E^(a/(b\*n))\*(c\*x^n)^n^(-1)\*ExpIntegralEi[-((a + b\*Log[c\*x^n])/(b\*n))])\*(a + b\*Log[c\*x^n])^2 + b\*n\*(a - b\*n + b\*Log[c\*x^n])/(2\*b^3\*n^3\*x\*(a + b\*Log[c\*x^n])^2)

**Maple [F]** time = 0.719, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a + b \ln(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+b\*ln(c\*x^n))^3, x)

[Out] int(1/x^2/(a+b\*ln(c\*x^n))^3, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{b(n - \log(c)) - b \log(x^n) - a}{2(b^4 n^2 x \log(x^n)^2 + 2(b^4 n^2 \log(c) + ab^3 n^2)x \log(x^n) + (b^4 n^2 \log(c)^2 + 2ab^3 n^2 \log(c) + a^2 b^2 n^2)x)} + \int \frac{1}{2(b^3 n^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*log(c\*x^n))^3, x, algorithm="maxima")

[Out] -1/2\*(b\*(n - log(c)) - b\*log(x^n) - a)/(b^4\*n^2\*x\*log(x^n)^2 + 2\*(b^4\*n^2\*log(c) + a\*b^3\*n^2)\*x\*log(x^n) + (b^4\*n^2\*log(c)^2 + 2\*a\*b^3\*n^2\*log(c) + a^2\*b^2\*n^2)\*x) + integrate(1/2/(b^3\*n^2\*x^2\*log(x^n) + (b^3\*n^2\*log(c) + a\*b

$^{2n^2}x^2), x)$

---

**Fricas [B]** time = 0.904241, size = 473, normalized size = 4.64

$$\frac{b^2 n^2 \log(x) - b^2 n^2 + b^2 n \log(c) + abn + (b^2 n^2 x \log(x)^2 + b^2 x \log(c)^2 + 2 abx \log(c) + a^2 x + 2 (b^2 n x \log(c) + abnx) \log(x))}{2 (b^5 n^5 x \log(x)^2 + b^5 n^3 x \log(c)^2 + 2 ab^4 n^3 x \log(c) + a^2 b^3 n^3 x + 2 (b^5 n^4 x \log(c) + ab^4 n^4 x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out] 1/2\*(b^2\*n^2\*log(x) - b^2\*n^2 + b^2\*n\*log(c) + a\*b\*n + (b^2\*n^2\*x\*log(x)^2 + b^2\*x\*log(c)^2 + 2\*a\*b\*x\*log(c) + a^2\*x + 2\*(b^2\*n\*x\*log(c) + a\*b\*n\*x)\*log(x))\*e^((b\*log(c) + a)/(b\*n))\*log\_integral(e^(-(b\*log(c) + a)/(b\*n))/x))/(b^5\*n^5\*x\*log(x)^2 + b^5\*n^3\*x\*log(c)^2 + 2\*a\*b^4\*n^3\*x\*log(c) + a^2\*b^3\*n^3\*x + 2\*(b^5\*n^4\*x\*log(c) + a\*b^4\*n^4\*x)\*log(x))

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a + b \log(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(a+b\*ln(c\*x\*\*n))\*\*3,x)

[Out] Integral(1/(x\*\*2\*(a + b\*log(c\*x\*\*n))\*\*3), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \log(cx^n) + a)^3 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out] integrate(1/((b\*log(c\*x^n) + a)^3\*x^2), x)

$$3.87 \quad \int \frac{1}{x^3(a+b \log(cx^n))^3} dx$$

**Optimal.** Leaf size=100

$$\frac{2e^{\frac{2a}{bn}} (cx^n)^{2/n} \operatorname{Ei}\left(-\frac{2(a+b \log(cx^n))}{bn}\right)}{b^3 n^3 x^2} + \frac{1}{b^2 n^2 x^2 (a+b \log(cx^n))} - \frac{1}{2bnx^2 (a+b \log(cx^n))^2}$$

[Out] (2\*E^((2\*a)/(b\*n))\*(c\*x^n)^(2/n)\*ExpIntegralEi[(-2\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b^3\*n^3\*x^2) - 1/(2\*b\*n\*x^2\*(a + b\*Log[c\*x^n])^2) + 1/(b^2\*n^2\*x^2\*(a + b\*Log[c\*x^n]))

**Rubi [A]** time = 0.108335, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {2306, 2310, 2178}

$$\frac{2e^{\frac{2a}{bn}} (cx^n)^{2/n} \operatorname{Ei}\left(-\frac{2(a+b \log(cx^n))}{bn}\right)}{b^3 n^3 x^2} + \frac{1}{b^2 n^2 x^2 (a+b \log(cx^n))} - \frac{1}{2bnx^2 (a+b \log(cx^n))^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*(a + b\*Log[c\*x^n])^3), x]

[Out] (2\*E^((2\*a)/(b\*n))\*(c\*x^n)^(2/n)\*ExpIntegralEi[(-2\*(a + b\*Log[c\*x^n]))/(b\*n)])/(b^3\*n^3\*x^2) - 1/(2\*b\*n\*x^2\*(a + b\*Log[c\*x^n])^2) + 1/(b^2\*n^2\*x^2\*(a + b\*Log[c\*x^n]))

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 (a + b \log(cx^n))^3} dx &= -\frac{1}{2bnx^2 (a + b \log(cx^n))^2} - \frac{\int \frac{1}{x^3 (a + b \log(cx^n))^2} dx}{bn} \\
&= -\frac{1}{2bnx^2 (a + b \log(cx^n))^2} + \frac{1}{b^2 n^2 x^2 (a + b \log(cx^n))} + \frac{2 \int \frac{1}{x^3 (a + b \log(cx^n))} dx}{b^2 n^2} \\
&= -\frac{1}{2bnx^2 (a + b \log(cx^n))^2} + \frac{1}{b^2 n^2 x^2 (a + b \log(cx^n))} + \frac{(2 (cx^n)^{2/n}) \text{Subst} \left( \int \frac{e^{-\frac{2x}{n}}}{a+bx} dx, x, \log(cx^n) \right)}{b^2 n^3 x^2} \\
&= \frac{2e^{\frac{2a}{bn}} (cx^n)^{2/n} \text{Ei} \left( -\frac{2(a+b \log(cx^n))}{bn} \right)}{b^3 n^3 x^2} - \frac{1}{2bnx^2 (a + b \log(cx^n))^2} + \frac{1}{b^2 n^2 x^2 (a + b \log(cx^n))}
\end{aligned}$$

**Mathematica [A]** time = 0.118754, size = 89, normalized size = 0.89

$$\frac{4e^{\frac{2a}{bn}} (cx^n)^{2/n} \text{Ei} \left( -\frac{2(a+b \log(cx^n))}{bn} \right) + \frac{bn(2a+2b \log(cx^n)-bn)}{(a+b \log(cx^n))^2}}{2b^3 n^3 x^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(a + b\*Log[c\*x^n])^3),x]

[Out] (4\*E^((2\*a)/(b\*n))\*(c\*x^n)^(2/n)\*ExpIntegralEi[(-2\*(a + b\*Log[c\*x^n]))/(b\*n)] + (b\*n\*(2\*a - b\*n + 2\*b\*Log[c\*x^n]))/(a + b\*Log[c\*x^n])^2)/(2\*b^3\*n^3\*x^2)

**Maple [F]** time = 0.709, size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (a + b \ln(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(a+b\*ln(c\*x^n))^3,x)

[Out] int(1/x^3/(a+b\*ln(c\*x^n))^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{b(n - 2 \log(c)) - 2b \log(x^n) - 2a}{2(b^4 n^2 x^2 \log(x^n)^2 + 2(b^4 n^2 \log(c) + ab^3 n^2)x^2 \log(x^n) + (b^4 n^2 \log(c)^2 + 2ab^3 n^2 \log(c) + a^2 b^2 n^2)x^2)} + 2 \int \frac{1}{b^3 n^2 x^3 \log(x^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out] -1/2\*(b\*(n - 2\*log(c)) - 2\*b\*log(x^n) - 2\*a)/(b^4\*n^2\*x^2\*log(x^n)^2 + 2\*(b^4\*n^2\*log(c) + a\*b^3\*n^2)\*x^2\*log(x^n) + (b^4\*n^2\*log(c)^2 + 2\*a\*b^3\*n^2\*log(c) + a^2\*b^2\*n^2)\*x^2) + 2\*integrate(1/(b^3\*n^2\*x^3\*log(x^n)) + (b^3\*n^2\*

$\log(c) + a*b^2*n^2*x^3, x)$

**Fricas [B]** time = 0.738526, size = 524, normalized size = 5.24

$$\frac{2b^2n^2 \log(x) - b^2n^2 + 2b^2n \log(c) + 2abn + 4(b^2n^2x^2 \log(x)^2 + b^2x^2 \log(c)^2 + 2abx^2 \log(c) + a^2x^2 + 2(b^2nx^2 \log(c) + a*b*n*x^2) \log(x)) * e^{(2*(b*\log(c) + a)/(b*n))} * \log\_integral(e^{-2*(b*\log(c) + a)/(b*n)}/x^2)} / (b^5n^5x^2 \log(x)^2 + b^5n^3x^2 \log(c)^2 + 2ab^4n^3x^2 \log(c) + a^2b^3n^3x^2 + 2(b^5n^4x^2 \log(c) + a*b^4n^4x^2) \log(x))}{2(b^5n^5x^2 \log(x)^2 + b^5n^3x^2 \log(c)^2 + 2ab^4n^3x^2 \log(c) + a^2b^3n^3x^2 + 2(b^5n^4x^2 \log(c) + a*b^4n^4x^2) \log(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out] 1/2\*(2\*b^2\*n^2\*log(x) - b^2\*n^2 + 2\*b^2\*n\*log(c) + 2\*a\*b\*n + 4\*(b^2\*n^2\*x^2\*log(x)^2 + b^2\*x^2\*log(c)^2 + 2\*a\*b\*x^2\*log(c) + a^2\*x^2 + 2\*(b^2\*n\*x^2\*log(c) + a\*b\*n\*x^2)\*log(x))\*e^(2\*(b\*log(c) + a)/(b\*n))\*log\_integral(e^(-2\*(b\*log(c) + a)/(b\*n))/x^2))/(b^5\*n^5\*x^2\*log(x)^2 + b^5\*n^3\*x^2\*log(c)^2 + 2\*a\*b^4\*n^3\*x^2\*log(c) + a^2\*b^3\*n^3\*x^2 + 2\*(b^5\*n^4\*x^2\*log(c) + a\*b^4\*n^4\*x^2)\*log(x))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (a + b \log(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/(a+b\*ln(c\*x\*\*n))\*\*3,x)

[Out] Integral(1/(x\*\*3\*(a + b\*log(c\*x\*\*n))\*\*3), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \log(cx^n) + a)^3 x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out] integrate(1/((b\*log(c\*x^n) + a)^3\*x^3), x)

$$3.88 \quad \int \frac{1}{x^4(a+b \log(cx^n))^3} dx$$

**Optimal.** Leaf size=105

$$\frac{9e^{\frac{3a}{bn}} (cx^n)^{3/n} \operatorname{Ei}\left(-\frac{3(a+b \log(cx^n))}{bn}\right)}{2b^3 n^3 x^3} + \frac{3}{2b^2 n^2 x^3 (a+b \log(cx^n))} - \frac{1}{2bnx^3 (a+b \log(cx^n))^2}$$

[Out] (9\*E^((3\*a)/(b\*n))\*(c\*x^n)^(3/n)\*ExpIntegralEi[(-3\*(a + b\*Log[c\*x^n]))/(b\*n)])/(2\*b^3\*n^3\*x^3) - 1/(2\*b\*n\*x^3\*(a + b\*Log[c\*x^n])^2) + 3/(2\*b^2\*n^2\*x^3\*(a + b\*Log[c\*x^n]))

**Rubi [A]** time = 0.106838, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {2306, 2310, 2178}

$$\frac{9e^{\frac{3a}{bn}} (cx^n)^{3/n} \operatorname{Ei}\left(-\frac{3(a+b \log(cx^n))}{bn}\right)}{2b^3 n^3 x^3} + \frac{3}{2b^2 n^2 x^3 (a+b \log(cx^n))} - \frac{1}{2bnx^3 (a+b \log(cx^n))^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a + b\*Log[c\*x^n])^3), x]

[Out] (9\*E^((3\*a)/(b\*n))\*(c\*x^n)^(3/n)\*ExpIntegralEi[(-3\*(a + b\*Log[c\*x^n]))/(b\*n)])/(2\*b^3\*n^3\*x^3) - 1/(2\*b\*n\*x^3\*(a + b\*Log[c\*x^n])^2) + 3/(2\*b^2\*n^2\*x^3\*(a + b\*Log[c\*x^n]))

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

#### Rubi steps



$$\begin{aligned}
\int \frac{1}{x^4 (a + b \log(cx^n))^3} dx &= -\frac{1}{2bnx^3 (a + b \log(cx^n))^2} - \frac{3 \int \frac{1}{x^4 (a + b \log(cx^n))^2} dx}{2bn} \\
&= -\frac{1}{2bnx^3 (a + b \log(cx^n))^2} + \frac{3}{2b^2 n^2 x^3 (a + b \log(cx^n))} + \frac{9 \int \frac{1}{x^4 (a + b \log(cx^n))} dx}{2b^2 n^2} \\
&= -\frac{1}{2bnx^3 (a + b \log(cx^n))^2} + \frac{3}{2b^2 n^2 x^3 (a + b \log(cx^n))} + \frac{(9 (cx^n)^{3/n}) \text{Subst} \left( \int \frac{e^{-\frac{3x}{n}}}{a+bx} dx, x, \right)}{2b^2 n^3 x^3} \\
&= \frac{9e^{\frac{3a}{bn}} (cx^n)^{3/n} \text{Ei} \left( -\frac{3(a+b \log(cx^n))}{bn} \right)}{2b^3 n^3 x^3} - \frac{1}{2bnx^3 (a + b \log(cx^n))^2} + \frac{3}{2b^2 n^2 x^3 (a + b \log(cx^n))}
\end{aligned}$$

**Mathematica [A]** time = 0.114823, size = 89, normalized size = 0.85

$$\frac{9e^{\frac{3a}{bn}} (cx^n)^{3/n} \text{Ei} \left( -\frac{3(a+b \log(cx^n))}{bn} \right) + \frac{bn(3a+3b \log(cx^n))-bn}{(a+b \log(cx^n))^2}}{2b^3 n^3 x^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(a + b\*Log[c\*x^n])^3), x]

[Out] (9\*E^((3\*a)/(b\*n))\*(c\*x^n)^(3/n)\*ExpIntegralEi[(-3\*(a + b\*Log[c\*x^n]))]/(b\*n)) + (b\*n\*(3\*a - b\*n + 3\*b\*Log[c\*x^n]))/(a + b\*Log[c\*x^n])^2/(2\*b^3\*n^3\*x^3)

**Maple [F]** time = 0.741, size = 0, normalized size = 0.

$$\int \frac{1}{x^4 (a + b \ln(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(a+b\*ln(c\*x^n))^3,x)

[Out] int(1/x^4/(a+b\*ln(c\*x^n))^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{b(n - 3 \log(c)) - 3 b \log(x^n) - 3 a}{2 (b^4 n^2 x^3 \log(x^n)^2 + 2 (b^4 n^2 \log(c) + ab^3 n^2) x^3 \log(x^n) + (b^4 n^2 \log(c)^2 + 2 ab^3 n^2 \log(c) + a^2 b^2 n^2) x^3)} + 9 \int \frac{1}{2 (b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out] -1/2\*(b\*(n - 3\*log(c)) - 3\*b\*log(x^n) - 3\*a)/(b^4\*n^2\*x^3\*log(x^n)^2 + 2\*(b^4\*n^2\*log(c) + a\*b^3\*n^2)\*x^3\*log(x^n) + (b^4\*n^2\*log(c)^2 + 2\*a\*b^3\*n^2\*log(c) + a^2\*b^2\*n^2)\*x^3) + 9\*integrate(1/2/(b^3\*n^2\*x^4\*log(x^n) + (b^3\*n^2

$2 \cdot \log(c) + a \cdot b^2 \cdot n^2 \cdot x^4$ , x)

**Fricas [B]** time = 0.738719, size = 524, normalized size = 4.99

$$\frac{3b^2n^2 \log(x) - b^2n^2 + 3b^2n \log(c) + 3abn + 9(b^2n^2x^3 \log(x)^2 + b^2x^3 \log(c)^2 + 2abx^3 \log(c) + a^2x^3 + 2(b^2nx^3 \log(c) - b^2n^2 \log(x) + b^2n^2) \cdot x^4)}{2(b^5n^5x^3 \log(x)^2 + b^5n^3x^3 \log(c)^2 + 2ab^4n^3x^3 \log(c) + a^2b^3n^3x^3 + 2(b^5n^4x^3 \log(c) - b^5n^3x^3 \log(x) + b^5n^3x^3 \log(c)^2) \cdot x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out]  $\frac{1}{2} \cdot (3b^2n^2 \log(x) - b^2n^2 + 3b^2n \log(c) + 3a \cdot b \cdot n + 9(b^2n^2x^3 \log(x)^2 + b^2x^3 \log(c)^2 + 2a \cdot b \cdot x^3 \log(c) + a^2x^3 + 2(b^2nx^3 \log(c) - b^2n^2 \log(x) + b^2n^2) \cdot x^4) \cdot e^{3(b \log(c) + a)/(b \cdot n)} \cdot \log\_integral(e^{-3(b \log(c) + a)/(b \cdot n)}/x^3)) / (b^5n^5x^3 \log(x)^2 + b^5n^3x^3 \log(c)^2 + 2a \cdot b^4n^3x^3 \log(c) + a^2b^3n^3x^3 + 2(b^5n^4x^3 \log(c) - b^5n^3x^3 \log(x) + b^5n^3x^3 \log(c)^2) \cdot x^4)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 (a + b \log(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*4/(a+b\*ln(c\*x\*\*n))\*\*3,x)

[Out] Integral(1/(x\*\*4\*(a + b\*log(c\*x\*\*n))\*\*3), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \log(cx^n) + a)^3 x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out] integrate(1/((b\*log(c\*x^n) + a)^3\*x^4), x)

### 3.89 $\int (dx)^{5/2} (a + b \log(cx^n)) dx$

**Optimal.** Leaf size=41

$$\frac{2(dx)^{7/2} (a + b \log(cx^n))}{7d} - \frac{4bn(dx)^{7/2}}{49d}$$

[Out]  $(-4*b*n*(d*x)^{(7/2)})/(49*d) + (2*(d*x)^{(7/2)}*(a + b*Log[c*x^n]))/(7*d)$

**Rubi [A]** time = 0.0158954, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$ , Rules used = {2304}

$$\frac{2(dx)^{7/2} (a + b \log(cx^n))}{7d} - \frac{4bn(dx)^{7/2}}{49d}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^(5/2)\*(a + b\*Log[c\*x^n]),x]

[Out]  $(-4*b*n*(d*x)^{(7/2)})/(49*d) + (2*(d*x)^{(7/2)}*(a + b*Log[c*x^n]))/(7*d)$

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :>  
Simp[(d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\int (dx)^{5/2} (a + b \log(cx^n)) dx = -\frac{4bn(dx)^{7/2}}{49d} + \frac{2(dx)^{7/2} (a + b \log(cx^n))}{7d}$$

**Mathematica [A]** time = 0.0135287, size = 29, normalized size = 0.71

$$\frac{2}{49}x(dx)^{5/2} (7a + 7b \log(cx^n) - 2bn)$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(5/2)\*(a + b\*Log[c\*x^n]),x]

[Out]  $(2*x*(d*x)^{(5/2)}*(7*a - 2*b*n + 7*b*Log[c*x^n]))/49$

**Maple [C]** time = 0.108, size = 128, normalized size = 3.1

$$\frac{2d^3bx^4 \ln(x^n)}{7} \frac{1}{\sqrt{dx}} + \frac{d^3(7ib\pi \operatorname{csgn}(ix^n)(\operatorname{csgn}(icx^n))^2 - 7ib\pi \operatorname{csgn}(ix^n) \operatorname{csgn}(icx^n) \operatorname{csgn}(ic) - 7ib\pi (\operatorname{csgn}(icx^n))^3)}{49}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^(5/2)*(a+b*ln(c*x^n)),x)`

[Out]  $2/7*d^3*b*x^4/(d*x)^{(1/2)}*\ln(x^n)+1/49*d^3*(7*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-7*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-7*I*b*Pi*csgn(I*c*x^n)^3+7*I*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+14*b*\ln(c)-4*b*n+14*a)*x^4/(d*x)^{(1/2)}$

**Maxima [A]** time = 1.19304, size = 55, normalized size = 1.34

$$-\frac{4(dx)^{\frac{7}{2}}bn}{49d} + \frac{2(dx)^{\frac{7}{2}}b\log(cx^n)}{7d} + \frac{2(dx)^{\frac{7}{2}}a}{7d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(5/2)*(a+b*log(c*x^n)),x, algorithm="maxima")`

[Out]  $-4/49*(d*x)^{(7/2)}*b*n/d + 2/7*(d*x)^{(7/2)}*b*\log(c*x^n)/d + 2/7*(d*x)^{(7/2)}*a/d$

**Fricas [A]** time = 0.913255, size = 119, normalized size = 2.9

$$\frac{2}{49} (7bd^2nx^3 \log(x) + 7bd^2x^3 \log(c) - (2bd^2n - 7ad^2)x^3) \sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(5/2)*(a+b*log(c*x^n)),x, algorithm="fricas")`

[Out]  $2/49*(7*b*d^2*n*x^3*\log(x) + 7*b*d^2*x^3*\log(c) - (2*b*d^2*n - 7*a*d^2)*x^3)*\sqrt{d*x}$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)**(5/2)*(a+b*ln(c*x**n)),x)`

[Out] Timed out

**Giac [C]** time = 1.43687, size = 158, normalized size = 3.85

$$\left(\frac{1}{7}i + \frac{1}{7}\right) \sqrt{2bd^2nx^{\frac{7}{2}}\sqrt{|d|}} \cos\left(\frac{1}{4}\pi\operatorname{sgn}(d)\right) \log(x) - \left(\frac{1}{7}i - \frac{1}{7}\right) \sqrt{2bd^2nx^{\frac{7}{2}}\sqrt{|d|}} \log(x) \sin\left(\frac{1}{4}\pi\operatorname{sgn}(d)\right) - \left(\frac{2}{49}i + \frac{2}{49}\right) \sqrt{2bd^2nx^{\frac{7}{2}}\sqrt{|d|}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(5/2)*(a+b*log(c*x^n)),x, algorithm="giac")`

```
[Out] (1/7*I + 1/7)*sqrt(2)*b*d^2*n*x^(7/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d))*log(x)
- (1/7*I - 1/7)*sqrt(2)*b*d^2*n*x^(7/2)*sqrt(abs(d))*log(x)*sin(1/4*pi*sgn(d))
- (2/49*I + 2/49)*sqrt(2)*b*d^2*n*x^(7/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d))
+ (2/49*I - 2/49)*sqrt(2)*b*d^2*n*x^(7/2)*sqrt(abs(d))*sin(1/4*pi*sgn(d))
+ 2/7*b*d^(5/2)*x^(7/2)*log(c) + 2/7*a*d^(5/2)*x^(7/2)
```

### 3.90 $\int (dx)^{3/2} (a + b \log(cx^n)) dx$

**Optimal.** Leaf size=41

$$\frac{2(dx)^{5/2} (a + b \log(cx^n))}{5d} - \frac{4bn(dx)^{5/2}}{25d}$$

[Out]  $(-4*b*n*(d*x)^{(5/2)})/(25*d) + (2*(d*x)^{(5/2)}*(a + b*Log[c*x^n]))/(5*d)$

**Rubi [A]** time = 0.0159575, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$ , Rules used = {2304}

$$\frac{2(dx)^{5/2} (a + b \log(cx^n))}{5d} - \frac{4bn(dx)^{5/2}}{25d}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^(3/2)\*(a + b\*Log[c\*x^n]),x]

[Out]  $(-4*b*n*(d*x)^{(5/2)})/(25*d) + (2*(d*x)^{(5/2)}*(a + b*Log[c*x^n]))/(5*d)$

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :=  
Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\int (dx)^{3/2} (a + b \log(cx^n)) dx = -\frac{4bn(dx)^{5/2}}{25d} + \frac{2(dx)^{5/2} (a + b \log(cx^n))}{5d}$$

**Mathematica [A]** time = 0.0100356, size = 29, normalized size = 0.71

$$\frac{2}{25} x(dx)^{3/2} (5a + 5b \log(cx^n) - 2bn)$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(3/2)\*(a + b\*Log[c\*x^n]),x]

[Out]  $(2*x*(d*x)^{(3/2)}*(5*a - 2*b*n + 5*b*Log[c*x^n]))/25$

**Maple [C]** time = 0.082, size = 128, normalized size = 3.1

$$\frac{2bd^2x^3 \ln(x^n)}{5} \frac{1}{\sqrt{dx}} + \frac{d^2 (5ib\pi \operatorname{csgn}(ix^n) (\operatorname{csgn}(icx^n))^2 - 5ib\pi \operatorname{csgn}(ix^n) \operatorname{csgn}(icx^n) \operatorname{csgn}(ic) - 5ib\pi (\operatorname{csgn}(icx^n))^3 + 5ib\pi \operatorname{csgn}(icx^n) \operatorname{csgn}(ic))}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^(3/2)*(a+b*ln(c*x^n)),x)`

[Out]  $\frac{2}{5}d^2bx^3/(d*x)^{(1/2)}*\ln(x^n)+1/25*d^2*(5*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-5*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-5*I*b*Pi*csgn(I*c*x^n)^3+5*I*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+10*b*\ln(c)-4*b*n+10*a)*x^3/(d*x)^{(1/2)}$

**Maxima [A]** time = 1.01124, size = 55, normalized size = 1.34

$$-\frac{4(dx)^{\frac{5}{2}}bn}{25d} + \frac{2(dx)^{\frac{5}{2}}b\log(cx^n)}{5d} + \frac{2(dx)^{\frac{5}{2}}a}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(3/2)*(a+b*log(c*x^n)),x, algorithm="maxima")`

[Out]  $-4/25*(d*x)^{(5/2)}*b*n/d + 2/5*(d*x)^{(5/2)}*b*\log(c*x^n)/d + 2/5*(d*x)^{(5/2)}*a/d$

**Fricas [A]** time = 0.934176, size = 108, normalized size = 2.63

$$\frac{2}{25} (5bdnx^2 \log(x) + 5bdx^2 \log(c) - (2bdn - 5ad)x^2) \sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(3/2)*(a+b*log(c*x^n)),x, algorithm="fricas")`

[Out]  $2/25*(5*b*d*n*x^2*\log(x) + 5*b*d*x^2*\log(c) - (2*b*d*n - 5*a*d)*x^2)*\sqrt{d*x}$

**Sympy [A]** time = 46.0505, size = 70, normalized size = 1.71

$$\frac{2ad^{\frac{3}{5}}x^{\frac{5}{2}}}{5} + \frac{2bd^{\frac{3}{5}}nx^{\frac{5}{2}}\log(x)}{5} - \frac{4bd^{\frac{3}{5}}nx^{\frac{5}{2}}}{25} + \frac{2bd^{\frac{3}{5}}x^{\frac{5}{2}}\log(c)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)**(3/2)*(a+b*ln(c*x**n)),x)`

[Out]  $2*a*d**(3/2)*x**(5/2)/5 + 2*b*d**(3/2)*n*x**(5/2)*\log(x)/5 - 4*b*d**(3/2)*n*x**(5/2)/25 + 2*b*d**(3/2)*x**(5/2)*\log(c)/5$

**Giac [C]** time = 1.52479, size = 146, normalized size = 3.56

$$-\frac{1}{25} \left( -(5i+5) \sqrt{2}bnx^{\frac{5}{2}}\sqrt{|d|} \cos\left(\frac{1}{4}\pi\operatorname{sgn}(d)\right) \log(x) + (5i-5) \sqrt{2}bnx^{\frac{5}{2}}\sqrt{|d|} \log(x) \sin\left(\frac{1}{4}\pi\operatorname{sgn}(d)\right) + (2i+2) \sqrt{2}bn \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] -1/25*(-(5*I + 5)*sqrt(2)*b*n*x^(5/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d))*log(x)
) + (5*I - 5)*sqrt(2)*b*n*x^(5/2)*sqrt(abs(d))*log(x)*sin(1/4*pi*sgn(d)) +
(2*I + 2)*sqrt(2)*b*n*x^(5/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d)) - (2*I - 2)*s
qrt(2)*b*n*x^(5/2)*sqrt(abs(d))*sin(1/4*pi*sgn(d)) - 10*b*sqrt(d)*x^(5/2)*l
og(c) - 10*a*sqrt(d)*x^(5/2))*d
```



### 3.91 $\int \sqrt{dx} (a + b \log(cx^n)) dx$

**Optimal.** Leaf size=41

$$\frac{2(dx)^{3/2} (a + b \log(cx^n))}{3d} - \frac{4bn(dx)^{3/2}}{9d}$$

[Out]  $(-4*b*n*(d*x)^{(3/2))/(9*d) + (2*(d*x)^{(3/2)}*(a + b*Log[c*x^n]))/(3*d)$

**Rubi [A]** time = 0.013953, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$ , Rules used = {2304}

$$\frac{2(dx)^{3/2} (a + b \log(cx^n))}{3d} - \frac{4bn(dx)^{3/2}}{9d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d\*x]\*(a + b\*Log[c\*x^n]), x]

[Out]  $(-4*b*n*(d*x)^{(3/2))/(9*d) + (2*(d*x)^{(3/2)}*(a + b*Log[c*x^n]))/(3*d)$

**Rule 2304**

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

**Rubi steps**

$$\int \sqrt{dx} (a + b \log(cx^n)) dx = -\frac{4bn(dx)^{3/2}}{9d} + \frac{2(dx)^{3/2} (a + b \log(cx^n))}{3d}$$

**Mathematica [A]** time = 0.0070536, size = 29, normalized size = 0.71

$$\frac{2}{9}x\sqrt{dx} (3a + 3b \log(cx^n) - 2bn)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d\*x]\*(a + b\*Log[c\*x^n]), x]

[Out]  $(2*x*Sqrt[d*x]*(3*a - 2*b*n + 3*b*Log[c*x^n]))/9$

**Maple [C]** time = 0.08, size = 124, normalized size = 3.

$$\frac{2bdx^2 \ln(x^n)}{3} \frac{1}{\sqrt{dx}} + \frac{d(3ib\pi \operatorname{csgn}(ix^n) (\operatorname{csgn}(icx^n))^2 - 3ib\pi \operatorname{csgn}(ix^n) \operatorname{csgn}(icx^n) \operatorname{csgn}(ic) - 3ib\pi (\operatorname{csgn}(icx^n))^3)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^(1/2)*(a+b*ln(c*x^n)),x)`

[Out]  $2/3*d*b*x^2/(d*x)^{(1/2)*\ln(x^n)+1/9*d*(3*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-3*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-3*I*b*Pi*csgn(I*c*x^n)^3+3*I*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+6*b*\ln(c)-4*b*n+6*a)*x^2/(d*x)^{(1/2)}$

**Maxima [A]** time = 1.13549, size = 55, normalized size = 1.34

$$-\frac{4(dx)^{\frac{3}{2}}bn}{9d} + \frac{2(dx)^{\frac{3}{2}}b\log(cx^n)}{3d} + \frac{2(dx)^{\frac{3}{2}}a}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(1/2)*(a+b*log(c*x^n)),x, algorithm="maxima")`

[Out]  $-4/9*(d*x)^{(3/2)*b*n/d + 2/3*(d*x)^{(3/2)*b*\log(c*x^n)/d + 2/3*(d*x)^{(3/2)*a/d}$

**Fricas [A]** time = 0.915775, size = 88, normalized size = 2.15

$$\frac{2}{9}(3bnx\log(x) + 3bx\log(c) - (2bn - 3a)x)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(1/2)*(a+b*log(c*x^n)),x, algorithm="fricas")`

[Out]  $2/9*(3*b*n*x*\log(x) + 3*b*x*\log(c) - (2*b*n - 3*a)*x)*\sqrt{d*x}$

**Sympy [A]** time = 2.5384, size = 70, normalized size = 1.71

$$\frac{2a\sqrt{dx^{\frac{3}{2}}}}{3} + \frac{2b\sqrt{d}nx^{\frac{3}{2}}\log(x)}{3} - \frac{4b\sqrt{d}nx^{\frac{3}{2}}}{9} + \frac{2b\sqrt{dx^{\frac{3}{2}}}\log(c)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)**(1/2)*(a+b*ln(c*x**n)),x)`

[Out]  $2*a*\sqrt{d}*x^{(3/2)}/3 + 2*b*\sqrt{d}*n*x^{(3/2)}*\log(x)/3 - 4*b*\sqrt{d}*n*x^{(3/2)}/9 + 2*b*\sqrt{d}*x^{(3/2)}*\log(c)/3$

**Giac [C]** time = 1.55264, size = 142, normalized size = 3.46

$$\left(\frac{1}{3}i + \frac{1}{3}\right)\sqrt{2}bnx^{\frac{3}{2}}\sqrt{|d|}\cos\left(\frac{1}{4}\pi\operatorname{sgn}(d)\right)\log(x) - \left(\frac{1}{3}i - \frac{1}{3}\right)\sqrt{2}bnx^{\frac{3}{2}}\sqrt{|d|}\log(x)\sin\left(\frac{1}{4}\pi\operatorname{sgn}(d)\right) - \left(\frac{2}{9}i + \frac{2}{9}\right)\sqrt{2}bnx^{\frac{3}{2}}\sqrt{|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(1/2)*(a+b*log(c*x^n)),x, algorithm="giac")`

```
[Out] (1/3*I + 1/3)*sqrt(2)*b*n*x^(3/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d))*log(x) -  
(1/3*I - 1/3)*sqrt(2)*b*n*x^(3/2)*sqrt(abs(d))*log(x)*sin(1/4*pi*sgn(d)) -  
(2/9*I + 2/9)*sqrt(2)*b*n*x^(3/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d)) + (2/9*I  
- 2/9)*sqrt(2)*b*n*x^(3/2)*sqrt(abs(d))*sin(1/4*pi*sgn(d)) + 2/3*b*sqrt(d)*  
x^(3/2)*log(c) + 2/3*a*sqrt(d)*x^(3/2)
```

$$3.92 \quad \int \frac{a+b \log(cx^n)}{\sqrt{dx}} dx$$

**Optimal.** Leaf size=37

$$\frac{2\sqrt{dx}(a+b \log(cx^n))}{d} - \frac{4bn\sqrt{dx}}{d}$$

[Out]  $(-4*b*n*\text{Sqrt}[d*x])/d + (2*\text{Sqrt}[d*x]*(a + b*\text{Log}[c*x^n]))/d$

**Rubi [A]** time = 0.0145035, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$ , Rules used = {2304}

$$\frac{2\sqrt{dx}(a+b \log(cx^n))}{d} - \frac{4bn\sqrt{dx}}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{Log}[c*x^n])/ \text{Sqrt}[d*x], x]$

[Out]  $(-4*b*n*\text{Sqrt}[d*x])/d + (2*\text{Sqrt}[d*x]*(a + b*\text{Log}[c*x^n]))/d$

#### Rule 2304

$\text{Int}[(a + \text{Log}[c*(x_)^{(n_)}])*(b_)*((d_)*(x_))^{(m_)}, x\_Symbol] :>$   
 $\text{Simp}[(d*x)^{(m+1)}*(a + b*\text{Log}[c*x^n])/(d*(m+1)), x] - \text{Simp}[(b*n*(d*x)^{(m+1)})/(d*(m+1)^2), x] /;$  FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\int \frac{a+b \log(cx^n)}{\sqrt{dx}} dx = -\frac{4bn\sqrt{dx}}{d} + \frac{2\sqrt{dx}(a+b \log(cx^n))}{d}$$

**Mathematica [A]** time = 0.005951, size = 24, normalized size = 0.65

$$\frac{2x(a+b \log(cx^n)) - 2bn}{\sqrt{dx}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a + b*\text{Log}[c*x^n])/ \text{Sqrt}[d*x], x]$

[Out]  $(2*x*(a - 2*b*n + b*\text{Log}[c*x^n]))/ \text{Sqrt}[d*x]$

**Maple [A]** time = 0.043, size = 42, normalized size = 1.1

$$2 \frac{\sqrt{dx} b \ln(cx^n)}{d} - 4 \frac{bn\sqrt{dx}}{d} + 2 \frac{\sqrt{dx} a}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*ln(c*x^n))/(d*x)^(1/2),x)`

[Out]  $2/d*(d*x)^{(1/2)}*b*\ln(c*x^n)-4*b*n*(d*x)^{(1/2)}/d+2/d*(d*x)^{(1/2)}*a$

**Maxima [A]** time = 1.17155, size = 55, normalized size = 1.49

$$-\frac{4\sqrt{d}xbn}{d} + \frac{2\sqrt{d}xb\log(cx^n)}{d} + \frac{2\sqrt{d}xa}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))/(d*x)^(1/2),x, algorithm="maxima")`

[Out]  $-4*\text{sqrt}(d*x)*b*n/d + 2*\text{sqrt}(d*x)*b*\log(c*x^n)/d + 2*\text{sqrt}(d*x)*a/d$

**Fricas [A]** time = 0.869924, size = 69, normalized size = 1.86

$$\frac{2(bn\log(x) - 2bn + b\log(c) + a)\sqrt{d}x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))/(d*x)^(1/2),x, algorithm="fricas")`

[Out]  $2*(b*n*\log(x) - 2*b*n + b*\log(c) + a)*\text{sqrt}(d*x)/d$

**Sympy [A]** time = 1.6799, size = 63, normalized size = 1.7

$$\frac{2a\sqrt{x}}{\sqrt{d}} + \frac{2bn\sqrt{x}\log(x)}{\sqrt{d}} - \frac{4bn\sqrt{x}}{\sqrt{d}} + \frac{2b\sqrt{x}\log(c)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*ln(c*x**n))/(d*x)**(1/2),x)`

[Out]  $2*a*\text{sqrt}(x)/\text{sqrt}(d) + 2*b*n*\text{sqrt}(x)*\log(x)/\text{sqrt}(d) - 4*b*n*\text{sqrt}(x)/\text{sqrt}(d) + 2*b*\text{sqrt}(x)*\log(c)/\text{sqrt}(d)$

**Giac [A]** time = 1.37406, size = 55, normalized size = 1.49

$$\frac{2\left(\left(\sqrt{d}x\log(x) - 2\sqrt{d}x\right)bn + \sqrt{d}xb\log(c) + \sqrt{d}xa\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))/(d*x)^(1/2),x, algorithm="giac")`

[Out]  $2*((\text{sqrt}(d*x)*\log(x) - 2*\text{sqrt}(d*x))*b*n + \text{sqrt}(d*x)*b*\log(c) + \text{sqrt}(d*x)*a)/d$

$$3.93 \quad \int \frac{a+b \log(cx^n)}{(dx)^{3/2}} dx$$

**Optimal.** Leaf size=37

$$-\frac{2(a+b \log(cx^n))}{d\sqrt{dx}} - \frac{4bn}{d\sqrt{dx}}$$

[Out]  $(-4*b*n)/(d*\text{Sqrt}[d*x]) - (2*(a + b*\text{Log}[c*x^n]))/(d*\text{Sqrt}[d*x])$

**Rubi [A]** time = 0.0159402, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$ , Rules used = {2304}

$$-\frac{2(a+b \log(cx^n))}{d\sqrt{dx}} - \frac{4bn}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{Log}[c*x^n])/(d*x)^{(3/2)}, x]$

[Out]  $(-4*b*n)/(d*\text{Sqrt}[d*x]) - (2*(a + b*\text{Log}[c*x^n]))/(d*\text{Sqrt}[d*x])$

#### Rule 2304

$\text{Int}[(a + \text{Log}[c*(x_)^{(n_*)}])*(b_*)*((d_*)*(x_))^{(m_*)}, x\_Symbol] :>$   
 $\text{Simp}[(d*x)^{(m+1)}*(a + b*\text{Log}[c*x^n])/(d*(m+1)), x] - \text{Simp}[(b*n*(d*x)^{(m+1)})/(d*(m+1)^2), x] /;$  FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\int \frac{a+b \log(cx^n)}{(dx)^{3/2}} dx = -\frac{4bn}{d\sqrt{dx}} - \frac{2(a+b \log(cx^n))}{d\sqrt{dx}}$$

**Mathematica [A]** time = 0.0067746, size = 24, normalized size = 0.65

$$-\frac{2x(a+b \log(cx^n)+2bn)}{(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a + b*\text{Log}[c*x^n])/(d*x)^{(3/2)}, x]$

[Out]  $(-2*x*(a + 2*b*n + b*\text{Log}[c*x^n]))/(d*x)^{(3/2)}$

**Maple [C]** time = 0.083, size = 122, normalized size = 3.3

$$-2 \frac{b \ln(x^n)}{d\sqrt{dx}} - \frac{ib\pi \text{csgn}(ix^n) (\text{csgn}(icx^n))^2 - ib\pi \text{csgn}(ix^n) \text{csgn}(icx^n) \text{csgn}(ic) - ib\pi (\text{csgn}(icx^n))^3 + ib\pi (\text{csgn}(icx^n))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^n))/(d\*x)^(3/2),x)

[Out] 
$$-2/d*b/(d*x)^{(1/2)}*\ln(x^n)-1/d*(I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*b*Pi*csgn(I*c*x^n)^3+I*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+2*b*\ln(c)+4*b*n+2*a)/(d*x)^{(1/2)}$$

**Maxima [A]** time = 1.17758, size = 55, normalized size = 1.49

$$-\frac{4bn}{\sqrt{dx}d} - \frac{2b\log(cx^n)}{\sqrt{dx}d} - \frac{2a}{\sqrt{dx}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))/(d\*x)^(3/2),x, algorithm="maxima")

[Out] 
$$-4*b*n/(\text{sqrt}(d*x)*d) - 2*b*\log(c*x^n)/(\text{sqrt}(d*x)*d) - 2*a/(\text{sqrt}(d*x)*d)$$

**Fricas [A]** time = 0.864841, size = 78, normalized size = 2.11

$$-\frac{2(bn\log(x) + 2bn + b\log(c) + a)\sqrt{dx}}{d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))/(d\*x)^(3/2),x, algorithm="fricas")

[Out] 
$$-2*(b*n*\log(x) + 2*b*n + b*\log(c) + a)*\text{sqrt}(d*x)/(d^2*x)$$

**Sympy [A]** time = 5.86456, size = 65, normalized size = 1.76

$$-\frac{2a}{d^{\frac{3}{2}}\sqrt{x}} - \frac{2bn\log(x)}{d^{\frac{3}{2}}\sqrt{x}} - \frac{4bn}{d^{\frac{3}{2}}\sqrt{x}} - \frac{2b\log(c)}{d^{\frac{3}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))/(d\*x)\*\*(3/2),x)

[Out] 
$$-2*a/(d^{(3/2)}*\text{sqrt}(x)) - 2*b*n*\log(x)/(d^{(3/2)}*\text{sqrt}(x)) - 4*b*n/(d^{(3/2)}*\text{sqrt}(x)) - 2*b*\log(c)/(d^{(3/2)}*\text{sqrt}(x))$$

**Giac [A]** time = 1.36171, size = 58, normalized size = 1.57

$$-\frac{2\left(\frac{bn\log(dx)}{\sqrt{dx}} - \frac{bn\log(d)-2bn-b\log(c)-a}{\sqrt{dx}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))/(d\*x)^(3/2),x, algorithm="giac")

```
[Out] -2*(b*n*log(d*x)/sqrt(d*x) - (b*n*log(d) - 2*b*n - b*log(c) - a)/sqrt(d*x))  
/d
```



$$3.94 \quad \int \frac{a+b \log(cx^n)}{(dx)^{5/2}} dx$$

**Optimal.** Leaf size=41

$$-\frac{2(a+b \log(cx^n))}{3d(dx)^{3/2}} - \frac{4bn}{9d(dx)^{3/2}}$$

[Out]  $(-4*b*n)/(9*d*(d*x)^{(3/2)}) - (2*(a + b*Log[c*x^n]))/(3*d*(d*x)^{(3/2)})$

**Rubi [A]** time = 0.0151008, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$ , Rules used = {2304}

$$-\frac{2(a+b \log(cx^n))}{3d(dx)^{3/2}} - \frac{4bn}{9d(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])/(d\*x)^(5/2), x]

[Out]  $(-4*b*n)/(9*d*(d*x)^{(3/2)}) - (2*(a + b*Log[c*x^n]))/(3*d*(d*x)^{(3/2)})$

**Rule 2304**

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :>  
Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

**Rubi steps**

$$\int \frac{a+b \log(cx^n)}{(dx)^{5/2}} dx = -\frac{4bn}{9d(dx)^{3/2}} - \frac{2(a+b \log(cx^n))}{3d(dx)^{3/2}}$$

**Mathematica [A]** time = 0.0086047, size = 29, normalized size = 0.71

$$-\frac{2x(3a + 3b \log(cx^n) + 2bn)}{9(dx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])/(d\*x)^(5/2), x]

[Out]  $(-2*x*(3*a + 2*b*n + 3*b*Log[c*x^n]))/(9*(d*x)^{(5/2)})$

**Maple [C]** time = 0.088, size = 128, normalized size = 3.1

$$\frac{2b \ln(x^n)}{3xd^2} \frac{1}{\sqrt{dx}} - \frac{3ib\pi \operatorname{csgn}(ix^n) (\operatorname{csgn}(icx^n))^2 - 3ib\pi \operatorname{csgn}(ix^n) \operatorname{csgn}(icx^n) \operatorname{csgn}(ic) - 3ib\pi (\operatorname{csgn}(icx^n))^3 + 3ib\pi}{9xd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*ln(c*x^n))/(d*x)^(5/2),x)`

[Out] 
$$-2/3/d^2*b/x/(d*x)^{(1/2)}*\ln(x^n)-1/9/d^2*(3*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-3*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-3*I*b*Pi*csgn(I*c*x^n)^3+3*I*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+6*b*\ln(c)+4*b*n+6*a)/x/(d*x)^{(1/2)}$$

**Maxima [A]** time = 1.04975, size = 55, normalized size = 1.34

$$-\frac{4bn}{9(dx)^{\frac{3}{2}}d} - \frac{2b\log(cx^n)}{3(dx)^{\frac{3}{2}}d} - \frac{2a}{3(dx)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))/(d*x)^(5/2),x, algorithm="maxima")`

[Out] 
$$-4/9*b*n/((d*x)^{(3/2)}*d) - 2/3*b*\log(c*x^n)/((d*x)^{(3/2)}*d) - 2/3*a/((d*x)^{(3/2)}*d)$$

**Fricas [A]** time = 0.866572, size = 92, normalized size = 2.24

$$\frac{2(3bn\log(x) + 2bn + 3b\log(c) + 3a)\sqrt{dx}}{9d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))/(d*x)^(5/2),x, algorithm="fricas")`

[Out] 
$$-2/9*(3*b*n*\log(x) + 2*b*n + 3*b*\log(c) + 3*a)*\sqrt{d*x}/(d^3*x^2)$$

**Sympy [A]** time = 39.234, size = 71, normalized size = 1.73

$$-\frac{2a}{3d^{\frac{5}{2}}x^{\frac{3}{2}}} - \frac{2bn\log(x)}{3d^{\frac{5}{2}}x^{\frac{3}{2}}} - \frac{4bn}{9d^{\frac{5}{2}}x^{\frac{3}{2}}} - \frac{2b\log(c)}{3d^{\frac{5}{2}}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*ln(c*x**n))/(d*x)**(5/2),x)`

[Out] 
$$-2*a/(3*d**(5/2)*x**(3/2)) - 2*b*n*\log(x)/(3*d**(5/2)*x**(3/2)) - 4*b*n/(9*d**(5/2)*x**(3/2)) - 2*b*\log(c)/(3*d**(5/2)*x**(3/2))$$

**Giac [A]** time = 1.40789, size = 90, normalized size = 2.2

$$2\left(\frac{3bdn\log(dx)}{\sqrt{dxx}} - \frac{3bd^2n\log(d)-2bd^2n-3bd^2\log(c)-3ad^2}{\sqrt{dxx}}\right)$$

$$\frac{\hspace{10em}}{9d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/9*(3*b*d*n*log(d*x)/(sqrt(d*x)*x) - (3*b*d^2*n*log(d) - 2*b*d^2*n - 3*b*d^2*log(c) - 3*a*d^2)/(sqrt(d*x)*d*x))/d^3
```

### 3.95 $\int (dx)^{5/2} (a + b \log(cx^n))^2 dx$

**Optimal.** Leaf size=73

$$\frac{2(dx)^{7/2} (a + b \log(cx^n))^2}{7d} - \frac{8bn(dx)^{7/2} (a + b \log(cx^n))}{49d} + \frac{16b^2n^2(dx)^{7/2}}{343d}$$

[Out]  $(16*b^2*n^2*(d*x)^{(7/2)})/(343*d) - (8*b*n*(d*x)^{(7/2)}*(a + b*Log[c*x^n]))/(49*d) + (2*(d*x)^{(7/2)}*(a + b*Log[c*x^n])^2)/(7*d)$

**Rubi [A]** time = 0.0450409, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2305, 2304}

$$\frac{2(dx)^{7/2} (a + b \log(cx^n))^2}{7d} - \frac{8bn(dx)^{7/2} (a + b \log(cx^n))}{49d} + \frac{16b^2n^2(dx)^{7/2}}{343d}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^(5/2)\*(a + b\*Log[c\*x^n])^2,x]

[Out]  $(16*b^2*n^2*(d*x)^{(7/2)})/(343*d) - (8*b*n*(d*x)^{(7/2)}*(a + b*Log[c*x^n]))/(49*d) + (2*(d*x)^{(7/2)}*(a + b*Log[c*x^n])^2)/(7*d)$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int (dx)^{5/2} (a + b \log(cx^n))^2 dx &= \frac{2(dx)^{7/2} (a + b \log(cx^n))^2}{7d} - \frac{1}{7}(4bn) \int (dx)^{5/2} (a + b \log(cx^n)) dx \\ &= \frac{16b^2n^2(dx)^{7/2}}{343d} - \frac{8bn(dx)^{7/2} (a + b \log(cx^n))}{49d} + \frac{2(dx)^{7/2} (a + b \log(cx^n))^2}{7d} \end{aligned}$$

**Mathematica [A]** time = 0.0202206, size = 61, normalized size = 0.84

$$\frac{2}{343}x(dx)^{5/2} (49a^2 + 14b(7a - 2bn) \log(cx^n) - 28abn + 49b^2 \log^2(cx^n) + 8b^2n^2)$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(5/2)\*(a + b\*Log[c\*x^n])^2,x]

[Out]  $(2*x*(d*x)^{(5/2)}*(49*a^2 - 28*a*b*n + 8*b^2*n^2 + 14*b*(7*a - 2*b*n)*\text{Log}[c*x^n] + 49*b^2*\text{Log}[c*x^n]^2))/343$

**Maple [C]** time = 0.139, size = 716, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((d*x)^{(5/2)}*(a+b*\ln(c*x^n))^2, x)$

[Out]  $2/7*d^3*b^2*x^4/(d*x)^{(1/2)}*\ln(x^n)^2+2/49*d^3*b*x^4*(7*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-7*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)^{-7*I*b*Pi*csgn(I*c*x^n)^3+7*I*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+14*b*\ln(c)-4*b*n+14*a)/(d*x)^{(1/2)}*\ln(x^n)+1/686*d^3*(-196*I*\ln(c)*Pi*b^2*csgn(I*c*x^n)^3+56*I*Pi*b^2*n*csgn(I*c*x^n)^3-196*I*Pi*a*b*csgn(I*c*x^n)^3+196*\ln(c)^2*b^2-49*Pi^2*b^2*csgn(I*c*x^n)^4*csgn(I*c)^2-112*a*b*n+32*b^2*n^2+196*a^2+98*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2+98*Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-49*Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2-196*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)+196*I*Pi*a*b*csgn(I*c*x^n)^2*csgn(I*c)-56*I*Pi*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)^2-56*I*Pi*b^2*n*csgn(I*c*x^n)^2*csgn(I*c)+196*I*\ln(c)*Pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2-49*Pi^2*b^2*csgn(I*c*x^n)^6+392*\ln(c)*a*b-112*\ln(c)*b^2*n-196*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+98*Pi^2*b^2*csgn(I*c*x^n)^5*csgn(I*c)+98*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^5+196*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)^2+196*I*\ln(c)*Pi*b^2*csgn(I*c*x^n)^2*csgn(I*c)+56*I*Pi*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-196*I*\ln(c)*Pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-49*Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4)*x^4/(d*x)^{(1/2)}$

**Maxima [A]** time = 1.15542, size = 138, normalized size = 1.89

$$\frac{2(dx)^{\frac{7}{2}}b^2\log(cx^n)^2}{7d} - \frac{8(dx)^{\frac{7}{2}}abn}{49d} + \frac{4(dx)^{\frac{7}{2}}ab\log(cx^n)}{7d} + \frac{2(dx)^{\frac{7}{2}}a^2}{7d} + \frac{8}{343} \left( \frac{2(dx)^{\frac{7}{2}}n^2}{d} - \frac{7(dx)^{\frac{7}{2}}n\log(cx^n)}{d} \right) b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((d*x)^{(5/2)}*(a+b*\log(c*x^n))^2, x, \text{algorithm}="maxima")$

[Out]  $2/7*(d*x)^{(7/2)}*b^2*\log(c*x^n)^2/d - 8/49*(d*x)^{(7/2)}*a*b*n/d + 4/7*(d*x)^{(7/2)}*a*b*\log(c*x^n)/d + 2/7*(d*x)^{(7/2)}*a^2/d + 8/343*(2*(d*x)^{(7/2)}*n^2/d - 7*(d*x)^{(7/2)}*n*\log(c*x^n)/d)*b^2$

**Fricas [B]** time = 0.796186, size = 321, normalized size = 4.4

$$\frac{2}{343} (49 b^2 d^2 n^2 x^3 \log(x)^2 + 49 b^2 d^2 x^3 \log(c)^2 - 14 (2 b^2 d^2 n - 7 a b d^2) x^3 \log(c) + (8 b^2 d^2 n^2 - 28 a b d^2 n + 49 a^2 d^2) x^3 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((d*x)^{(5/2)}*(a+b*\log(c*x^n))^2, x, \text{algorithm}="fricas")$

```
[Out] 2/343*(49*b^2*d^2*n^2*x^3*log(x)^2 + 49*b^2*d^2*x^3*log(c)^2 - 14*(2*b^2*d^2*n - 7*a*b*d^2)*x^3*log(c) + (8*b^2*d^2*n^2 - 28*a*b*d^2*n + 49*a^2*d^2)*x^3 + 14*(7*b^2*d^2*n*x^3*log(c) - (2*b^2*d^2*n^2 - 7*a*b*d^2*n)*x^3)*log(x))*sqrt(d*x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(a+b*ln(c*x**n))**2,x)
```

```
[Out] Timed out
```

**Giac [C]** time = 1.99799, size = 574, normalized size = 7.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] (1/7*I + 1/7)*sqrt(2)*b^2*d^2*n^2*x^(7/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d))*log(x)^2 - (1/7*I - 1/7)*sqrt(2)*b^2*d^2*n^2*x^(7/2)*sqrt(abs(d))*log(x)^2*sin(1/4*pi*sgn(d)) - (4/49*I + 4/49)*sqrt(2)*b^2*d^2*n^2*x^(7/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d))*log(x) + (2/7*I + 2/7)*sqrt(2)*b^2*d^2*n*x^(7/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d))*log(c)*log(x) + (4/49*I - 4/49)*sqrt(2)*b^2*d^2*n^2*x^(7/2)*sqrt(abs(d))*log(x)*sin(1/4*pi*sgn(d)) - (2/7*I - 2/7)*sqrt(2)*b^2*d^2*n*x^(7/2)*sqrt(abs(d))*log(c)*log(x)*sin(1/4*pi*sgn(d)) + (8/343*I + 8/343)*sqrt(2)*b^2*d^2*n^2*x^(7/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d)) - (4/49*I + 4/49)*sqrt(2)*b^2*d^2*n*x^(7/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d))*log(c) + (2/7*I + 2/7)*sqrt(2)*a*b*d^2*n*x^(7/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d))*log(x) - (8/343*I - 8/343)*sqrt(2)*b^2*d^2*n^2*x^(7/2)*sqrt(abs(d))*sin(1/4*pi*sgn(d)) + (4/49*I - 4/49)*sqrt(2)*b^2*d^2*n*x^(7/2)*sqrt(abs(d))*log(c)*sin(1/4*pi*sgn(d)) - (2/7*I - 2/7)*sqrt(2)*a*b*d^2*n*x^(7/2)*sqrt(abs(d))*log(x)*sin(1/4*pi*sgn(d)) - (4/49*I + 4/49)*sqrt(2)*a*b*d^2*n*x^(7/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d)) + (4/49*I - 4/49)*sqrt(2)*a*b*d^2*n*x^(7/2)*sqrt(abs(d))*sin(1/4*pi*sgn(d)) + 2/7*b^2*d^(5/2)*x^(7/2)*log(c)^2 + 4/7*a*b*d^(5/2)*x^(7/2)*log(c) + 2/7*a^2*d^(5/2)*x^(7/2)
```

### 3.96 $\int (dx)^{3/2} (a + b \log(cx^n))^2 dx$

**Optimal.** Leaf size=73

$$\frac{2(dx)^{5/2} (a + b \log(cx^n))^2}{5d} - \frac{8bn(dx)^{5/2} (a + b \log(cx^n))}{25d} + \frac{16b^2n^2(dx)^{5/2}}{125d}$$

[Out]  $(16*b^2*n^2*(d*x)^{(5/2)})/(125*d) - (8*b*n*(d*x)^{(5/2)}*(a + b*Log[c*x^n]))/(25*d) + (2*(d*x)^{(5/2)}*(a + b*Log[c*x^n])^2)/(5*d)$

**Rubi [A]** time = 0.0464597, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2305, 2304}

$$\frac{2(dx)^{5/2} (a + b \log(cx^n))^2}{5d} - \frac{8bn(dx)^{5/2} (a + b \log(cx^n))}{25d} + \frac{16b^2n^2(dx)^{5/2}}{125d}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^(3/2)\*(a + b\*Log[c\*x^n])^2,x]

[Out]  $(16*b^2*n^2*(d*x)^{(5/2)})/(125*d) - (8*b*n*(d*x)^{(5/2)}*(a + b*Log[c*x^n]))/(25*d) + (2*(d*x)^{(5/2)}*(a + b*Log[c*x^n])^2)/(5*d)$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int (dx)^{3/2} (a + b \log(cx^n))^2 dx &= \frac{2(dx)^{5/2} (a + b \log(cx^n))^2}{5d} - \frac{1}{5}(4bn) \int (dx)^{3/2} (a + b \log(cx^n)) dx \\ &= \frac{16b^2n^2(dx)^{5/2}}{125d} - \frac{8bn(dx)^{5/2} (a + b \log(cx^n))}{25d} + \frac{2(dx)^{5/2} (a + b \log(cx^n))^2}{5d} \end{aligned}$$

**Mathematica [A]** time = 0.0177044, size = 61, normalized size = 0.84

$$\frac{2}{125}x(dx)^{3/2} (25a^2 + 10b(5a - 2bn) \log(cx^n) - 20abn + 25b^2 \log^2(cx^n) + 8b^2n^2)$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(3/2)\*(a + b\*Log[c\*x^n])^2,x]

[Out]  $(2*x*(d*x)^{(3/2)}*(25*a^2 - 20*a*b*n + 8*b^2*n^2 + 10*b*(5*a - 2*b*n)*\text{Log}[c*x^n] + 25*b^2*\text{Log}[c*x^n]^2))/125$

**Maple [C]** time = 0.134, size = 716, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^(3/2)*(a+b*ln(c*x^n))^2,x)`

[Out]  $2/5*d^2*b^2*x^3/(d*x)^{(1/2)}*\ln(x^n)^2+2/25*d^2*b*x^3*(5*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-5*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-5*I*b*Pi*csgn(I*c*x^n)^3+5*I*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+10*b*\ln(c)-4*b*n+10*a)/(d*x)^{(1/2)}*\ln(x^n)+1/250*d^2*(-100*I*\ln(c)*Pi*b^2*csgn(I*c*x^n)^3+40*I*Pi*b^2*n*csgn(I*c*x^n)^3-100*I*Pi*a*b*csgn(I*c*x^n)^3+100*\ln(c)^2*b^2-25*Pi^2*b^2*csgn(I*c*x^n)^4*csgn(I*c)^2-80*a*b*n+32*b^2*n^2+100*a^2+50*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2+50*Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-25*Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2-100*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)+100*I*Pi*a*b*csgn(I*c*x^n)^2*csgn(I*c)-40*I*Pi*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)^2-40*I*Pi*b^2*n*csgn(I*c*x^n)^2*csgn(I*c)+100*I*\ln(c)*Pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2-25*Pi^2*b^2*csgn(I*c*x^n)^6+200*\ln(c)*a*b-80*\ln(c)*b^2*n+50*Pi^2*b^2*csgn(I*c*x^n)^5*csgn(I*c)+50*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^5-100*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+100*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)^2+100*I*\ln(c)*Pi*b^2*csgn(I*c*x^n)^2*csgn(I*c)+40*I*Pi*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-100*I*\ln(c)*Pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-25*Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4)*x^3/(d*x)^{(1/2)}$

**Maxima [A]** time = 1.10065, size = 138, normalized size = 1.89

$$\frac{2(dx)^{\frac{5}{2}}b^2\log(cx^n)^2}{5d} - \frac{8(dx)^{\frac{5}{2}}abn}{25d} + \frac{4(dx)^{\frac{5}{2}}ab\log(cx^n)}{5d} + \frac{2(dx)^{\frac{5}{2}}a^2}{5d} + \frac{8}{125} \left( \frac{2(dx)^{\frac{5}{2}}n^2}{d} - \frac{5(dx)^{\frac{5}{2}}n\log(cx^n)}{d} \right) b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(3/2)*(a+b*log(c*x^n))^2,x, algorithm="maxima")`

[Out]  $2/5*(d*x)^{(5/2)}*b^2*\log(c*x^n)^2/d - 8/25*(d*x)^{(5/2)}*a*b*n/d + 4/5*(d*x)^{(5/2)}*a*b*\log(c*x^n)/d + 2/5*(d*x)^{(5/2)}*a^2/d + 8/125*(2*(d*x)^{(5/2)}*n^2/d - 5*(d*x)^{(5/2)}*n*\log(c*x^n)/d)*b^2$

**Fricas [A]** time = 0.898863, size = 294, normalized size = 4.03

$$\frac{2}{125} (25b^2dn^2x^2\log(x)^2 + 25b^2dx^2\log(c)^2 - 10(2b^2dn - 5abd)x^2\log(c) + (8b^2dn^2 - 20abdn + 25a^2d)x^2 + 10(5b^2dn^2 - 10abdn + 5a^2d)\log(c) + 10(5b^2dn^2 - 10abdn + 5a^2d)\log(c)^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(3/2)*(a+b*log(c*x^n))^2,x, algorithm="fricas")`



```
[Out] 2/125*(25*b^2*d*n^2*x^2*log(x)^2 + 25*b^2*d*x^2*log(c)^2 - 10*(2*b^2*d*n - 5*a*b*d)*x^2*log(c) + (8*b^2*d*n^2 - 20*a*b*d*n + 25*a^2*d)*x^2 + 10*(5*b^2*d*n*x^2*log(c) - (2*b^2*d*n^2 - 5*a*b*d*n)*x^2)*log(x))*sqrt(d*x)
```

**Sympy [B]** time = 124.624, size = 216, normalized size = 2.96

$$\frac{2a^2d^{\frac{3}{2}}x^{\frac{5}{2}}}{5} + \frac{4abd^{\frac{3}{2}}nx^{\frac{5}{2}}\log(x)}{5} - \frac{8abd^{\frac{3}{2}}nx^{\frac{5}{2}}}{25} + \frac{4abd^{\frac{3}{2}}x^{\frac{5}{2}}\log(c)}{5} + \frac{2b^2d^{\frac{3}{2}}n^2x^{\frac{5}{2}}\log(x)^2}{5} - \frac{8b^2d^{\frac{3}{2}}n^2x^{\frac{5}{2}}\log(x)}{25} + \frac{16b^2d^{\frac{3}{2}}n^2x^{\frac{5}{2}}\log(x)^2}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(a+b*ln(c*x**n))**2,x)
```

```
[Out] 2*a**2*d**(3/2)*x**(5/2)/5 + 4*a*b*d**(3/2)*n*x**(5/2)*log(x)/5 - 8*a*b*d**(3/2)*n*x**(5/2)/25 + 4*a*b*d**(3/2)*x**(5/2)*log(c)/5 + 2*b**2*d**(3/2)*n**2*x**(5/2)*log(x)**2/5 - 8*b**2*d**(3/2)*n**2*x**(5/2)*log(x)/25 + 16*b**2*d**(3/2)*n**2*x**(5/2)/125 + 4*b**2*d**(3/2)*n*x**(5/2)*log(c)*log(x)/5 - 8*b**2*d**(3/2)*n*x**(5/2)*log(c)/25 + 2*b**2*d**(3/2)*x**(5/2)*log(c)**2/5
```

**Giac [C]** time = 1.94437, size = 521, normalized size = 7.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] -1/125*(-(25*I + 25)*sqrt(2)*b^2*n^2*x^(5/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d))*log(x)^2 + (25*I - 25)*sqrt(2)*b^2*n^2*x^(5/2)*sqrt(abs(d))*log(x)^2*sin(1/4*pi*sgn(d)) + (20*I + 20)*sqrt(2)*b^2*n^2*x^(5/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d))*log(x) - (50*I + 50)*sqrt(2)*b^2*n*x^(5/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d))*log(c)*log(x) - (20*I - 20)*sqrt(2)*b^2*n^2*x^(5/2)*sqrt(abs(d))*log(x)*sin(1/4*pi*sgn(d)) + (50*I - 50)*sqrt(2)*b^2*n*x^(5/2)*sqrt(abs(d))*log(c)*log(x)*sin(1/4*pi*sgn(d)) - (8*I + 8)*sqrt(2)*b^2*n^2*x^(5/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d)) + (20*I + 20)*sqrt(2)*b^2*n*x^(5/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d))*log(c) - (50*I + 50)*sqrt(2)*a*b*n*x^(5/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d))*log(x) + (8*I - 8)*sqrt(2)*b^2*n^2*x^(5/2)*sqrt(abs(d))*sin(1/4*pi*sgn(d)) - (20*I - 20)*sqrt(2)*b^2*n*x^(5/2)*sqrt(abs(d))*log(c)*sin(1/4*pi*sgn(d)) + (50*I - 50)*sqrt(2)*a*b*n*x^(5/2)*sqrt(abs(d))*log(x)*sin(1/4*pi*sgn(d)) + (20*I + 20)*sqrt(2)*a*b*n*x^(5/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d)) - (20*I - 20)*sqrt(2)*a*b*n*x^(5/2)*sqrt(abs(d))*sin(1/4*pi*sgn(d)) - 50*b^2*sqrt(d)*x^(5/2)*log(c)^2 - 100*a*b*sqrt(d)*x^(5/2)*log(c) - 50*a^2*sqrt(d)*x^(5/2))*d
```

### 3.97 $\int \sqrt{dx} (a + b \log(cx^n))^2 dx$

**Optimal.** Leaf size=73

$$-\frac{8bn(dx)^{3/2}(a+b\log(cx^n))}{9d} + \frac{2(dx)^{3/2}(a+b\log(cx^n))^2}{3d} + \frac{16b^2n^2(dx)^{3/2}}{27d}$$

[Out]  $(16*b^2*n^2*(d*x)^{(3/2)})/(27*d) - (8*b*n*(d*x)^{(3/2)*(a + b*Log[c*x^n])})/(9*d) + (2*(d*x)^{(3/2)*(a + b*Log[c*x^n])^2})/(3*d)$

**Rubi [A]** time = 0.0409941, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2305, 2304}

$$-\frac{8bn(dx)^{3/2}(a+b\log(cx^n))}{9d} + \frac{2(dx)^{3/2}(a+b\log(cx^n))^2}{3d} + \frac{16b^2n^2(dx)^{3/2}}{27d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d\*x]\*(a + b\*Log[c\*x^n])^2,x]

[Out]  $(16*b^2*n^2*(d*x)^{(3/2)})/(27*d) - (8*b*n*(d*x)^{(3/2)*(a + b*Log[c*x^n])})/(9*d) + (2*(d*x)^{(3/2)*(a + b*Log[c*x^n])^2})/(3*d)$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \sqrt{dx} (a + b \log(cx^n))^2 dx &= \frac{2(dx)^{3/2}(a + b \log(cx^n))^2}{3d} - \frac{1}{3}(4bn) \int \sqrt{dx} (a + b \log(cx^n)) dx \\ &= \frac{16b^2n^2(dx)^{3/2}}{27d} - \frac{8bn(dx)^{3/2}(a + b \log(cx^n))}{9d} + \frac{2(dx)^{3/2}(a + b \log(cx^n))^2}{3d} \end{aligned}$$

**Mathematica [A]** time = 0.016092, size = 61, normalized size = 0.84

$$\frac{2}{27}x\sqrt{dx}(9a^2 + 6b(3a - 2bn)\log(cx^n) - 12abn + 9b^2\log^2(cx^n) + 8b^2n^2)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d\*x]\*(a + b\*Log[c\*x^n])^2,x]

[Out]  $(2*x*\sqrt{d*x}*(9*a^2 - 12*a*b*n + 8*b^2*n^2 + 6*b*(3*a - 2*b*n)*\text{Log}[c*x^n] + 9*b^2*\text{Log}[c*x^n]^2))/27$

**Maple [C]** time = 0.126, size = 710, normalized size = 9.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((d*x)^{(1/2)}*(a+b*\ln(c*x^n))^2, x)$

[Out]  $2/3*d*b^2*x^2/(d*x)^{(1/2)}*\ln(x^n)^2+2/9*d*b*x^2*(3*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-3*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-3*I*b*Pi*csgn(I*c*x^n)^3+3*I*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+6*b*\ln(c)-4*b*n+6*a)/(d*x)^{(1/2)}*\ln(x^n)+1/54*d*(36*\ln(c)^2*b^2-9*Pi^2*b^2*csgn(I*c*x^n)^4*csgn(I*c)^2-48*a*b*n+32*b^2*n^2+36*a^2+36*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)^2+36*I*Pi*a*b*csgn(I*c*x^n)^2*csgn(I*c)+18*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2+18*Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-9*Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2-36*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)+24*I*Pi*b^2*n*csgn(I*c*x^n)^3-36*I*\ln(c)*Pi*b^2*csgn(I*c*x^n)^3-36*I*Pi*a*b*csgn(I*c*x^n)^3-24*I*Pi*b^2*n*csgn(I*c*x^n)^2*csgn(I*c)+36*I*\ln(c)*Pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2+36*I*\ln(c)*Pi*b^2*csgn(I*c*x^n)^2*csgn(I*c)-9*Pi^2*b^2*csgn(I*c*x^n)^6+72*\ln(c)*a*b-48*\ln(c)*b^2*n-36*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+18*Pi^2*b^2*csgn(I*c*x^n)^5*csgn(I*c)+18*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^5-24*I*Pi*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)^2-36*I*\ln(c)*Pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+24*I*Pi*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-9*Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4)*x^2/(d*x)^{(1/2)}$

**Maxima [A]** time = 1.10184, size = 138, normalized size = 1.89

$$\frac{2(dx)^{\frac{3}{2}}b^2\log(cx^n)^2}{3d} - \frac{8(dx)^{\frac{3}{2}}abn}{9d} + \frac{4(dx)^{\frac{3}{2}}ab\log(cx^n)}{3d} + \frac{8}{27}\left(\frac{2(dx)^{\frac{3}{2}}n^2}{d} - \frac{3(dx)^{\frac{3}{2}}n\log(cx^n)}{d}\right)b^2 + \frac{2(dx)^{\frac{3}{2}}a^2}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((d*x)^{(1/2)}*(a+b*\log(c*x^n))^2, x, \text{algorithm}="maxima")$

[Out]  $2/3*(d*x)^{(3/2)}*b^2*\log(c*x^n)^2/d - 8/9*(d*x)^{(3/2)}*a*b*n/d + 4/3*(d*x)^{(3/2)}*a*b*\log(c*x^n)/d + 8/27*(2*(d*x)^{(3/2)}*n^2/d - 3*(d*x)^{(3/2)}*n*\log(c*x^n)/d)*b^2 + 2/3*(d*x)^{(3/2)}*a^2/d$

**Fricas [A]** time = 0.895862, size = 243, normalized size = 3.33

$$\frac{2}{27}(9b^2n^2x\log(x)^2 + 9b^2x\log(c)^2 - 6(2b^2n - 3ab)x\log(c) + (8b^2n^2 - 12abn + 9a^2)x + 6(3b^2nx\log(c) - (2b^2n$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((d*x)^{(1/2)}*(a+b*\log(c*x^n))^2, x, \text{algorithm}="fricas")$

[Out]  $2/27*(9*b^2*n^2*x*\log(x)^2 + 9*b^2*x*\log(c)^2 - 6*(2*b^2*n - 3*a*b)*x*\log(c) + (8*b^2*n^2 - 12*a*b*n + 9*a^2)*x + 6*(3*b^2*n*x*\log(c) - (2*b^2*n^2 - 3*a*b*n)*x)*\log(x))*\sqrt{d*x}$

**Sympy [B]** time = 7.88766, size = 216, normalized size = 2.96

$$\frac{2a^2\sqrt{dx^3}}{3} + \frac{4ab\sqrt{dnx^3}\log(x)}{3} - \frac{8ab\sqrt{dnx^3}}{9} + \frac{4ab\sqrt{dx^3}\log(c)}{3} + \frac{2b^2\sqrt{dn^2x^3}\log(x)^2}{3} - \frac{8b^2\sqrt{dn^2x^3}\log(x)}{9} + \frac{16b^2\sqrt{dn^2x^3}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(1/2)\*(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out]  $2*a**2*\sqrt{d}*x**(3/2)/3 + 4*a*b*\sqrt{d}*n*x**(3/2)*\log(x)/3 - 8*a*b*\sqrt{d}*n*x**(3/2)/9 + 4*a*b*\sqrt{d}*x**(3/2)*\log(c)/3 + 2*b**2*\sqrt{d}*n**2*x**(3/2)*\log(x)**2/3 - 8*b**2*\sqrt{d}*n**2*x**(3/2)*\log(x)/9 + 16*b**2*\sqrt{d}*n**2*x**(3/2)/27 + 4*b**2*\sqrt{d}*n*x**(3/2)*\log(c)*\log(x)/3 - 8*b**2*\sqrt{d}*n*x**(3/2)*\log(c)/9 + 2*b**2*\sqrt{d}*x**(3/2)*\log(c)**2/3$

**Giac [C]** time = 1.97107, size = 517, normalized size = 7.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out]  $(1/3*I + 1/3)*\sqrt{2}*b^2*n^2*x^(3/2)*\sqrt{\text{abs}(d)}*\cos(1/4*\pi*\text{sgn}(d))*\log(x)^2 - (1/3*I - 1/3)*\sqrt{2}*b^2*n^2*x^(3/2)*\sqrt{\text{abs}(d)}*\log(x)^2*\sin(1/4*\pi*\text{sgn}(d)) - (4/9*I + 4/9)*\sqrt{2}*b^2*n^2*x^(3/2)*\sqrt{\text{abs}(d)}*\cos(1/4*\pi*\text{sgn}(d))*\log(x) + (2/3*I + 2/3)*\sqrt{2}*b^2*n*x^(3/2)*\sqrt{\text{abs}(d)}*\cos(1/4*\pi*\text{sgn}(d))*\log(c)*\log(x) + (4/9*I - 4/9)*\sqrt{2}*b^2*n^2*x^(3/2)*\sqrt{\text{abs}(d)}*\log(x)*\sin(1/4*\pi*\text{sgn}(d)) - (2/3*I - 2/3)*\sqrt{2}*b^2*n*x^(3/2)*\sqrt{\text{abs}(d)}*\log(c)*\log(x)*\sin(1/4*\pi*\text{sgn}(d)) + (8/27*I + 8/27)*\sqrt{2}*b^2*n^2*x^(3/2)*\sqrt{\text{abs}(d)}*\cos(1/4*\pi*\text{sgn}(d)) - (4/9*I + 4/9)*\sqrt{2}*b^2*n*x^(3/2)*\sqrt{\text{abs}(d)}*\cos(1/4*\pi*\text{sgn}(d))*\log(c) + (2/3*I + 2/3)*\sqrt{2}*a*b*n*x^(3/2)*\sqrt{\text{abs}(d)}*\cos(1/4*\pi*\text{sgn}(d))*\log(x) - (8/27*I - 8/27)*\sqrt{2}*b^2*n^2*x^(3/2)*\sqrt{\text{abs}(d)}*\sin(1/4*\pi*\text{sgn}(d)) + (4/9*I - 4/9)*\sqrt{2}*b^2*n*x^(3/2)*\sqrt{\text{abs}(d)}*\log(c)*\sin(1/4*\pi*\text{sgn}(d)) - (2/3*I - 2/3)*\sqrt{2}*a*b*n*x^(3/2)*\sqrt{\text{abs}(d)}*\log(x)*\sin(1/4*\pi*\text{sgn}(d)) - (4/9*I + 4/9)*\sqrt{2}*a*b*n*x^(3/2)*\sqrt{\text{abs}(d)}*\cos(1/4*\pi*\text{sgn}(d)) + (4/9*I - 4/9)*\sqrt{2}*a*b*n*x^(3/2)*\sqrt{\text{abs}(d)}*\sin(1/4*\pi*\text{sgn}(d)) + 2/3*b^2*\sqrt{d}*x^(3/2)*\log(c)^2 + 4/3*a*b*\sqrt{d}*x^(3/2)*\log(c) + 2/3*a^2*\sqrt{d}*x^(3/2)$

$$3.98 \quad \int \frac{(a+b \log(cx^n))^2}{\sqrt{dx}} dx$$

**Optimal.** Leaf size=67

$$-\frac{8bn\sqrt{dx}(a+b \log(cx^n))}{d} + \frac{2\sqrt{dx}(a+b \log(cx^n))^2}{d} + \frac{16b^2n^2\sqrt{dx}}{d}$$

[Out]  $(16*b^2*n^2*sqrt[d*x])/d - (8*b*n*sqrt[d*x]*(a + b*Log[c*x^n]))/d + (2*sqrt[d*x]*(a + b*Log[c*x^n])^2)/d$

**Rubi [A]** time = 0.0409136, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2305, 2304}

$$-\frac{8bn\sqrt{dx}(a+b \log(cx^n))}{d} + \frac{2\sqrt{dx}(a+b \log(cx^n))^2}{d} + \frac{16b^2n^2\sqrt{dx}}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^2/Sqrt[d\*x], x]

[Out]  $(16*b^2*n^2*sqrt[d*x])/d - (8*b*n*sqrt[d*x]*(a + b*Log[c*x^n]))/d + (2*sqrt[d*x]*(a + b*Log[c*x^n])^2)/d$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{(a+b \log(cx^n))^2}{\sqrt{dx}} dx &= \frac{2\sqrt{dx}(a+b \log(cx^n))^2}{d} - (4bn) \int \frac{a+b \log(cx^n)}{\sqrt{dx}} dx \\ &= \frac{16b^2n^2\sqrt{dx}}{d} - \frac{8bn\sqrt{dx}(a+b \log(cx^n))}{d} + \frac{2\sqrt{dx}(a+b \log(cx^n))^2}{d} \end{aligned}$$

**Mathematica [A]** time = 0.0132299, size = 54, normalized size = 0.81

$$\frac{2x(a^2 + 2b(a - 2bn) \log(cx^n) - 4abn + b^2 \log^2(cx^n) + 8b^2n^2)}{\sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^2/Sqrt[d\*x], x]

[Out]  $(2*x*(a^2 - 4*a*b*n + 8*b^2*n^2 + 2*b*(a - 2*b*n)*\text{Log}[c*x^n] + b^2*\text{Log}[c*x^n]^2))/\text{Sqrt}[d*x]$

**Maple [A]** time = 0.054, size = 107, normalized size = 1.6

$$2 \frac{b^2 \sqrt{dx} (\ln(ce^{n \ln(x)}))^2}{d} - 8 \frac{b^2 n \sqrt{dx} \ln(ce^{n \ln(x)})}{d} + 16 \frac{b^2 n^2 \sqrt{dx}}{d} + 4 \frac{\sqrt{dx} ab \ln(cx^n)}{d} - 8 \frac{\sqrt{dx} abn}{d} + 2 \frac{\sqrt{dx} a^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*ln(c*x^n))^2/(d*x)^(1/2),x)`

[Out]  $2/d*b^2*(d*x)^(1/2)*\ln(c*\exp(n*\ln(x)))^2-8/d*b^2*n*(d*x)^(1/2)*\ln(c*\exp(n*\ln(x)))+16*b^2*n^2*(d*x)^(1/2)/d+4/d*(d*x)^(1/2)*a*b*\ln(c*x^n)-8/d*(d*x)^(1/2)*a*b*n+2/d*(d*x)^(1/2)*a^2$

**Maxima [A]** time = 1.18133, size = 138, normalized size = 2.06

$$\frac{2 \sqrt{dx} b^2 \log(cx^n)^2}{d} + 8 \left( \frac{2 \sqrt{dx} n^2}{d} - \frac{\sqrt{dx} n \log(cx^n)}{d} \right) b^2 - \frac{8 \sqrt{dx} abn}{d} + \frac{4 \sqrt{dx} ab \log(cx^n)}{d} + \frac{2 \sqrt{dx} a^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))^2/(d*x)^(1/2),x, algorithm="maxima")`

[Out]  $2*\text{sqrt}(d*x)*b^2*\log(c*x^n)^2/d + 8*(2*\text{sqrt}(d*x)*n^2/d - \text{sqrt}(d*x)*n*\log(c*x^n)/d)*b^2 - 8*\text{sqrt}(d*x)*a*b*n/d + 4*\text{sqrt}(d*x)*a*b*\log(c*x^n)/d + 2*\text{sqrt}(d*x)*a^2/d$

**Fricas [A]** time = 0.956475, size = 203, normalized size = 3.03

$$\frac{2(b^2 n^2 \log(x)^2 + 8 b^2 n^2 + b^2 \log(c)^2 - 4 abn + a^2 - 2(2 b^2 n - ab) \log(c) - 2(2 b^2 n^2 - b^2 n \log(c) - abn) \log(x)) \sqrt{dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))^2/(d*x)^(1/2),x, algorithm="fricas")`

[Out]  $2*(b^2*n^2*\log(x)^2 + 8*b^2*n^2 + b^2*\log(c)^2 - 4*a*b*n + a^2 - 2*(2*b^2*n - a*b)*\log(c) - 2*(2*b^2*n^2 - b^2*n*\log(c) - a*b*n)*\log(x))*\text{sqrt}(d*x)/d$

**Sympy [B]** time = 4.27413, size = 199, normalized size = 2.97

$$\frac{2a^2\sqrt{x}}{\sqrt{d}} + \frac{4abn\sqrt{x}\log(x)}{\sqrt{d}} - \frac{8abn\sqrt{x}}{\sqrt{d}} + \frac{4ab\sqrt{x}\log(c)}{\sqrt{d}} + \frac{2b^2n^2\sqrt{x}\log(x)^2}{\sqrt{d}} - \frac{8b^2n^2\sqrt{x}\log(x)}{\sqrt{d}} + \frac{16b^2n^2\sqrt{x}}{\sqrt{d}} + \frac{4b^2n\sqrt{x}\log(x)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*ln(c*x**n))**2/(d*x)**(1/2),x)`

```
[Out] 2*a**2*sqrt(x)/sqrt(d) + 4*a*b*n*sqrt(x)*log(x)/sqrt(d) - 8*a*b*n*sqrt(x)/s
qrt(d) + 4*a*b*sqrt(x)*log(c)/sqrt(d) + 2*b**2*n**2*sqrt(x)*log(x)**2/sqrt(
d) - 8*b**2*n**2*sqrt(x)*log(x)/sqrt(d) + 16*b**2*n**2*sqrt(x)/sqrt(d) + 4*
b**2*n*sqrt(x)*log(c)*log(x)/sqrt(d) - 8*b**2*n*sqrt(x)*log(c)/sqrt(d) + 2*
b**2*sqrt(x)*log(c)**2/sqrt(d)
```

**Giac [A]** time = 1.30331, size = 159, normalized size = 2.37

$$\frac{2\left(\sqrt{dx}\log(x)^2 - 4\sqrt{dx}\log(x) + 8\sqrt{dx}\right)b^2n^2 + 2\left(\sqrt{dx}\log(x) - 2\sqrt{dx}\right)b^2n\log(c) + \sqrt{dx}b^2\log(c)^2 + 2\left(\sqrt{dx}\log(x) - 2\sqrt{dx}\right)a*b*n + 2\sqrt{dx}a*b\log(c) + \sqrt{dx}a^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*((sqrt(d*x)*log(x)^2 - 4*sqrt(d*x)*log(x) + 8*sqrt(d*x))*b^2*n^2 + 2*(sq
rt(d*x)*log(x) - 2*sqrt(d*x))*b^2*n*log(c) + sqrt(d*x)*b^2*log(c)^2 + 2*(sq
rt(d*x)*log(x) - 2*sqrt(d*x))*a*b*n + 2*sqrt(d*x)*a*b*log(c) + sqrt(d*x)*a^2
)/d
```

$$3.99 \quad \int \frac{(a+b \log(cx^n))^2}{(dx)^{3/2}} dx$$

**Optimal.** Leaf size=67

$$-\frac{8bn(a+b \log(cx^n))}{d\sqrt{dx}} - \frac{2(a+b \log(cx^n))^2}{d\sqrt{dx}} - \frac{16b^2n^2}{d\sqrt{dx}}$$

[Out]  $(-16*b^2*n^2)/(d*\text{Sqrt}[d*x]) - (8*b*n*(a + b*\text{Log}[c*x^n]))/(d*\text{Sqrt}[d*x]) - (2*(a + b*\text{Log}[c*x^n])^2)/(d*\text{Sqrt}[d*x])$

**Rubi [A]** time = 0.0468623, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2305, 2304}

$$-\frac{8bn(a+b \log(cx^n))}{d\sqrt{dx}} - \frac{2(a+b \log(cx^n))^2}{d\sqrt{dx}} - \frac{16b^2n^2}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{Log}[c*x^n])^2/(d*x)^{(3/2)}, x]$

[Out]  $(-16*b^2*n^2)/(d*\text{Sqrt}[d*x]) - (8*b*n*(a + b*\text{Log}[c*x^n]))/(d*\text{Sqrt}[d*x]) - (2*(a + b*\text{Log}[c*x^n])^2)/(d*\text{Sqrt}[d*x])$

#### Rule 2305

$\text{Int}[(a_. + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.))^{(p_.)}*((d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*(a + b*\text{Log}[c*x^n])^p/(d*(m+1)), x] - \text{Dist}[(b*n*p)/(m+1), \text{Int}[(d*x)^m*(a + b*\text{Log}[c*x^n])^{(p-1)}, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{NeQ}[m, -1] \&\& \text{GtQ}[p, 0]$

#### Rule 2304

$\text{Int}[(a_. + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.))^{(p_.)}*((d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*(a + b*\text{Log}[c*x^n])^p/(d*(m+1)), x] - \text{Simp}[(b*n*(d*x)^{(m+1)})/(d*(m+1)^2), x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{NeQ}[m, -1]$

#### Rubi steps

$$\begin{aligned} \int \frac{(a+b \log(cx^n))^2}{(dx)^{3/2}} dx &= -\frac{2(a+b \log(cx^n))^2}{d\sqrt{dx}} + (4bn) \int \frac{a+b \log(cx^n)}{(dx)^{3/2}} dx \\ &= -\frac{16b^2n^2}{d\sqrt{dx}} - \frac{8bn(a+b \log(cx^n))}{d\sqrt{dx}} - \frac{2(a+b \log(cx^n))^2}{d\sqrt{dx}} \end{aligned}$$

**Mathematica [A]** time = 0.0127942, size = 54, normalized size = 0.81

$$-\frac{2x(a^2 + 2b(a + 2bn) \log(cx^n) + 4abn + b^2 \log^2(cx^n) + 8b^2n^2)}{(dx)^{3/2}}$$

Antiderivative was successfully verified.



[In] Integrate[(a + b\*Log[c\*x^n])^2/(d\*x)^(3/2), x]

[Out] (-2\*x\*(a^2 + 4\*a\*b\*n + 8\*b^2\*n^2 + 2\*b\*(a + 2\*b\*n)\*Log[c\*x^n] + b^2\*Log[c\*x^n]^2))/(d\*x)^(3/2)

**Maple [C]** time = 0.131, size = 707, normalized size = 10.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^n))^2/(d\*x)^(3/2), x)

[Out] 
$$\begin{aligned} & -2/d*b^2/(d*x)^{(1/2)}*\ln(x^n)^2-2/d*b*(I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I* \\ & b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*b*Pi*csgn(I*c*x^n)^3+I*b*Pi*csgn \\ & (I*c*x^n)^2*csgn(I*c)+2*b*\ln(c)+4*b*n+2*a)/(d*x)^{(1/2)}*\ln(x^n)-1/2/d*(4*I* \\ & \ln(c)*Pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2+4*\ln(c)^2*b^2-Pi^2*b^2*csgn(I*c*x^n) \\ & )^4*csgn(I*c)^2+16*a*b*n+32*b^2*n^2+4*a^2+2*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x \\ & ^n)^3*csgn(I*c)^2+2*Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-Pi^2*b \\ & ^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2-4*Pi^2*b^2*csgn(I*x^n)*csgn(I* \\ & c*x^n)^4*csgn(I*c)+4*I*\ln(c)*Pi*b^2*csgn(I*c*x^n)^2*csgn(I*c)+4*I*Pi*a*b*c \\ & sgn(I*x^n)*csgn(I*c*x^n)^2+4*I*Pi*a*b*csgn(I*c*x^n)^2*csgn(I*c)-4*I*Pi*a*b*c \\ & sgn(I*c*x^n)^3-4*I*\ln(c)*Pi*b^2*csgn(I*c*x^n)^3+8*I*Pi*b^2*n*csgn(I*x^n)*c \\ & sgn(I*c*x^n)^2+8*I*Pi*b^2*n*csgn(I*c*x^n)^2*csgn(I*c)-4*I*\ln(c)*Pi*b^2*csgn( \\ & I*x^n)*csgn(I*c*x^n)*csgn(I*c)-4*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I* \\ & c)-Pi^2*b^2*csgn(I*c*x^n)^6+8*\ln(c)*a*b+16*\ln(c)*b^2*n-8*I*Pi*b^2*n*csgn(I* \\ & c*x^n)^3+2*Pi^2*b^2*csgn(I*c*x^n)^5*csgn(I*c)+2*Pi^2*b^2*csgn(I*x^n)*csgn(I \\ & *c*x^n)^5-8*I*Pi*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-Pi^2*b^2*csgn(I* \\ & x^n)^2*csgn(I*c*x^n)^4)/(d*x)^{(1/2)} \end{aligned}$$

**Maxima [A]** time = 1.08424, size = 136, normalized size = 2.03

$$-8b^2\left(\frac{2n^2}{\sqrt{dxd}} + \frac{n \log(cx^n)}{\sqrt{dxd}}\right) - \frac{2b^2 \log(cx^n)^2}{\sqrt{dxd}} - \frac{8abn}{\sqrt{dxd}} - \frac{4ab \log(cx^n)}{\sqrt{dxd}} - \frac{2a^2}{\sqrt{dxd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^2/(d\*x)^(3/2), x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -8*b^2*(2*n^2/(\sqrt{d*x}*d) + n*\log(c*x^n)/(\sqrt{d*x}*d)) - 2*b^2*\log(c*x^n) \\ & )^2/(\sqrt{d*x}*d) - 8*a*b*n/(\sqrt{d*x}*d) - 4*a*b*\log(c*x^n)/(\sqrt{d*x}*d) \\ & - 2*a^2/(\sqrt{d*x}*d) \end{aligned}$$

**Fricas [A]** time = 0.938875, size = 212, normalized size = 3.16

$$\frac{2(b^2n^2 \log(x)^2 + 8b^2n^2 + b^2 \log(c)^2 + 4abn + a^2 + 2(2b^2n + ab) \log(c) + 2(2b^2n^2 + b^2n \log(c) + abn) \log(x))}{d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^2/(d\*x)^(3/2), x, algorithm="fricas")

[Out]  $-2*(b^2*n^2*\log(x)^2 + 8*b^2*n^2 + b^2*\log(c)^2 + 4*a*b*n + a^2 + 2*(2*b^2*n + a*b)*\log(c) + 2*(2*b^2*n^2 + b^2*n*\log(c) + a*b*n)*\log(x))*\sqrt{d*x}/(d^2*x)$

**Sympy [B]** time = 5.55021, size = 201, normalized size = 3.

$$\frac{2a^2}{d^{\frac{3}{2}}\sqrt{x}} - \frac{4abn \log(x)}{d^{\frac{3}{2}}\sqrt{x}} - \frac{8abn}{d^{\frac{3}{2}}\sqrt{x}} - \frac{4ab \log(c)}{d^{\frac{3}{2}}\sqrt{x}} - \frac{2b^2n^2 \log(x)^2}{d^{\frac{3}{2}}\sqrt{x}} - \frac{8b^2n^2 \log(x)}{d^{\frac{3}{2}}\sqrt{x}} - \frac{16b^2n^2}{d^{\frac{3}{2}}\sqrt{x}} - \frac{4b^2n \log(c) \log(x)}{d^{\frac{3}{2}}\sqrt{x}} - \frac{8b^2n \log(c)}{d^{\frac{3}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*ln(c*x**n))**2/(d*x)**(3/2),x)`

[Out]  $-2*a**2/(d**(3/2)*\sqrt{x}) - 4*a*b*n*\log(x)/(d**(3/2)*\sqrt{x}) - 8*a*b*n/(d**(3/2)*\sqrt{x}) - 4*a*b*\log(c)/(d**(3/2)*\sqrt{x}) - 2*b**2*n**2*\log(x)**2/(d**(3/2)*\sqrt{x}) - 8*b**2*n**2*\log(x)/(d**(3/2)*\sqrt{x}) - 16*b**2*n**2/(d**(3/2)*\sqrt{x}) - 4*b**2*n*\log(c)*\log(x)/(d**(3/2)*\sqrt{x}) - 8*b**2*n*\log(c)/(d**(3/2)*\sqrt{x}) - 2*b**2*\log(c)**2/(d**(3/2)*\sqrt{x})$

**Giac [B]** time = 1.26972, size = 201, normalized size = 3.

$$2 \left( \frac{b^2n^2 \log(dx)^2}{\sqrt{dx}} - \frac{2(b^2n^2 \log(d) - 2b^2n^2 - b^2n \log(c) - abn) \log(dx)}{\sqrt{dx}} + \frac{b^2n^2 \log(d)^2 - 4b^2n^2 \log(d) - 2b^2n \log(c) \log(d) + 8b^2n^2 + 4b^2n \log(c) + b^2 \log(c)^2 - 2abn}{\sqrt{dx}} \right) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x^n))^2/(d*x)^(3/2),x, algorithm="giac")`

[Out]  $-2*(b^2*n^2*\log(d*x)^2/\sqrt{d*x} - 2*(b^2*n^2*\log(d) - 2*b^2*n^2 - b^2*n*\log(c) - a*b*n)*\log(d*x)/\sqrt{d*x} + (b^2*n^2*\log(d)^2 - 4*b^2*n^2*\log(d) - 2*b^2*n*\log(c)*\log(d) + 8*b^2*n^2 + 4*b^2*n*\log(c) + b^2*\log(c)^2 - 2*a*b*n*\log(d) + 4*a*b*n + 2*a*b*\log(c) + a^2)/\sqrt{d*x})/d$

$$3.100 \quad \int \frac{(a+b \log(cx^n))^2}{(dx)^{5/2}} dx$$

**Optimal.** Leaf size=73

$$-\frac{8bn(a+b \log(cx^n))}{9d(dx)^{3/2}} - \frac{2(a+b \log(cx^n))^2}{3d(dx)^{3/2}} - \frac{16b^2n^2}{27d(dx)^{3/2}}$$

[Out]  $(-16*b^2*n^2)/(27*d*(d*x)^(3/2)) - (8*b*n*(a + b*Log[c*x^n]))/(9*d*(d*x)^(3/2)) - (2*(a + b*Log[c*x^n])^2)/(3*d*(d*x)^(3/2))$

**Rubi [A]** time = 0.0464623, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2305, 2304}

$$-\frac{8bn(a+b \log(cx^n))}{9d(dx)^{3/2}} - \frac{2(a+b \log(cx^n))^2}{3d(dx)^{3/2}} - \frac{16b^2n^2}{27d(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^2/(d\*x)^(5/2), x]

[Out]  $(-16*b^2*n^2)/(27*d*(d*x)^(3/2)) - (8*b*n*(a + b*Log[c*x^n]))/(9*d*(d*x)^(3/2)) - (2*(a + b*Log[c*x^n])^2)/(3*d*(d*x)^(3/2))$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{(a+b \log(cx^n))^2}{(dx)^{5/2}} dx &= -\frac{2(a+b \log(cx^n))^2}{3d(dx)^{3/2}} + \frac{1}{3}(4bn) \int \frac{a+b \log(cx^n)}{(dx)^{5/2}} dx \\ &= -\frac{16b^2n^2}{27d(dx)^{3/2}} - \frac{8bn(a+b \log(cx^n))}{9d(dx)^{3/2}} - \frac{2(a+b \log(cx^n))^2}{3d(dx)^{3/2}} \end{aligned}$$

**Mathematica [A]** time = 0.0146199, size = 61, normalized size = 0.84

$$-\frac{2x(9a^2 + 6b(3a + 2bn) \log(cx^n) + 12abn + 9b^2 \log^2(cx^n) + 8b^2n^2)}{27(dx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^2/(d\*x)^(5/2), x]

[Out]  $(-2*x*(9*a^2 + 12*a*b*n + 8*b^2*n^2 + 6*b*(3*a + 2*b*n)*\text{Log}[c*x^n] + 9*b^2*\text{Log}[c*x^n]^2))/(27*(d*x)^{(5/2)})$

**Maple [C]** time = 0.137, size = 716, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a+b*\ln(c*x^n))^2/(d*x)^{(5/2)}, x)$

[Out]  $-2/3/d^2*b^2/x/(d*x)^{(1/2)}*\ln(x^n)^2-2/9/d^2*b*(3*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-3*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-3*I*b*Pi*csgn(I*c*x^n)^3+3*I*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)+6*b*\ln(c)+4*b*n+6*a)/x/(d*x)^{(1/2)}*\ln(x^n)-1/54/d^2*(36*\ln(c)^2*b^2-9*Pi^2*b^2*csgn(I*c*x^n)^4*csgn(I*c)^2+48*a*b*n+32*b^2*n^2+36*a^2+36*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)^2+36*I*Pi*a*b*csgn(I*c*x^n)^2*csgn(I*c)+18*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2+18*Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-9*Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2-36*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)-24*I*Pi*b^2*n*csgn(I*c*x^n)^3-36*I*\ln(c)*Pi*b^2*csgn(I*c*x^n)^3-36*I*Pi*a*b*csgn(I*c*x^n)^3+36*I*\ln(c)*Pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2+36*I*\ln(c)*Pi*b^2*csgn(I*c*x^n)^2*csgn(I*c)-9*Pi^2*b^2*csgn(I*c*x^n)^6+72*\ln(c)*a*b+48*\ln(c)*b^2*n-36*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+18*Pi^2*b^2*csgn(I*c*x^n)^5*csgn(I*c)+18*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^5-24*I*Pi*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-36*I*\ln(c)*Pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+24*I*Pi*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)^2+24*I*Pi*b^2*n*csgn(I*c*x^n)^2*csgn(I*c)-9*Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4)/x/(d*x)^{(1/2)}$

**Maxima [A]** time = 1.13115, size = 138, normalized size = 1.89

$$-\frac{8}{27}b^2\left(\frac{2n^2}{(dx)^{\frac{3}{2}}d} + \frac{3n\log(cx^n)}{(dx)^{\frac{3}{2}}d}\right) - \frac{2b^2\log(cx^n)^2}{3(dx)^{\frac{3}{2}}d} - \frac{8abn}{9(dx)^{\frac{3}{2}}d} - \frac{4ab\log(cx^n)}{3(dx)^{\frac{3}{2}}d} - \frac{2a^2}{3(dx)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a+b*\log(c*x^n))^2/(d*x)^{(5/2)}, x, \text{algorithm}="maxima")$

[Out]  $-8/27*b^2*(2*n^2/((d*x)^{(3/2)}*d) + 3*n*\log(c*x^n)/((d*x)^{(3/2)}*d)) - 2/3*b^2*\log(c*x^n)^2/((d*x)^{(3/2)}*d) - 8/9*a*b*n/((d*x)^{(3/2)}*d) - 4/3*a*b*\log(c*x^n)/((d*x)^{(3/2)}*d) - 2/3*a^2/((d*x)^{(3/2)}*d)$

**Fricas [A]** time = 0.848769, size = 236, normalized size = 3.23

$$\frac{2(9b^2n^2\log(x)^2 + 8b^2n^2 + 9b^2\log(c)^2 + 12abn + 9a^2 + 6(2b^2n + 3ab)\log(c) + 6(2b^2n^2 + 3b^2n\log(c) + 3abn)\log(c))}{27d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a+b*\log(c*x^n))^2/(d*x)^{(5/2)}, x, \text{algorithm}="fricas")$

```
[Out] -2/27*(9*b^2*n^2*log(x)^2 + 8*b^2*n^2 + 9*b^2*log(c)^2 + 12*a*b*n + 9*a^2 +
6*(2*b^2*n + 3*a*b)*log(c) + 6*(2*b^2*n^2 + 3*b^2*n*log(c) + 3*a*b*n)*log(
x))*sqrt(d*x)/(d^3*x^2)
```

**Sympy [B]** time = 42.5333, size = 218, normalized size = 2.99

$$\frac{2a^2}{3d^{\frac{5}{2}}x^{\frac{3}{2}}} - \frac{4abn \log(x)}{3d^{\frac{5}{2}}x^{\frac{3}{2}}} - \frac{8abn}{9d^{\frac{5}{2}}x^{\frac{3}{2}}} - \frac{4ab \log(c)}{3d^{\frac{5}{2}}x^{\frac{3}{2}}} - \frac{2b^2n^2 \log(x)^2}{3d^{\frac{5}{2}}x^{\frac{3}{2}}} - \frac{8b^2n^2 \log(x)}{9d^{\frac{5}{2}}x^{\frac{3}{2}}} - \frac{16b^2n^2}{27d^{\frac{5}{2}}x^{\frac{3}{2}}} - \frac{4b^2n \log(c) \log(x)}{3d^{\frac{5}{2}}x^{\frac{3}{2}}} - \frac{8b^2n}{27d^{\frac{5}{2}}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**2/(d*x)**(5/2),x)
```

```
[Out] -2*a**2/(3*d**(5/2)*x**(3/2)) - 4*a*b*n*log(x)/(3*d**(5/2)*x**(3/2)) - 8*a*
b*n/(9*d**(5/2)*x**(3/2)) - 4*a*b*log(c)/(3*d**(5/2)*x**(3/2)) - 2*b**2*n**
2*log(x)**2/(3*d**(5/2)*x**(3/2)) - 8*b**2*n**2*log(x)/(9*d**(5/2)*x**(3/2))
) - 16*b**2*n**2/(27*d**(5/2)*x**(3/2)) - 4*b**2*n*log(c)*log(x)/(3*d**(5/2)
)*x**(3/2)) - 8*b**2*n*log(c)/(9*d**(5/2)*x**(3/2)) - 2*b**2*log(c)**2/(3*d
**(5/2)*x**(3/2))
```

**Giac [B]** time = 1.30257, size = 288, normalized size = 3.95

$$2 \left( \frac{9b^2dn^2 \log(dx)^2}{\sqrt{dxx}} - \frac{6(3b^2d^2n^2 \log(d) - 2b^2d^2n^2 - 3b^2d^2n \log(c) - 3abd^2n) \log(dx)}{\sqrt{dxdx}} + \frac{9b^2d^2n^2 \log(d)^2 - 12b^2d^2n^2 \log(d) - 18b^2d^2n \log(c) \log(d) + 8b^2d^2n}{27d^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^2/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/27*(9*b^2*d*n^2*log(d*x)^2/(sqrt(d*x)*x) - 6*(3*b^2*d^2*n^2*log(d) - 2*b
^2*d^2*n^2 - 3*b^2*d^2*n*log(c) - 3*a*b*d^2*n)*log(d*x)/(sqrt(d*x)*d*x) + (
9*b^2*d^2*n^2*log(d)^2 - 12*b^2*d^2*n^2*log(d) - 18*b^2*d^2*n*log(c)*log(d)
+ 8*b^2*d^2*n^2 + 12*b^2*d^2*n*log(c) + 9*b^2*d^2*log(c)^2 - 18*a*b*d^2*n*
log(d) + 12*a*b*d^2*n + 18*a*b*d^2*log(c) + 9*a^2*d^2)/(sqrt(d*x)*d*x))/d^3
```

$$3.101 \quad \int \frac{(dx)^{5/2}}{a+b \log(cx^n)} dx$$

**Optimal.** Leaf size=64

$$\frac{(dx)^{7/2} e^{-\frac{7a}{2bn}} (cx^n)^{-\frac{7}{2}/n} \operatorname{Ei}\left(\frac{7(a+b \log(cx^n))}{2bn}\right)}{bdn}$$

[Out]  $((d*x)^{(7/2)}*ExpIntegralEi[(7*(a + b*Log[c*x^n]))/(2*b*n)])/(b*d*E^{((7*a)/(2*b*n))}*n*(c*x^n)^{(7/(2*n))})$

**Rubi [A]** time = 0.0637514, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2310, 2178}

$$\frac{(dx)^{7/2} e^{-\frac{7a}{2bn}} (cx^n)^{-\frac{7}{2}/n} \operatorname{Ei}\left(\frac{7(a+b \log(cx^n))}{2bn}\right)}{bdn}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(d*x)^{(5/2)}/(a + b*\operatorname{Log}[c*x^n]), x]$

[Out]  $((d*x)^{(7/2)}*ExpIntegralEi[(7*(a + b*Log[c*x^n]))/(2*b*n)])/(b*d*E^{((7*a)/(2*b*n))}*n*(c*x^n)^{(7/(2*n))})$

#### Rule 2310

$\operatorname{Int}[(a_. + \operatorname{Log}[c_.*(x_.)^{n_.}]*b_.)^{(p_.)}*((d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(d*x)^{(m+1)}/(d*n*(c*x^n)^{((m+1)/n)}), \operatorname{Subst}[\operatorname{Int}[E^{((m+1)*x)/n}*(a + b*x)^p, x], x, \operatorname{Log}[c*x^n]], x] /; \operatorname{FreeQ}\{a, b, c, d, m, n, p\}, x]$

#### Rule 2178

$\operatorname{Int}[(F_.)^{(g_.)*((e_.) + (f_.)*(x_.))}/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \operatorname{Simp}[(F^{(g*(e - (c*f)/d)})*ExpIntegralEi[(f*g*(c + d*x)*\operatorname{Log}[F])/d])/d, x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\amp; \operatorname{!}\$UseGamma == True$

#### Rubi steps

$$\begin{aligned} \int \frac{(dx)^{5/2}}{a+b \log(cx^n)} dx &= \frac{\left((dx)^{7/2} (cx^n)^{-\frac{7}{2}/n}\right) \operatorname{Subst}\left(\int \frac{e^{\frac{7x}{a+bx}}}{a+bx} dx, x, \log(cx^n)\right)}{dn} \\ &= \frac{e^{-\frac{7a}{2bn}} (dx)^{7/2} (cx^n)^{-\frac{7}{2}/n} \operatorname{Ei}\left(\frac{7(a+b \log(cx^n))}{2bn}\right)}{bdn} \end{aligned}$$

**Mathematica [A]** time = 0.0811146, size = 62, normalized size = 0.97

$$\frac{x(dx)^{5/2} e^{-\frac{7a}{2bn}} (cx^n)^{-\frac{7}{2}/n} \operatorname{Ei}\left(\frac{7(a+b \log(cx^n))}{2bn}\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(5/2)/(a + b\*Log[c\*x^n]), x]

[Out] (x\*(d\*x)^(5/2)\*ExpIntegralEi[(7\*(a + b\*Log[c\*x^n]))/(2\*b\*n)]/(b\*E^((7\*a)/(2\*b\*n))\*n\*(c\*x^n)^(7/(2\*n))))

**Maple [F]** time = 0.118, size = 0, normalized size = 0.

$$\int \frac{1}{a + b \ln(cx^n)} (dx)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(5/2)/(a+b\*ln(c\*x^n)), x)

[Out] int((d\*x)^(5/2)/(a+b\*ln(c\*x^n)), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$2bd^{\frac{5}{2}}n \int \frac{x^{\frac{5}{2}}}{7(b^2 \log(c)^2 + b^2 \log(x^n)^2 + 2ab \log(c) + a^2 + 2(b^2 \log(c) + ab) \log(x^n))} dx + \frac{2d^{\frac{5}{2}}x^{\frac{7}{2}}}{7(b \log(c) + b \log(x^n) + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)/(a+b\*log(c\*x^n)), x, algorithm="maxima")

[Out] 2\*b\*d^(5/2)\*n\*integrate(1/7\*x^(5/2)/(b^2\*log(c)^2 + b^2\*log(x^n)^2 + 2\*a\*b\*log(c) + a^2 + 2\*(b^2\*log(c) + a\*b)\*log(x^n)), x) + 2/7\*d^(5/2)\*x^(7/2)/(b\*log(c) + b\*log(x^n) + a)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}d^2x^2}{b \log(cx^n) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)/(a+b\*log(c\*x^n)), x, algorithm="fricas")

[Out] integral(sqrt(d\*x)\*d^2\*x^2/(b\*log(c\*x^n) + a), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(5/2)/(a+b\*ln(c\*x\*\*n)), x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{5}{2}}}{b \log(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)/(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] integrate((d\*x)^(5/2)/(b\*log(c\*x^n) + a), x)



$$3.102 \quad \int \frac{(dx)^{3/2}}{a+b \log(cx^n)} dx$$

**Optimal.** Leaf size=64

$$\frac{(dx)^{5/2} e^{-\frac{5a}{2bn}} (cx^n)^{-\frac{5}{2n}} \operatorname{Ei}\left(\frac{5(a+b \log(cx^n))}{2bn}\right)}{bdn}$$

[Out]  $((d*x)^{(5/2)*\operatorname{ExpIntegralEi}[(5*(a + b*\operatorname{Log}[c*x^n])]/(2*b*n)])/(b*d*E^{((5*a)/(2*b*n))*n*(c*x^n)^{(5/(2*n))})}$

**Rubi [A]** time = 0.0643295, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2310, 2178}

$$\frac{(dx)^{5/2} e^{-\frac{5a}{2bn}} (cx^n)^{-\frac{5}{2n}} \operatorname{Ei}\left(\frac{5(a+b \log(cx^n))}{2bn}\right)}{bdn}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(d*x)^{(3/2)}/(a + b*\operatorname{Log}[c*x^n]), x]$

[Out]  $((d*x)^{(5/2)*\operatorname{ExpIntegralEi}[(5*(a + b*\operatorname{Log}[c*x^n])]/(2*b*n)])/(b*d*E^{((5*a)/(2*b*n))*n*(c*x^n)^{(5/(2*n))})}$

#### Rule 2310

$\operatorname{Int}[(a + \operatorname{Log}[c*(x)^n]*b)^p*(d*(x))^m, x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[(d*x)^{(m+1)}/(d*n*(c*x^n)^{(m+1)/n}), \operatorname{Subst}[\operatorname{Int}[E^{((m+1)*x)/n}*(a + b*x)^p, x], x, \operatorname{Log}[c*x^n], x] /; \operatorname{FreeQ}\{a, b, c, d, m, n, p\}, x]$

#### Rule 2178

$\operatorname{Int}[(F)^{(g*(e + (c*f)/d))}/((c + d*(x))), x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(F^{g*(e - (c*f)/d)}*\operatorname{ExpIntegralEi}[(f*g*(c + d*x)*\operatorname{Log}[F])/d])/d, x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\& \text{!}\$UseGamma == True$

#### Rubi steps

$$\begin{aligned} \int \frac{(dx)^{3/2}}{a + b \log(cx^n)} dx &= \frac{\left((dx)^{5/2} (cx^n)^{-\frac{5}{2n}}\right) \operatorname{Subst}\left(\int \frac{e^{\frac{5x}{a+bx}} dx, x, \log(cx^n)}\right)}{dn} \\ &= \frac{e^{-\frac{5a}{2bn}} (dx)^{5/2} (cx^n)^{-\frac{5}{2n}} \operatorname{Ei}\left(\frac{5(a+b \log(cx^n))}{2bn}\right)}{bdn} \end{aligned}$$

**Mathematica [A]** time = 0.0706092, size = 62, normalized size = 0.97

$$\frac{x(dx)^{3/2} e^{-\frac{5a}{2bn}} (cx^n)^{-\frac{5}{2n}} \operatorname{Ei}\left(\frac{5(a+b \log(cx^n))}{2bn}\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(3/2)/(a + b\*Log[c\*x^n]),x]

[Out] (x\*(d\*x)^(3/2)\*ExpIntegralEi[(5\*(a + b\*Log[c\*x^n]))/(2\*b\*n)]/(b\*E^((5\*a)/(2\*b\*n)))\*n\*(c\*x^n)^(5/(2\*n)))

**Maple [F]** time = 0.116, size = 0, normalized size = 0.

$$\int \frac{1}{a + b \ln(cx^n)} (dx)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(3/2)/(a+b\*ln(c\*x^n)),x)

[Out] int((d\*x)^(3/2)/(a+b\*ln(c\*x^n)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$2bd^{\frac{3}{2}}n \int \frac{x^{\frac{3}{2}}}{5(b^2 \log(c)^2 + b^2 \log(x^n)^2 + 2ab \log(c) + a^2 + 2(b^2 \log(c) + ab) \log(x^n))} dx + \frac{2d^{\frac{3}{2}}x^{\frac{5}{2}}}{5(b \log(c) + b \log(x^n) + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)/(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out] 2\*b\*d^(3/2)\*n\*integrate(1/5\*x^(3/2)/(b^2\*log(c)^2 + b^2\*log(x^n)^2 + 2\*a\*b\*log(c) + a^2 + 2\*(b^2\*log(c) + a\*b)\*log(x^n)), x) + 2/5\*d^(3/2)\*x^(5/2)/(b\*log(c) + b\*log(x^n) + a)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}dx}{b \log(cx^n) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)/(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] integral(sqrt(d\*x)\*d\*x/(b\*log(c\*x^n) + a), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{3}{2}}}{a + b \log(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(3/2)/(a+b\*ln(c\*x\*\*n)),x)

[Out] Integral((d\*x)\*\*(3/2)/(a + b\*log(c\*x\*\*n)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{3}{2}}}{b \log(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)/(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] integrate((d\*x)^(3/2)/(b\*log(c\*x^n) + a), x)

$$3.103 \quad \int \frac{\sqrt{dx}}{a+b \log(cx^n)} dx$$

**Optimal.** Leaf size=64

$$\frac{(dx)^{3/2} e^{-\frac{3a}{2bn}} (cx^n)^{-\frac{3}{2}/n} \operatorname{Ei}\left(\frac{3(a+b \log(cx^n))}{2bn}\right)}{bdn}$$

[Out] ((d\*x)^(3/2)\*ExpIntegralEi[(3\*(a + b\*Log[c\*x^n]))/(2\*b\*n)])/(b\*d\*E^((3\*a)/(2\*b\*n))\*n\*(c\*x^n)^(3/(2\*n)))

**Rubi [A]** time = 0.060284, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2310, 2178}

$$\frac{(dx)^{3/2} e^{-\frac{3a}{2bn}} (cx^n)^{-\frac{3}{2}/n} \operatorname{Ei}\left(\frac{3(a+b \log(cx^n))}{2bn}\right)}{bdn}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d\*x]/(a + b\*Log[c\*x^n]), x]

[Out] ((d\*x)^(3/2)\*ExpIntegralEi[(3\*(a + b\*Log[c\*x^n]))/(2\*b\*n)])/(b\*d\*E^((3\*a)/(2\*b\*n))\*n\*(c\*x^n)^(3/(2\*n)))

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{dx}}{a+b \log(cx^n)} dx &= \frac{\left( (dx)^{3/2} (cx^n)^{-\frac{3}{2}/n} \right) \operatorname{Subst}\left( \int \frac{e^{\frac{3x}{a+bx}}}{a+bx} dx, x, \log(cx^n) \right)}{dn} \\ &= \frac{e^{-\frac{3a}{2bn}} (dx)^{3/2} (cx^n)^{-\frac{3}{2}/n} \operatorname{Ei}\left(\frac{3(a+b \log(cx^n))}{2bn}\right)}{bdn} \end{aligned}$$

**Mathematica [A]** time = 0.0685466, size = 62, normalized size = 0.97

$$\frac{x\sqrt{dx} e^{-\frac{3a}{2bn}} (cx^n)^{-\frac{3}{2}/n} \operatorname{Ei}\left(\frac{3(a+b \log(cx^n))}{2bn}\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d\*x]/(a + b\*Log[c\*x^n]), x]

[Out] (x\*Sqrt[d\*x]\*ExpIntegralEi[(3\*(a + b\*Log[c\*x^n]))/(2\*b\*n)])/(b\*E^((3\*a)/(2\*b\*n))\*n\*(c\*x^n)^(3/(2\*n)))

**Maple [F]** time = 0.115, size = 0, normalized size = 0.

$$\int \frac{1}{a + b \ln(cx^n)} \sqrt{dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(1/2)/(a+b\*ln(c\*x^n)), x)

[Out] int((d\*x)^(1/2)/(a+b\*ln(c\*x^n)), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$2b\sqrt{dn} \int \frac{\sqrt{x}}{3(b^2 \log(c)^2 + b^2 \log(x^n)^2 + 2ab \log(c) + a^2 + 2(b^2 \log(c) + ab) \log(x^n))} dx + \frac{2\sqrt{dx}^{\frac{3}{2}}}{3(b \log(c) + b \log(x^n) + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)/(a+b\*log(c\*x^n)), x, algorithm="maxima")

[Out] 2\*b\*sqrt(d)\*n\*integrate(1/3\*sqrt(x)/(b^2\*log(c)^2 + b^2\*log(x^n)^2 + 2\*a\*b\*log(c) + a^2 + 2\*(b^2\*log(c) + a\*b)\*log(x^n)), x) + 2/3\*sqrt(d)\*x^(3/2)/(b\*log(c) + b\*log(x^n) + a)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}}{b \log(cx^n) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)/(a+b\*log(c\*x^n)), x, algorithm="fricas")

[Out] integral(sqrt(d\*x)/(b\*log(c\*x^n) + a), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx}}{a + b \log(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(1/2)/(a+b\*ln(c\*x\*\*n)), x)

```
[Out] Integral(sqrt(d*x)/(a + b*log(c*x**n)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx}}{b \log(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)/(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)/(b*log(c*x^n) + a), x)
```

$$3.104 \quad \int \frac{1}{\sqrt{dx}(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=64

$$\frac{\sqrt{dx} e^{-\frac{a}{2bn}} (cx^n)^{-\frac{1}{2}/n} \text{Ei}\left(\frac{a+b \log(cx^n)}{2bn}\right)}{bdn}$$

[Out] (Sqrt[d\*x]\*ExpIntegralEi[(a + b\*Log[c\*x^n])/(2\*b\*n)])/(b\*d\*E^(a/(2\*b\*n))\*n\*(c\*x^n)^(1/(2\*n)))

**Rubi [A]** time = 0.0605463, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2310, 2178}

$$\frac{\sqrt{dx} e^{-\frac{a}{2bn}} (cx^n)^{-\frac{1}{2}/n} \text{Ei}\left(\frac{a+b \log(cx^n)}{2bn}\right)}{bdn}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[d\*x]\*(a + b\*Log[c\*x^n])), x]

[Out] (Sqrt[d\*x]\*ExpIntegralEi[(a + b\*Log[c\*x^n])/(2\*b\*n)])/(b\*d\*E^(a/(2\*b\*n))\*n\*(c\*x^n)^(1/(2\*n)))

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{dx}(a+b \log(cx^n))} dx &= \frac{\left(\sqrt{dx}(cx^n)^{-\frac{1}{2}/n}\right) \text{Subst}\left(\int \frac{e^{\frac{x}{a+bx}}}{a+bx} dx, x, \log(cx^n)\right)}{dn} \\ &= \frac{e^{-\frac{a}{2bn}} \sqrt{dx}(cx^n)^{-\frac{1}{2}/n} \text{Ei}\left(\frac{a+b \log(cx^n)}{2bn}\right)}{bdn} \end{aligned}$$

**Mathematica [A]** time = 0.0628889, size = 62, normalized size = 0.97

$$\frac{x e^{-\frac{a}{2bn}} (cx^n)^{-\frac{1}{2}/n} \text{Ei}\left(\frac{a+b \log(cx^n)}{2bn}\right)}{bn\sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[d\*x]\*(a + b\*Log[c\*x^n])),x]

[Out] (x\*ExpIntegralEi[(a + b\*Log[c\*x^n])/(2\*b\*n)])/(b\*E^(a/(2\*b\*n))\*n\*Sqrt[d\*x]\*(c\*x^n)^(1/(2\*n)))

**Maple [F]** time = 0.108, size = 0, normalized size = 0.

$$\int \frac{1}{a + b \ln(cx^n)} \frac{1}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x)^(1/2)/(a+b\*ln(c\*x^n)),x)

[Out] int(1/(d\*x)^(1/2)/(a+b\*ln(c\*x^n)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$2bn \int \frac{1}{(b^2\sqrt{d} \log(c)^2 + b^2\sqrt{d} \log(x^n)^2 + 2ab\sqrt{d} \log(c) + a^2\sqrt{d} + 2(b^2\sqrt{d} \log(c) + ab\sqrt{d}) \log(x^n))\sqrt{x}} dx + \frac{1}{b\sqrt{d} \log(c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(1/2)/(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out] 2\*b\*n\*integrate(1/((b^2\*sqrt(d)\*log(c)^2 + b^2\*sqrt(d)\*log(x^n)^2 + 2\*a\*b\*sqrt(d)\*log(c) + a^2\*sqrt(d) + 2\*(b^2\*sqrt(d)\*log(c) + a\*b\*sqrt(d))\*log(x^n))\*sqrt(x)), x) + 2\*sqrt(x)/(b\*sqrt(d)\*log(c) + b\*sqrt(d)\*log(x^n) + a\*sqrt(d))

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}}{bdx \log(cx^n) + adx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(1/2)/(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] integral(sqrt(d\*x)/(b\*d\*x\*log(c\*x^n) + a\*d\*x), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx}(a + b \log(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(1/(d*x)**(1/2)/(a+b*ln(c*x**n)),x)
```

```
[Out] Integral(1/(sqrt(d*x)*(a + b*log(c*x**n))), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx}(b \log(cx^n) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x)*(b*log(c*x^n) + a)), x)
```

$$3.105 \quad \int \frac{1}{(dx)^{3/2}(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=64

$$\frac{e^{\frac{a}{2bn}} (cx^n)^{\frac{1}{2n}} \operatorname{Ei}\left(-\frac{a+b \log(cx^n)}{2bn}\right)}{bdn\sqrt{dx}}$$

[Out] (E^(a/(2\*b\*n))\*(c\*x^n)^(1/(2\*n))\*ExpIntegralEi[-(a + b\*Log[c\*x^n])/(2\*b\*n)])/(b\*d\*n\*Sqrt[d\*x])

**Rubi [A]** time = 0.0607804, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2310, 2178}

$$\frac{e^{\frac{a}{2bn}} (cx^n)^{\frac{1}{2n}} \operatorname{Ei}\left(-\frac{a+b \log(cx^n)}{2bn}\right)}{bdn\sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Int[1/((d\*x)^(3/2)\*(a + b\*Log[c\*x^n])),x]

[Out] (E^(a/(2\*b\*n))\*(c\*x^n)^(1/(2\*n))\*ExpIntegralEi[-(a + b\*Log[c\*x^n])/(2\*b\*n)])/(b\*d\*n\*Sqrt[d\*x])

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{1}{(dx)^{3/2}(a+b \log(cx^n))} dx &= \frac{(cx^n)^{\frac{1}{2n}} \operatorname{Subst}\left(\int \frac{e^{-\frac{x}{2n}}}{a+bx} dx, x, \log(cx^n)\right)}{dn\sqrt{dx}} \\ &= \frac{e^{\frac{a}{2bn}} (cx^n)^{\frac{1}{2n}} \operatorname{Ei}\left(-\frac{a+b \log(cx^n)}{2bn}\right)}{bdn\sqrt{dx}} \end{aligned}$$

**Mathematica [A]** time = 0.0712396, size = 62, normalized size = 0.97

$$\frac{x e^{\frac{a}{2bn}} (cx^n)^{\frac{1}{2n}} \operatorname{Ei}\left(-\frac{a+b \log(cx^n)}{2bn}\right)}{bn(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d\*x)^(3/2)\*(a + b\*Log[c\*x^n])),x]

[Out] (E^(a/(2\*b\*n))\*x\*(c\*x^n)^(1/(2\*n))\*ExpIntegralEi[-(a + b\*Log[c\*x^n])/(2\*b\*n)])/(b\*n\*(d\*x)^(3/2))

**Maple [F]** time = 0.109, size = 0, normalized size = 0.

$$\int \frac{1}{a + b \ln(cx^n)} (dx)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x)^(3/2)/(a+b\*ln(c\*x^n)),x)

[Out] int(1/(d\*x)^(3/2)/(a+b\*ln(c\*x^n)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-2bn \int \frac{1}{\left(b^2 d^{\frac{3}{2}} \log(c)^2 + b^2 d^{\frac{3}{2}} \log(x^n)^2 + 2abd^{\frac{3}{2}} \log(c) + a^2 d^{\frac{3}{2}} + 2\left(b^2 d^{\frac{3}{2}} \log(c) + abd^{\frac{3}{2}}\right) \log(x^n)\right) x^{\frac{3}{2}}} dx - \frac{1}{\left(bd^{\frac{3}{2}} \log(c) + bd^{\frac{3}{2}} \log(x^n) + ad^{\frac{3}{2}}\right) \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(3/2)/(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out] -2\*b\*n\*integrate(1/((b^2\*d^(3/2)\*log(c)^2 + b^2\*d^(3/2)\*log(x^n)^2 + 2\*a\*b\*d^(3/2)\*log(c) + a^2\*d^(3/2) + 2\*(b^2\*d^(3/2)\*log(c) + a\*b\*d^(3/2))\*log(x^n))\*x^(3/2)), x) - 2/((b\*d^(3/2)\*log(c) + b\*d^(3/2)\*log(x^n) + a\*d^(3/2))\*sqrt(x))

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}}{bd^2x^2 \log(cx^n) + ad^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(3/2)/(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] integral(sqrt(d\*x)/(b\*d^2\*x^2\*log(c\*x^n) + a\*d^2\*x^2), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{3}{2}} (a + b \log(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)\*\*(3/2)/(a+b\*ln(c\*x\*\*n)),x)

[Out] Integral(1/((d\*x)\*\*(3/2)\*(a + b\*log(c\*x\*\*n))), x)

**Giac [A]** time = 1.28626, size = 66, normalized size = 1.03

$$\frac{c^{\frac{1}{2n}} \operatorname{Ei}\left(-\frac{\log(c)}{2n} - \frac{a}{2bn} - \frac{1}{2} \log(x)\right) e^{\left(\frac{a}{2bn}\right)}}{bd^{\frac{3}{2}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(3/2)/(a+b\*log(c\*x^n)),x, algorithm="giac")

[Out] c^(1/2/n)\*Ei(-1/2\*log(c)/n - 1/2\*a/(b\*n) - 1/2\*log(x))\*e^(1/2\*a/(b\*n))/(b\*d^(3/2)\*n)

$$3.106 \quad \int \frac{1}{(dx)^{5/2}(a+b \log(cx^n))} dx$$

**Optimal.** Leaf size=64

$$\frac{e^{\frac{3a}{2bn}} (cx^n)^{\frac{3}{2n}} \operatorname{Ei}\left(-\frac{3(a+b \log(cx^n))}{2bn}\right)}{bdn(dx)^{3/2}}$$

[Out] (E^((3\*a)/(2\*b\*n)))\*(c\*x^n)^(3/(2\*n))\*ExpIntegralEi[(-3\*(a + b\*Log[c\*x^n]))/(2\*b\*n)]/(b\*d\*n\*(d\*x)^(3/2))

**Rubi [A]** time = 0.0591011, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2310, 2178}

$$\frac{e^{\frac{3a}{2bn}} (cx^n)^{\frac{3}{2n}} \operatorname{Ei}\left(-\frac{3(a+b \log(cx^n))}{2bn}\right)}{bdn(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((d\*x)^(5/2)\*(a + b\*Log[c\*x^n])), x]

[Out] (E^((3\*a)/(2\*b\*n)))\*(c\*x^n)^(3/(2\*n))\*ExpIntegralEi[(-3\*(a + b\*Log[c\*x^n]))/(2\*b\*n)]/(b\*d\*n\*(d\*x)^(3/2))

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x)/n]\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{1}{(dx)^{5/2}(a+b \log(cx^n))} dx &= \frac{(cx^n)^{\frac{3}{2n}} \operatorname{Subst}\left(\int \frac{e^{-\frac{3x}{2n}}}{a+bx} dx, x, \log(cx^n)\right)}{dn(dx)^{3/2}} \\ &= \frac{e^{\frac{3a}{2bn}} (cx^n)^{\frac{3}{2n}} \operatorname{Ei}\left(-\frac{3(a+b \log(cx^n))}{2bn}\right)}{bdn(dx)^{3/2}} \end{aligned}$$

**Mathematica [A]** time = 0.0729569, size = 62, normalized size = 0.97

$$\frac{xe^{\frac{3a}{2bn}} (cx^n)^{\frac{3}{2n}} \operatorname{Ei}\left(-\frac{3(a+b \log(cx^n))}{2bn}\right)}{bn(dx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d\*x)^(5/2)\*(a + b\*Log[c\*x^n])),x]

[Out] (E^((3\*a)/(2\*b\*n))\*x\*(c\*x^n)^(3/(2\*n))\*ExpIntegralEi[(-3\*(a + b\*Log[c\*x^n]))/(2\*b\*n)])/(b\*n\*(d\*x)^(5/2))

**Maple [F]** time = 0.108, size = 0, normalized size = 0.

$$\int \frac{1}{a + b \ln(cx^n)} (dx)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x)^(5/2)/(a+b\*ln(c\*x^n)),x)

[Out] int(1/(d\*x)^(5/2)/(a+b\*ln(c\*x^n)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-2bn \int \frac{1}{3 \left( b^2 d^{\frac{5}{2}} \log(c)^2 + b^2 d^{\frac{5}{2}} \log(x^n)^2 + 2abd^{\frac{5}{2}} \log(c) + a^2 d^{\frac{5}{2}} + 2 \left( b^2 d^{\frac{5}{2}} \log(c) + abd^{\frac{5}{2}} \right) \log(x^n) \right) x^{\frac{5}{2}}} dx - \frac{1}{3 \left( b^2 d^{\frac{5}{2}} \log(c)^2 + b^2 d^{\frac{5}{2}} \log(x^n)^2 + 2abd^{\frac{5}{2}} \log(c) + a^2 d^{\frac{5}{2}} + 2 \left( b^2 d^{\frac{5}{2}} \log(c) + abd^{\frac{5}{2}} \right) \log(x^n) \right) x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(5/2)/(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out] -2\*b\*n\*integrate(1/3/((b^2\*d^(5/2)\*log(c)^2 + b^2\*d^(5/2)\*log(x^n)^2 + 2\*a\*b\*d^(5/2)\*log(c) + a^2\*d^(5/2) + 2\*(b^2\*d^(5/2)\*log(c) + a\*b\*d^(5/2))\*log(x^n))\*x^(5/2)), x) - 2/3/((b\*d^(5/2)\*log(c) + b\*d^(5/2)\*log(x^n) + a\*d^(5/2))\*x^(3/2))

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\sqrt{dx}}{bd^3x^3 \log(cx^n) + ad^3x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(5/2)/(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] integral(sqrt(d\*x)/(b\*d^3\*x^3\*log(c\*x^n) + a\*d^3\*x^3), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{5}{2}} (a + b \log(cx^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(5/2)/(a+b*ln(c*x**n)),x)
```

```
[Out] Integral(1/((d*x)**(5/2)*(a + b*log(c*x**n))), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{5}{2}} (b \log(cx^n) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(5/2)/(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x)^(5/2)*(b*log(c*x^n) + a)), x)
```

$$3.107 \quad \int \frac{(dx)^{5/2}}{(a+b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=98

$$\frac{7(dx)^{7/2} e^{-\frac{7a}{2bn}} (cx^n)^{-\frac{7}{2}/n} \operatorname{Ei}\left(\frac{7(a+b \log(cx^n))}{2bn}\right)}{2b^2 dn^2} - \frac{(dx)^{7/2}}{bdn(a+b \log(cx^n))}$$

[Out] (7\*(d\*x)^(7/2)\*ExpIntegralEi[(7\*(a + b\*Log[c\*x^n]))/(2\*b\*n)])/(2\*b^2\*d\*E^((7\*a)/(2\*b\*n))\*n^2\*(c\*x^n)^(7/(2\*n))) - (d\*x)^(7/2)/(b\*d\*n\*(a + b\*Log[c\*x^n]))

**Rubi [A]** time = 0.0913091, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$ , Rules used = {2306, 2310, 2178}

$$\frac{7(dx)^{7/2} e^{-\frac{7a}{2bn}} (cx^n)^{-\frac{7}{2}/n} \operatorname{Ei}\left(\frac{7(a+b \log(cx^n))}{2bn}\right)}{2b^2 dn^2} - \frac{(dx)^{7/2}}{bdn(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^(5/2)/(a + b\*Log[c\*x^n])^2,x]

[Out] (7\*(d\*x)^(7/2)\*ExpIntegralEi[(7\*(a + b\*Log[c\*x^n]))/(2\*b\*n)])/(2\*b^2\*d\*E^((7\*a)/(2\*b\*n))\*n^2\*(c\*x^n)^(7/(2\*n))) - (d\*x)^(7/2)/(b\*d\*n\*(a + b\*Log[c\*x^n]))

#### Rule 2306

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_)^(m_.), x_Symbol]
  := Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^(p + 1))/(b*d*n*(p + 1)), x] -
  Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
  /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

#### Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_)^(m_.), x_Symbol]
  := Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
  /n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

#### Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Si
  mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
  reeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

#### Rubi steps



$$\begin{aligned} \int \frac{(dx)^{5/2}}{(a + b \log(cx^n))^2} dx &= -\frac{(dx)^{7/2}}{bdn(a + b \log(cx^n))} + \frac{7 \int \frac{(dx)^{5/2}}{a+b \log(cx^n)} dx}{2bn} \\ &= -\frac{(dx)^{7/2}}{bdn(a + b \log(cx^n))} + \frac{\left(7(dx)^{7/2}(cx^n)^{-7/2/n}\right) \text{Subst}\left(\int \frac{e^{7x/2n}}{a+bx} dx, x, \log(cx^n)\right)}{2bdn^2} \\ &= \frac{7e^{-7a/2bn}(dx)^{7/2}(cx^n)^{-7/2/n} \text{Ei}\left(\frac{7(a+b \log(cx^n))}{2bn}\right)}{2b^2dn^2} - \frac{(dx)^{7/2}}{bdn(a + b \log(cx^n))} \end{aligned}$$

**Mathematica [A]** time = 0.153984, size = 84, normalized size = 0.86

$$\frac{x(dx)^{5/2} \left(7e^{-\frac{7a}{2bn}}(cx^n)^{-7/2/n} \text{Ei}\left(\frac{7(a+b \log(cx^n))}{2bn}\right) - \frac{2bn}{a+b \log(cx^n)}\right)}{2b^2n^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(5/2)/(a + b\*Log[c\*x^n])^2,x]

[Out] (x\*(d\*x)^(5/2)\*((7\*ExpIntegralEi[(7\*(a + b\*Log[c\*x^n]))]/(2\*b\*n)])/(E^((7\*a)/(2\*b\*n))\*(c\*x^n)^(7/(2\*n))) - (2\*b\*n)/(a + b\*Log[c\*x^n]))/(2\*b^2\*n^2)

**Maple [F]** time = 4.648, size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \ln(cx^n))^2} (dx)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(5/2)/(a+b\*ln(c\*x^n))^2,x)

[Out] int((d\*x)^(5/2)/(a+b\*ln(c\*x^n))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$4bd^{\frac{5}{2}}n \int \frac{x^{\frac{5}{2}}}{7(b^3 \log(c)^3 + b^3 \log(x^n)^3 + 3ab^2 \log(c)^2 + 3a^2b \log(c) + a^3 + 3(b^3 \log(c) + ab^2) \log(x^n)^2 + 3(b^3 \log(c) + ab^2) \log(x^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)/(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] 4\*b\*d^(5/2)\*n\*integrate(1/7\*x^(5/2)/(b^3\*log(c)^3 + b^3\*log(x^n)^3 + 3\*a\*b^2\*log(c)^2 + 3\*a^2\*b\*log(c) + a^3 + 3\*(b^3\*log(c) + a\*b^2)\*log(x^n)^2 + 3\*(b^3\*log(c)^2 + 2\*a\*b^2\*log(c) + a^2\*b)\*log(x^n)), x) + 2/7\*d^(5/2)\*x^(7/2)/(b^2\*log(c)^2 + b^2\*log(x^n)^2 + 2\*a\*b\*log(c) + a^2 + 2\*(b^2\*log(c) + a\*b)\*log(x^n))

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}d^2x^2}{b^2 \log(cx^n)^2 + 2ab \log(cx^n) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)/(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] integral(sqrt(d\*x)\*d^2\*x^2/(b^2\*log(c\*x^n)^2 + 2\*a\*b\*log(c\*x^n) + a^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(5/2)/(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{5}{2}}}{(b \log(cx^n) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)/(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] integrate((d\*x)^(5/2)/(b\*log(c\*x^n) + a)^2, x)

$$3.108 \quad \int \frac{(dx)^{3/2}}{(a+b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=98

$$\frac{5(dx)^{5/2} e^{-\frac{5a}{2bn}} (cx^n)^{-\frac{5}{2}/n} \operatorname{Ei}\left(\frac{5(a+b \log(cx^n))}{2bn}\right)}{2b^2 dn^2} - \frac{(dx)^{5/2}}{bdn(a+b \log(cx^n))}$$

[Out] (5\*(d\*x)^(5/2)\*ExpIntegralEi[(5\*(a + b\*Log[c\*x^n]))/(2\*b\*n)])/(2\*b^2\*d\*E^((5\*a)/(2\*b\*n))\*n^2\*(c\*x^n)^(5/(2\*n))) - (d\*x)^(5/2)/(b\*d\*n\*(a + b\*Log[c\*x^n]))

**Rubi [A]** time = 0.0957078, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$ , Rules used = {2306, 2310, 2178}

$$\frac{5(dx)^{5/2} e^{-\frac{5a}{2bn}} (cx^n)^{-\frac{5}{2}/n} \operatorname{Ei}\left(\frac{5(a+b \log(cx^n))}{2bn}\right)}{2b^2 dn^2} - \frac{(dx)^{5/2}}{bdn(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^(3/2)/(a + b\*Log[c\*x^n])^2, x]

[Out] (5\*(d\*x)^(5/2)\*ExpIntegralEi[(5\*(a + b\*Log[c\*x^n]))/(2\*b\*n)])/(2\*b^2\*d\*E^((5\*a)/(2\*b\*n))\*n^2\*(c\*x^n)^(5/(2\*n))) - (d\*x)^(5/2)/(b\*d\*n\*(a + b\*Log[c\*x^n]))

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\begin{aligned} \int \frac{(dx)^{3/2}}{(a + b \log(cx^n))^2} dx &= -\frac{(dx)^{5/2}}{bdn(a + b \log(cx^n))} + \frac{5 \int \frac{(dx)^{3/2}}{a+b \log(cx^n)} dx}{2bn} \\ &= -\frac{(dx)^{5/2}}{bdn(a + b \log(cx^n))} + \frac{\left(5(dx)^{5/2} (cx^n)^{-5/2/n}\right) \text{Subst}\left(\int \frac{e^{5x/2n}}{a+bx} dx, x, \log(cx^n)\right)}{2bdn^2} \\ &= \frac{5e^{-5a/2bn} (dx)^{5/2} (cx^n)^{-5/2/n} \text{Ei}\left(\frac{5(a+b \log(cx^n))}{2bn}\right)}{2b^2dn^2} - \frac{(dx)^{5/2}}{bdn(a + b \log(cx^n))} \end{aligned}$$

**Mathematica [A]** time = 0.143716, size = 84, normalized size = 0.86

$$\frac{x(dx)^{3/2} \left(5e^{-\frac{5a}{2bn}} (cx^n)^{-\frac{5}{2/n}} \text{Ei}\left(\frac{5(a+b \log(cx^n))}{2bn}\right) - \frac{2bn}{a+b \log(cx^n)}\right)}{2b^2n^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(3/2)/(a + b\*Log[c\*x^n])^2,x]

[Out] (x\*(d\*x)^(3/2)\*((5\*ExpIntegralEi[(5\*(a + b\*Log[c\*x^n]))/(2\*b\*n)])/(E^((5\*a)/(2\*b\*n)))\*(c\*x^n)^(5/(2\*n))) - (2\*b\*n)/(a + b\*Log[c\*x^n]))/(2\*b^2\*n^2)

**Maple [F]** time = 4.5, size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \ln(cx^n))^2} (dx)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(3/2)/(a+b\*ln(c\*x^n))^2,x)

[Out] int((d\*x)^(3/2)/(a+b\*ln(c\*x^n))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$4bd^{\frac{3}{2}}n \int \frac{x^{\frac{3}{2}}}{5(b^3 \log(c)^3 + b^3 \log(x^n)^3 + 3ab^2 \log(c)^2 + 3a^2b \log(c) + a^3 + 3(b^3 \log(c) + ab^2) \log(x^n)^2 + 3(b^3 \log(c)^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)/(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] 4\*b\*d^(3/2)\*n\*integrate(1/5\*x^(3/2)/(b^3\*log(c)^3 + b^3\*log(x^n)^3 + 3\*a\*b^2\*log(c)^2 + 3\*a^2\*b\*log(c) + a^3 + 3\*(b^3\*log(c) + a\*b^2)\*log(x^n)^2 + 3\*(b^3\*log(c)^2 + 2\*a\*b^2\*log(c) + a^2\*b)\*log(x^n)), x) + 2/5\*d^(3/2)\*x^(5/2)/(b^2\*log(c)^2 + b^2\*log(x^n)^2 + 2\*a\*b\*log(c) + a^2 + 2\*(b^2\*log(c) + a\*b)\*log(x^n))

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}dx}{b^2 \log(cx^n)^2 + 2ab \log(cx^n) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)/(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] integral(sqrt(d\*x)\*d\*x/(b^2\*log(c\*x^n)^2 + 2\*a\*b\*log(c\*x^n) + a^2), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{3}{2}}}{(a + b \log(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(3/2)/(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Integral((d\*x)\*\*(3/2)/(a + b\*log(c\*x\*\*n))\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{3}{2}}}{(b \log(cx^n) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)/(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] integrate((d\*x)^(3/2)/(b\*log(c\*x^n) + a)^2, x)

$$3.109 \quad \int \frac{\sqrt{dx}}{(a+b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=98

$$\frac{3(dx)^{3/2} e^{-\frac{3a}{2bn}} (cx^n)^{-\frac{3}{2}/n} \operatorname{Ei}\left(\frac{3(a+b \log(cx^n))}{2bn}\right)}{2b^2 dn^2} - \frac{(dx)^{3/2}}{bdn(a+b \log(cx^n))}$$

[Out] (3\*(d\*x)^(3/2)\*ExpIntegralEi[(3\*(a + b\*Log[c\*x^n]))/(2\*b\*n)])/(2\*b^2\*d\*E^((3\*a)/(2\*b\*n))\*n^2\*(c\*x^n)^(3/(2\*n))) - (d\*x)^(3/2)/(b\*d\*n\*(a + b\*Log[c\*x^n]))

**Rubi [A]** time = 0.0870387, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$ , Rules used = {2306, 2310, 2178}

$$\frac{3(dx)^{3/2} e^{-\frac{3a}{2bn}} (cx^n)^{-\frac{3}{2}/n} \operatorname{Ei}\left(\frac{3(a+b \log(cx^n))}{2bn}\right)}{2b^2 dn^2} - \frac{(dx)^{3/2}}{bdn(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d\*x]/(a + b\*Log[c\*x^n])^2,x]

[Out] (3\*(d\*x)^(3/2)\*ExpIntegralEi[(3\*(a + b\*Log[c\*x^n]))/(2\*b\*n)])/(2\*b^2\*d\*E^((3\*a)/(2\*b\*n))\*n^2\*(c\*x^n)^(3/(2\*n))) - (d\*x)^(3/2)/(b\*d\*n\*(a + b\*Log[c\*x^n]))

#### Rule 2306

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^(p + 1))/(b*d*n*(p + 1)), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

#### Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol]
:= Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)*x)
/n]*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

#### Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

#### Rubi steps

$$\int \frac{\sqrt{dx}}{(a + b \log(cx^n))^2} dx = -\frac{(dx)^{3/2}}{bdn(a + b \log(cx^n))} + \frac{3 \int \frac{\sqrt{dx}}{a + b \log(cx^n)} dx}{2bn}$$

$$= -\frac{(dx)^{3/2}}{bdn(a + b \log(cx^n))} + \frac{\left(3(dx)^{3/2} (cx^n)^{-\frac{3}{2n}}\right) \text{Subst}\left(\int \frac{e^{\frac{3x}{a+bx}}}{a+bx} dx, x, \log(cx^n)\right)}{2bdn^2}$$

$$= \frac{3e^{-\frac{3a}{2bn}} (dx)^{3/2} (cx^n)^{-\frac{3}{2n}} \text{Ei}\left(\frac{3(a+b \log(cx^n))}{2bn}\right)}{2b^2dn^2} - \frac{(dx)^{3/2}}{bdn(a + b \log(cx^n))}$$

**Mathematica [A]** time = 0.132666, size = 84, normalized size = 0.86

$$\frac{x\sqrt{dx} \left(3e^{-\frac{3a}{2bn}} (cx^n)^{-\frac{3}{2n}} \text{Ei}\left(\frac{3(a+b \log(cx^n))}{2bn}\right) - \frac{2bn}{a+b \log(cx^n)}\right)}{2b^2n^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d\*x]/(a + b\*Log[c\*x^n])^2,x]

[Out] (x\*Sqrt[d\*x]\*((3\*ExpIntegralEi[(3\*(a + b\*Log[c\*x^n]))]/(2\*b\*n)])/(E^((3\*a)/(2\*b\*n))\*(c\*x^n)^(3/(2\*n))) - (2\*b\*n)/(a + b\*Log[c\*x^n])))/(2\*b^2\*n^2)

**Maple [F]** time = 4.658, size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \ln(cx^n))^2} \sqrt{dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(1/2)/(a+b\*ln(c\*x^n))^2,x)

[Out] int((d\*x)^(1/2)/(a+b\*ln(c\*x^n))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$4b\sqrt{dn} \int \frac{\sqrt{x}}{3(b^3 \log(c)^3 + b^3 \log(x^n)^3 + 3ab^2 \log(c)^2 + 3a^2b \log(c) + a^3 + 3(b^3 \log(c) + ab^2) \log(x^n)^2 + 3(b^3 \log(c) + ab^2) \log(x^n))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)/(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] 4\*b\*sqrt(d)\*n\*integrate(1/3\*sqrt(x)/(b^3\*log(c)^3 + b^3\*log(x^n)^3 + 3\*a\*b^2\*log(c)^2 + 3\*a^2\*b\*log(c) + a^3 + 3\*(b^3\*log(c) + a\*b^2)\*log(x^n)^2 + 3\*(b^3\*log(c)^2 + 2\*a\*b^2\*log(c) + a^2\*b)\*log(x^n)), x) + 2/3\*sqrt(d)\*x^(3/2)/(b^2\*log(c)^2 + b^2\*log(x^n)^2 + 2\*a\*b\*log(c) + a^2 + 2\*(b^2\*log(c) + a\*b)\*log(x^n))

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}}{b^2 \log(cx^n)^2 + 2ab \log(cx^n) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)/(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] integral(sqrt(d\*x)/(b^2\*log(c\*x^n)^2 + 2\*a\*b\*log(c\*x^n) + a^2), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx}}{(a + b \log(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(1/2)/(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Integral(sqrt(d\*x)/(a + b\*log(c\*x\*\*n))\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx}}{(b \log(cx^n) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)/(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] integrate(sqrt(d\*x)/(b\*log(c\*x^n) + a)^2, x)



$$3.110 \quad \int \frac{1}{\sqrt{dx}(a+b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=98

$$\frac{\sqrt{dx} e^{-\frac{a}{2bn}} (cx^n)^{-\frac{1}{2n}} \operatorname{Ei}\left(\frac{a+b \log(cx^n)}{2bn}\right)}{2b^2 d n^2} - \frac{\sqrt{dx}}{bdn (a + b \log(cx^n))}$$

[Out] (Sqrt[d\*x]\*ExpIntegralEi[(a + b\*Log[c\*x^n])/(2\*b\*n)])/(2\*b^2\*d\*E^(a/(2\*b\*n))\*n^2\*(c\*x^n)^(1/(2\*n))) - Sqrt[d\*x]/(b\*d\*n\*(a + b\*Log[c\*x^n]))

**Rubi [A]** time = 0.0879756, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$ , Rules used = {2306, 2310, 2178}

$$\frac{\sqrt{dx} e^{-\frac{a}{2bn}} (cx^n)^{-\frac{1}{2n}} \operatorname{Ei}\left(\frac{a+b \log(cx^n)}{2bn}\right)}{2b^2 d n^2} - \frac{\sqrt{dx}}{bdn (a + b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[d\*x]\*(a + b\*Log[c\*x^n])^2), x]

[Out] (Sqrt[d\*x]\*ExpIntegralEi[(a + b\*Log[c\*x^n])/(2\*b\*n)])/(2\*b^2\*d\*E^(a/(2\*b\*n))\*n^2\*(c\*x^n)^(1/(2\*n))) - Sqrt[d\*x]/(b\*d\*n\*(a + b\*Log[c\*x^n]))

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^ (p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^ (p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

#### Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{dx}(a+b \log(cx^n))^2} dx &= -\frac{\sqrt{dx}}{bdn(a+b \log(cx^n))} + \frac{\int \frac{1}{\sqrt{dx}(a+b \log(cx^n))} dx}{2bn} \\ &= -\frac{\sqrt{dx}}{bdn(a+b \log(cx^n))} + \frac{\left(\sqrt{dx}(cx^n)^{-\frac{1}{2n}}\right) \text{Subst}\left(\int \frac{e^{\frac{x}{a+bx}}}{a+bx} dx, x, \log(cx^n)\right)}{2bdn^2} \\ &= \frac{e^{-\frac{a}{2bn}} \sqrt{dx}(cx^n)^{-\frac{1}{2n}} \text{Ei}\left(\frac{a+b \log(cx^n)}{2bn}\right)}{2b^2dn^2} - \frac{\sqrt{dx}}{bdn(a+b \log(cx^n))} \end{aligned}$$

**Mathematica [A]** time = 0.114841, size = 83, normalized size = 0.85

$$\frac{x \left( e^{-\frac{a}{2bn}} (cx^n)^{-\frac{1}{2n}} \text{Ei}\left(\frac{a+b \log(cx^n)}{2bn}\right) - \frac{2bn}{a+b \log(cx^n)} \right)}{2b^2n^2 \sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[d\*x]\*(a + b\*Log[c\*x^n])^2), x]

[Out] (x\*(ExpIntegralEi[(a + b\*Log[c\*x^n])/(2\*b\*n)]/(E^(a/(2\*b\*n))\*(c\*x^n)^(1/(2\*n)))) - (2\*b\*n)/(a + b\*Log[c\*x^n]))/(2\*b^2\*n^2\*Sqrt[d\*x])

**Maple [C]** time = 1.596, size = 427, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x)^(1/2)/(a+b\*ln(c\*x^n))^2,x)

[Out] 
$$\begin{aligned} & -2/b/n*x/(d*x)^{(1/2)}/(2*a+2*b*\ln(c)+2*b*\ln(\exp(n*\ln(x)))+I*b*Pi*csgn(I*\exp(n*\ln(x))) \\ & *csgn(I*c*\exp(n*\ln(x)))^2-I*b*Pi*csgn(I*\exp(n*\ln(x)))*csgn(I*c*\exp(n*\ln(x))) \\ & *csgn(I*c)-I*b*Pi*csgn(I*c*\exp(n*\ln(x)))^3+I*b*Pi*csgn(I*c*\exp(n*\ln(x)))^2 \\ & *csgn(I*c))-1/2/d/b^2/n^2*\exp(1/4*I*(b*Pi*csgn(I*\exp(n*\ln(x))))*csgn(I*c*\exp(n*\ln(x))) \\ & *csgn(I*c)-b*Pi*csgn(I*c*\exp(n*\ln(x)))^2*csgn(I*c)-b*Pi*csgn(I*\exp(n*\ln(x))) \\ & *csgn(I*c*\exp(n*\ln(x)))^2+b*Pi*csgn(I*c*\exp(n*\ln(x)))^3+2*I*b*n*(\ln(x)-\ln(d*x)) \\ & +2*I*b*\ln(c)+2*I*b*(\ln(\exp(n*\ln(x)))-n*\ln(x))+2*I*a)/b/n)*\text{Ei}(1,-1/2*\ln(d*x) \\ & +1/4*I*(b*Pi*csgn(I*\exp(n*\ln(x))))*csgn(I*c*\exp(n*\ln(x))))*csgn(I*c)-b*Pi*csgn(I*c*\exp(n*\ln(x))) \\ & ^2*csgn(I*c)-b*Pi*csgn(I*\exp(n*\ln(x)))^2*csgn(I*c)+2*I*b*n*(\ln(x)-\ln(d*x))+2*I*b*\ln(c) \\ & +2*I*b*(\ln(\exp(n*\ln(x)))-n*\ln(x))+2*I*a)/b/n \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$4bn \int \frac{1}{(b^3\sqrt{d} \log(c)^3 + b^3\sqrt{d} \log(x^n)^3 + 3ab^2\sqrt{d} \log(c)^2 + 3a^2b\sqrt{d} \log(c) + a^3\sqrt{d} + 3(b^3\sqrt{d} \log(c) + ab^2\sqrt{d}) \log(x^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(1/2)/(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

```
[Out] 4*b*n*integrate(1/((b^3*sqrt(d)*log(c)^3 + b^3*sqrt(d)*log(x^n)^3 + 3*a*b^2*sqrt(d)*log(c)^2 + 3*a^2*b*sqrt(d)*log(c) + a^3*sqrt(d) + 3*(b^3*sqrt(d)*log(c) + a*b^2*sqrt(d))*log(x^n)^2 + 3*(b^3*sqrt(d)*log(c)^2 + 2*a*b^2*sqrt(d)*log(c) + a^2*b*sqrt(d))*log(x^n))*sqrt(x)), x) + 2*sqrt(x)/(b^2*sqrt(d)*log(c)^2 + b^2*sqrt(d)*log(x^n)^2 + 2*a*b*sqrt(d)*log(c) + a^2*sqrt(d) + 2*(b^2*sqrt(d)*log(c) + a*b*sqrt(d))*log(x^n))
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}}{b^2 dx \log(cx^n)^2 + 2 ab dx \log(cx^n) + a^2 dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)/(b^2*d*x*log(c*x^n)^2 + 2*a*b*d*x*log(c*x^n) + a^2*d*x), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx} (a + b \log(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(1/2)/(a+b*ln(c*x**n))**2,x)
```

```
[Out] Integral(1/(sqrt(d*x)*(a + b*log(c*x**n))**2), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx} (b \log(cx^n) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)^(1/2)/(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x)*(b*log(c*x^n) + a)^2), x)
```

$$3.111 \quad \int \frac{1}{(dx)^{3/2}(a+b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=98

$$-\frac{e^{\frac{a}{2bn}} (cx^n)^{\frac{1}{2n}} \operatorname{Ei}\left(-\frac{a+b \log(cx^n)}{2bn}\right)}{2b^2 dn^2 \sqrt{dx}} - \frac{1}{bdn \sqrt{dx} (a + b \log(cx^n))}$$

[Out]  $-(E^{(a/(2*b*n))}*(c*x^n)^{(1/(2*n))}*\operatorname{ExpIntegralEi}[-(a + b*\operatorname{Log}[c*x^n])]/(2*b*n))/((2*b^2*d*n^2*\operatorname{Sqrt}[d*x]) - 1/(b*d*n*\operatorname{Sqrt}[d*x]*(a + b*\operatorname{Log}[c*x^n]))$

**Rubi [A]** time = 0.0925152, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$ , Rules used = {2306, 2310, 2178}

$$-\frac{e^{\frac{a}{2bn}} (cx^n)^{\frac{1}{2n}} \operatorname{Ei}\left(-\frac{a+b \log(cx^n)}{2bn}\right)}{2b^2 dn^2 \sqrt{dx}} - \frac{1}{bdn \sqrt{dx} (a + b \log(cx^n))}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[1/((d*x)^{(3/2)}*(a + b*\operatorname{Log}[c*x^n])^2), x]$

[Out]  $-(E^{(a/(2*b*n))}*(c*x^n)^{(1/(2*n))}*\operatorname{ExpIntegralEi}[-(a + b*\operatorname{Log}[c*x^n])]/(2*b*n))/((2*b^2*d*n^2*\operatorname{Sqrt}[d*x]) - 1/(b*d*n*\operatorname{Sqrt}[d*x]*(a + b*\operatorname{Log}[c*x^n]))$

#### Rule 2306

$\operatorname{Int}[(a_. + \operatorname{Log}[c_.*(x_.)^{n_.}]*b_.)^{(p_.)*((d_.)*(x_.))^{m_.}}, x\_Symbol] \rightarrow \operatorname{Simp}[(d*x)^{(m+1)}*(a + b*\operatorname{Log}[c*x^n])^{(p+1)}/(b*d*n*(p+1)), x] - \operatorname{Dist}[(m+1)/(b*n*(p+1)), \operatorname{Int}[(d*x)^m*(a + b*\operatorname{Log}[c*x^n])^{(p+1)}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m, n\}, x\} \ \&\& \ \operatorname{NeQ}[m, -1] \ \&\& \ \operatorname{LtQ}[p, -1]$

#### Rule 2310

$\operatorname{Int}[(a_. + \operatorname{Log}[c_.*(x_.)^{n_.}]*b_.)^{(p_.)*((d_.)*(x_.))^{m_.}}, x\_Symbol] \rightarrow \operatorname{Dist}[(d*x)^{(m+1)}/(d*n*(c*x^n)^{((m+1)/n)}), \operatorname{Subst}[\operatorname{Int}[E^{((m+1)*x/n)}*(a + b*x)^p, x], x, \operatorname{Log}[c*x^n]], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m, n, p\}, x\}$

#### Rule 2178

$\operatorname{Int}[(F_.)^{((g_.)*((e_.) + (f_.)*(x_.)))/((c_.) + (d_.)*(x_.))}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^{(g*(e - (c*f)/d)}*\operatorname{ExpIntegralEi}[(f*g*(c + d*x)*\operatorname{Log}[F])/d])/d, x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x\} \ \&\& \ \operatorname{!}\$UseGamma == \operatorname{True}$

#### Rubi steps

$$\begin{aligned} \int \frac{1}{(dx)^{3/2} (a + b \log(cx^n))^2} dx &= -\frac{1}{bdn\sqrt{dx} (a + b \log(cx^n))} - \frac{\int \frac{1}{(dx)^{3/2}(a+b \log(cx^n))} dx}{2bn} \\ &= -\frac{1}{bdn\sqrt{dx} (a + b \log(cx^n))} - \frac{(cx^n)^{\frac{1}{2}/n} \text{Subst}\left(\int \frac{e^{-\frac{x}{2n}}}{a+bx} dx, x, \log(cx^n)\right)}{2bdn^2\sqrt{dx}} \\ &= -\frac{e^{\frac{a}{2bn}} (cx^n)^{\frac{1}{2}/n} \text{Ei}\left(-\frac{a+b \log(cx^n)}{2bn}\right)}{2b^2dn^2\sqrt{dx}} - \frac{1}{bdn\sqrt{dx} (a + b \log(cx^n))} \end{aligned}$$

**Mathematica [A]** time = 0.120076, size = 93, normalized size = 0.95

$$\frac{x \left( e^{\frac{a}{2bn}} (cx^n)^{\frac{1}{2}/n} (a + b \log(cx^n)) \text{Ei}\left(-\frac{a+b \log(cx^n)}{2bn}\right) + 2bn \right)}{2b^2n^2(dx)^{3/2} (a + b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d\*x)^(3/2)\*(a + b\*Log[c\*x^n])^2), x]

[Out] -(x\*(2\*b\*n + E^(a/(2\*b\*n)))\*(c\*x^n)^(1/(2\*n))\*ExpIntegralEi[-(a + b\*Log[c\*x^n])/(2\*b\*n)]\*(a + b\*Log[c\*x^n]))/(2\*b^2\*n^2\*(d\*x)^(3/2)\*(a + b\*Log[c\*x^n]))

**Maple [F]** time = 4.557, size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \ln(cx^n))^2} (dx)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x)^(3/2)/(a+b\*ln(c\*x^n))^2, x)

[Out] int(1/(d\*x)^(3/2)/(a+b\*ln(c\*x^n))^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-4bn \int \frac{1}{\left(b^3d^{\frac{3}{2}} \log(c)^3 + b^3d^{\frac{3}{2}} \log(x^n)^3 + 3ab^2d^{\frac{3}{2}} \log(c)^2 + 3a^2bd^{\frac{3}{2}} \log(c) + a^3d^{\frac{3}{2}} + 3\left(b^3d^{\frac{3}{2}} \log(c) + ab^2d^{\frac{3}{2}}\right) \log(x^n)\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(3/2)/(a+b\*log(c\*x^n))^2, x, algorithm="maxima")

[Out] -4\*b\*n\*integrate(1/((b^3\*d^(3/2)\*log(c)^3 + b^3\*d^(3/2)\*log(x^n)^3 + 3\*a\*b^2\*d^(3/2)\*log(c)^2 + 3\*a^2\*b\*d^(3/2)\*log(c) + a^3\*d^(3/2) + 3\*(b^3\*d^(3/2)\*log(c) + a\*b^2\*d^(3/2))\*log(x^n)^2 + 3\*(b^3\*d^(3/2)\*log(c)^2 + 2\*a\*b^2\*d^(3/2)\*log(c) + a^2\*b\*d^(3/2))\*log(x^n))\*x^(3/2)), x) - 2/((b^2\*d^(3/2)\*log(c)^2 + b^2\*d^(3/2)\*log(x^n)^2 + 2\*a\*b\*d^(3/2)\*log(c) + a^2\*d^(3/2) + 2\*(b^2\*d

$$\sqrt[3]{d} \log(c) + a b \sqrt[3]{d} \log(x^n) \sqrt{x}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\sqrt{dx}}{b^2 d^2 x^2 \log(cx^n)^2 + 2 a b d^2 x^2 \log(cx^n) + a^2 d^2 x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(3/2)/(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] integral(sqrt(d\*x)/(b^2\*d^2\*x^2\*log(c\*x^n)^2 + 2\*a\*b\*d^2\*x^2\*log(c\*x^n) + a^2\*d^2\*x^2), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{3}{2}} (a + b \log(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)\*\*(3/2)/(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Integral(1/((d\*x)\*\*(3/2)\*(a + b\*log(c\*x\*\*n))\*\*2), x)

**Giac [B]** time = 1.31324, size = 379, normalized size = 3.87

$$\frac{\frac{1}{bc^{2n}} \sqrt{d} n \text{Ei} \left( -\frac{\log(c)}{2n} - \frac{a}{2bn} - \frac{1}{2} \log(x) \right) e^{\left( \frac{a}{2bn} \right) \log(c)} \log(x)}{b^3 d n^3 \log(x) + b^3 d n^2 \log(c) + a b^2 d n^2} + \frac{\frac{1}{bc^{2n}} \sqrt{d} \text{Ei} \left( -\frac{\log(c)}{2n} - \frac{a}{2bn} - \frac{1}{2} \log(x) \right) e^{\left( \frac{a}{2bn} \right) \log(c)}}{b^3 d n^3 \log(x) + b^3 d n^2 \log(c) + a b^2 d n^2} + \frac{\frac{1}{ac^{2n}} \sqrt{d} \text{Ei} \left( -\frac{\log(c)}{2n} - \frac{a}{2bn} - \frac{1}{2} \log(x) \right) e^{\left( \frac{a}{2bn} \right) \log(c)}}{b^3 d n^3 \log(x) + b^3 d n^2 \log(c) + a b^2 d n^2} + \frac{1}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(3/2)/(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] -1/2\*(b\*c^(1/2/n)\*sqrt(d)\*n\*Ei(-1/2\*log(c)/n - 1/2\*a/(b\*n) - 1/2\*log(x))\*e^(1/2\*a/(b\*n))\*log(x)/(b^3\*d\*n^3\*log(x) + b^3\*d\*n^2\*log(c) + a\*b^2\*d\*n^2) + b\*c^(1/2/n)\*sqrt(d)\*Ei(-1/2\*log(c)/n - 1/2\*a/(b\*n) - 1/2\*log(x))\*e^(1/2\*a/(b\*n))\*log(c)/(b^3\*d\*n^3\*log(x) + b^3\*d\*n^2\*log(c) + a\*b^2\*d\*n^2) + a\*c^(1/2/n)\*sqrt(d)\*Ei(-1/2\*log(c)/n - 1/2\*a/(b\*n) - 1/2\*log(x))\*e^(1/2\*a/(b\*n))/(b^3\*d\*n^3\*log(x) + b^3\*d\*n^2\*log(c) + a\*b^2\*d\*n^2) + 2\*b\*sqrt(d)\*n/((b^3\*d\*n^3\*log(x) + b^3\*d\*n^2\*log(c) + a\*b^2\*d\*n^2)\*sqrt(x))/d

$$3.112 \quad \int \frac{1}{(dx)^{5/2}(a+b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=98

$$-\frac{3e^{\frac{3a}{2bn}}(cx^n)^{\frac{3}{2n}} \operatorname{Ei}\left(-\frac{3(a+b \log(cx^n))}{2bn}\right)}{2b^2dn^2(dx)^{3/2}} - \frac{1}{bdn(dx)^{3/2}(a+b \log(cx^n))}$$

[Out] (-3\*E^((3\*a)/(2\*b\*n))\*(c\*x^n)^(3/(2\*n))\*ExpIntegralEi[(-3\*(a + b\*Log[c\*x^n])/(2\*b\*n))]/(2\*b^2\*d\*n^2\*(d\*x)^(3/2)) - 1/(b\*d\*n\*(d\*x)^(3/2)\*(a + b\*Log[c\*x^n]))

**Rubi [A]** time = 0.0924105, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$ , Rules used = {2306, 2310, 2178}

$$-\frac{3e^{\frac{3a}{2bn}}(cx^n)^{\frac{3}{2n}} \operatorname{Ei}\left(-\frac{3(a+b \log(cx^n))}{2bn}\right)}{2b^2dn^2(dx)^{3/2}} - \frac{1}{bdn(dx)^{3/2}(a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Int[1/((d\*x)^(5/2)\*(a + b\*Log[c\*x^n])^2), x]

[Out] (-3\*E^((3\*a)/(2\*b\*n))\*(c\*x^n)^(3/(2\*n))\*ExpIntegralEi[(-3\*(a + b\*Log[c\*x^n])/(2\*b\*n))]/(2\*b^2\*d\*n^2\*(d\*x)^(3/2)) - 1/(b\*d\*n\*(d\*x)^(3/2)\*(a + b\*Log[c\*x^n]))

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x)/n\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rubi steps

$$\int \frac{1}{(dx)^{5/2} (a + b \log(cx^n))^2} dx = -\frac{1}{bdn(dx)^{3/2} (a + b \log(cx^n))} - \frac{3 \int \frac{1}{(dx)^{5/2} (a + b \log(cx^n))} dx}{2bn}$$

$$= -\frac{1}{bdn(dx)^{3/2} (a + b \log(cx^n))} - \frac{\left(3 (cx^n)^{\frac{3}{2n}}\right) \text{Subst}\left(\int \frac{e^{-\frac{3x}{a+bx}}}{a+bx} dx, x, \log(cx^n)\right)}{2bdn^2(dx)^{3/2}}$$

$$= -\frac{3e^{\frac{3a}{2bn}} (cx^n)^{\frac{3}{2n}} \text{Ei}\left(-\frac{3(a+b \log(cx^n))}{2bn}\right)}{2b^2dn^2(dx)^{3/2}} - \frac{1}{bdn(dx)^{3/2} (a + b \log(cx^n))}$$

**Mathematica [A]** time = 0.123451, size = 94, normalized size = 0.96

$$-\frac{x \left( 3e^{\frac{3a}{2bn}} (cx^n)^{\frac{3}{2n}} (a + b \log(cx^n)) \text{Ei}\left(-\frac{3(a+b \log(cx^n))}{2bn}\right) + 2bn \right)}{2b^2n^2(dx)^{5/2} (a + b \log(cx^n))}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d\*x)^(5/2)\*(a + b\*Log[c\*x^n])^2),x]

[Out] -(x\*(2\*b\*n + 3\*E^((3\*a)/(2\*b\*n))\*(c\*x^n)^(3/(2\*n))\*ExpIntegralEi[(-3\*(a + b\*Log[c\*x^n]))/(2\*b\*n)]\*(a + b\*Log[c\*x^n]))/(2\*b^2\*n^2\*(d\*x)^(5/2)\*(a + b\*Log[c\*x^n]))

**Maple [F]** time = 4.664, size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \ln(cx^n))^2} (dx)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x)^(5/2)/(a+b\*ln(c\*x^n))^2,x)

[Out] int(1/(d\*x)^(5/2)/(a+b\*ln(c\*x^n))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-4bn \int \frac{1}{3 \left( b^3 d^{\frac{5}{2}} \log(c)^3 + b^3 d^{\frac{5}{2}} \log(x^n)^3 + 3ab^2 d^{\frac{5}{2}} \log(c)^2 + 3a^2 b d^{\frac{5}{2}} \log(c) + a^3 d^{\frac{5}{2}} + 3 \left( b^3 d^{\frac{5}{2}} \log(c) + ab^2 d^{\frac{5}{2}} \right) \log(x^n)^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(5/2)/(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] -4\*b\*n\*integrate(1/3/((b^3\*d^(5/2)\*log(c)^3 + b^3\*d^(5/2)\*log(x^n)^3 + 3\*a\*b^2\*d^(5/2)\*log(c)^2 + 3\*a^2\*b\*d^(5/2)\*log(c) + a^3\*d^(5/2) + 3\*(b^3\*d^(5/2)\*log(c) + a\*b^2\*d^(5/2))\*log(x^n)^2 + 3\*(b^3\*d^(5/2)\*log(c)^2 + 2\*a\*b^2\*d^(5/2)\*log(c) + a^2\*b\*d^(5/2))\*log(x^n))\*x^(5/2)), x) - 2/3/((b^2\*d^(5/2)\*log(c)^2 + b^2\*d^(5/2)\*log(x^n)^2 + 2\*a\*b\*d^(5/2)\*log(c) + a^2\*d^(5/2) + 2\*(b



$$\sqrt{d}^{5/2} \log(c) + a \cdot b \cdot d^{5/2} \log(x^n) \cdot x^{3/2}$$


---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}}{b^2 d^3 x^3 \log(cx^n)^2 + 2abd^3 x^3 \log(cx^n) + a^2 d^3 x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(5/2)/(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] integral(sqrt(d\*x)/(b^2\*d^3\*x^3\*log(c\*x^n)^2 + 2\*a\*b\*d^3\*x^3\*log(c\*x^n) + a^2\*d^3\*x^3), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)\*\*(5/2)/(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{5/2} (b \log(cx^n) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x)^(5/2)/(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] integrate(1/((d\*x)^(5/2)\*(b\*log(c\*x^n) + a)^2), x)

### 3.113 $\int \sqrt{a + b \log(cx^n)} dx$

**Optimal.** Leaf size=85

$$x\sqrt{a + b \log(cx^n)} - \frac{1}{2}\sqrt{\pi}\sqrt{b}\sqrt{n}xe^{-\frac{a}{bn}}(cx^n)^{-1/n} \operatorname{Erfi}\left(\frac{\sqrt{a + b \log(cx^n)}}{\sqrt{b}\sqrt{n}}\right)$$

[Out]  $-(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[n]*\operatorname{Sqrt}[\operatorname{Pi}]*x*\operatorname{Erfi}[\operatorname{Sqrt}[a + b*\operatorname{Log}[c*x^n]]]/(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[n]))/(2*E^{(a/(b*n))}*(c*x^n)^n)^{-1}) + x*\operatorname{Sqrt}[a + b*\operatorname{Log}[c*x^n]]$

**Rubi [A]** time = 0.0722577, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2296, 2300, 2180, 2204}

$$x\sqrt{a + b \log(cx^n)} - \frac{1}{2}\sqrt{\pi}\sqrt{b}\sqrt{n}xe^{-\frac{a}{bn}}(cx^n)^{-1/n} \operatorname{Erfi}\left(\frac{\sqrt{a + b \log(cx^n)}}{\sqrt{b}\sqrt{n}}\right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sqrt}[a + b*\operatorname{Log}[c*x^n]], x]$

[Out]  $-(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[n]*\operatorname{Sqrt}[\operatorname{Pi}]*x*\operatorname{Erfi}[\operatorname{Sqrt}[a + b*\operatorname{Log}[c*x^n]]]/(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[n]))/(2*E^{(a/(b*n))}*(c*x^n)^n)^{-1}) + x*\operatorname{Sqrt}[a + b*\operatorname{Log}[c*x^n]]$

#### Rule 2296

$\operatorname{Int}[(a_. + \operatorname{Log}[c_.*(x_.)^{n_.}]*b_.)^{p_.}, x\_Symbol] \rightarrow \operatorname{Simp}[x*(a + b*\operatorname{Log}[c*x^n])^p, x] - \operatorname{Dist}[b*n*p, \operatorname{Int}[(a + b*\operatorname{Log}[c*x^n])^{p-1}, x], x] /;$   
 $\operatorname{FreeQ}\{a, b, c, n, x\} \ \&\& \ \operatorname{GtQ}[p, 0] \ \&\& \ \operatorname{IntegerQ}[2*p]$

#### Rule 2300

$\operatorname{Int}[(a_. + \operatorname{Log}[c_.*(x_.)^{n_.}]*b_.)^{p_.}, x\_Symbol] \rightarrow \operatorname{Dist}[x/(n*(c*x^n)^{1/n}), \operatorname{Subst}[\operatorname{Int}[E^{(x/n)}*(a + b*x)^p, x], x, \operatorname{Log}[c*x^n]], x] /;$   
 $\operatorname{FreeQ}\{a, b, c, n, p, x\}$

#### Rule 2180

$\operatorname{Int}[(F_.)^{(g_.)*((e_.) + (f_.)*(x_.))}/\operatorname{Sqrt}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$   
 $\operatorname{FreeQ}\{F, c, d, e, f, g, x\} \ \&\& \ !\$UseGamma == True$

#### Rule 2204

$\operatorname{Int}[(F_.)^{(a_.) + (b_.)*((c_.) + (d_.)*(x_.))^2}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   
 $\operatorname{FreeQ}\{F, a, b, c, d, x\} \ \&\& \ \operatorname{PosQ}[b]$

#### Rubi steps

$$\begin{aligned}
\int \sqrt{a + b \log(cx^n)} dx &= x\sqrt{a + b \log(cx^n)} - \frac{1}{2}(bn) \int \frac{1}{\sqrt{a + b \log(cx^n)}} dx \\
&= x\sqrt{a + b \log(cx^n)} - \frac{1}{2} (bx (cx^n)^{-1/n}) \text{Subst} \left( \int \frac{e^{\frac{x}{bn}}}{\sqrt{a + bx}} dx, x, \log(cx^n) \right) \\
&= x\sqrt{a + b \log(cx^n)} - (x (cx^n)^{-1/n}) \text{Subst} \left( \int e^{-\frac{a}{bn} + \frac{x^2}{bn}} dx, x, \sqrt{a + b \log(cx^n)} \right) \\
&= -\frac{1}{2} \sqrt{be^{-\frac{a}{bn}}} \sqrt{n} \sqrt{\pi} x (cx^n)^{-1/n} \operatorname{erfi} \left( \frac{\sqrt{a + b \log(cx^n)}}{\sqrt{b} \sqrt{n}} \right) + x\sqrt{a + b \log(cx^n)}
\end{aligned}$$

**Mathematica [A]** time = 0.0376772, size = 85, normalized size = 1.

$$x\sqrt{a + b \log(cx^n)} - \frac{1}{2} \sqrt{\pi} \sqrt{b} \sqrt{n} x e^{-\frac{a}{bn}} (cx^n)^{-1/n} \operatorname{Erfi} \left( \frac{\sqrt{a + b \log(cx^n)}}{\sqrt{b} \sqrt{n}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*Log[c\*x^n]], x]

[Out] -(Sqrt[b]\*Sqrt[n]\*Sqrt[Pi]\*x\*Erfi[Sqrt[a + b\*Log[c\*x^n]]]/(Sqrt[b]\*Sqrt[n]))/(2\*E^(a/(b\*n))\*(c\*x^n)^n^(-1)) + x\*Sqrt[a + b\*Log[c\*x^n]]

**Maple [F]** time = 0.485, size = 0, normalized size = 0.

$$\int \sqrt{a + b \ln(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^n))^(1/2), x)

[Out] int((a+b\*ln(c\*x^n))^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \log(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(b\*log(c\*x^n) + a), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \log(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*log(c*x**n)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \log(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*log(c*x^n) + a), x)
```

### 3.114 $\int x^3 \sqrt{\log(ax^n)} dx$

**Optimal.** Leaf size=64

$$\frac{1}{4}x^4\sqrt{\log(ax^n)} - \frac{1}{16}\sqrt{\pi}\sqrt{n}x^4(ax^n)^{-4/n}\operatorname{Erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)$$

[Out]  $-(\operatorname{Sqrt}[n]*\operatorname{Sqrt}[\operatorname{Pi}]*x^4*\operatorname{Erfi}[(2*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/\operatorname{Sqrt}[n]])/(16*(a*x^n)^{(4/n)}) + (x^4*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/4$

**Rubi [A]** time = 0.0537024, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2305, 2310, 2180, 2204}

$$\frac{1}{4}x^4\sqrt{\log(ax^n)} - \frac{1}{16}\sqrt{\pi}\sqrt{n}x^4(ax^n)^{-4/n}\operatorname{Erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^3*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]], x]$

[Out]  $-(\operatorname{Sqrt}[n]*\operatorname{Sqrt}[\operatorname{Pi}]*x^4*\operatorname{Erfi}[(2*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/\operatorname{Sqrt}[n]])/(16*(a*x^n)^{(4/n)}) + (x^4*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/4$

#### Rule 2305

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]^{(p_.)}*((d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(d*x)^{(m+1)}*(a + b*\operatorname{Log}[c*x^n])^p/(d*(m+1)), x] - \operatorname{Dist}[(b*n*p)/(m+1), \operatorname{Int}[(d*x)^m*(a + b*\operatorname{Log}[c*x^n])^{(p-1)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \operatorname{NeQ}[m, -1] \&\& \operatorname{GtQ}[p, 0]$

#### Rule 2310

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]^{(p_.)}*((d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(d*x)^{(m+1)}/(d*n*(c*x^n)^{(m+1)/n}), \operatorname{Subst}[\operatorname{Int}[E^{((m+1)*x)/n}*(a + b*x)^p, x], x, \operatorname{Log}[c*x^n]], x] /; \operatorname{FreeQ}\{a, b, c, d, m, n, p\}, x]$

#### Rule 2180

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))}/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)], x\_Symbol] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\& !\$UseGamma == True$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2)}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x] \&\& \operatorname{PosQ}[b]$

#### Rubi steps

$$\begin{aligned}
\int x^3 \sqrt{\log(ax^n)} dx &= \frac{1}{4} x^4 \sqrt{\log(ax^n)} - \frac{1}{8} n \int \frac{x^3}{\sqrt{\log(ax^n)}} dx \\
&= \frac{1}{4} x^4 \sqrt{\log(ax^n)} - \frac{1}{8} (x^4 (ax^n)^{-4/n}) \text{Subst} \left( \int \frac{e^{\frac{4x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n) \right) \\
&= \frac{1}{4} x^4 \sqrt{\log(ax^n)} - \frac{1}{4} (x^4 (ax^n)^{-4/n}) \text{Subst} \left( \int e^{\frac{4x^2}{n}} dx, x, \sqrt{\log(ax^n)} \right) \\
&= -\frac{1}{16} \sqrt{n} \sqrt{\pi} x^4 (ax^n)^{-4/n} \operatorname{erfi} \left( \frac{2\sqrt{\log(ax^n)}}{\sqrt{n}} \right) + \frac{1}{4} x^4 \sqrt{\log(ax^n)}
\end{aligned}$$

**Mathematica [A]** time = 0.0221965, size = 61, normalized size = 0.95

$$\frac{1}{16} x^4 \left( 4\sqrt{\log(ax^n)} - \sqrt{\pi} \sqrt{n} (ax^n)^{-4/n} \operatorname{Erfi} \left( \frac{2\sqrt{\log(ax^n)}}{\sqrt{n}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Sqrt[Log[a\*x^n]],x]

[Out] (x^4\*((Sqrt[n]\*Sqrt[Pi]\*Erfi[(2\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(a\*x^n)^(4/n) + 4\*Sqrt[Log[a\*x^n]]))/16

**Maple [F]** time = 0.293, size = 0, normalized size = 0.

$$\int x^3 \sqrt{\ln(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*ln(a\*x^n)^(1/2),x)

[Out] int(x^3\*ln(a\*x^n)^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*log(a\*x^n)^(1/2),x, algorithm="maxima")

[Out] integrate(x^3\*sqrt(log(a\*x^n)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(a*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(a*x**n)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(log(a*x**n)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(a*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*sqrt(log(a*x^n)), x)
```

### 3.115 $\int x^2 \sqrt{\log(ax^n)} dx$

**Optimal.** Leaf size=72

$$\frac{1}{3}x^3\sqrt{\log(ax^n)} - \frac{1}{6}\sqrt{\frac{\pi}{3}}\sqrt{n}x^3(ax^n)^{-3/n}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)$$

[Out]  $-(\operatorname{Sqrt}[n]*\operatorname{Sqrt}[\pi/3]*x^3*\operatorname{Erfi}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/\operatorname{Sqrt}[n]])/(6*(a*x^n)^{(3/n)}) + (x^3*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/3$

**Rubi [A]** time = 0.0649955, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2305, 2310, 2180, 2204}

$$\frac{1}{3}x^3\sqrt{\log(ax^n)} - \frac{1}{6}\sqrt{\frac{\pi}{3}}\sqrt{n}x^3(ax^n)^{-3/n}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]], x]$

[Out]  $-(\operatorname{Sqrt}[n]*\operatorname{Sqrt}[\pi/3]*x^3*\operatorname{Erfi}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/\operatorname{Sqrt}[n]])/(6*(a*x^n)^{(3/n)}) + (x^3*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/3$

#### Rule 2305

$\operatorname{Int}[(a_. + \operatorname{Log}[c_.*(x_.)^{(n_.)}]*(b_.))^{(p_.)}*((d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(d*x)^{(m+1)}*(a + b*\operatorname{Log}[c*x^n])^p/(d*(m+1)), x] - \operatorname{Dist}[(b*n*p)/(m+1), \operatorname{Int}[(d*x)^m*(a + b*\operatorname{Log}[c*x^n])^{(p-1)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \operatorname{NeQ}[m, -1] \&\& \operatorname{GtQ}[p, 0]$

#### Rule 2310

$\operatorname{Int}[(a_. + \operatorname{Log}[c_.*(x_.)^{(n_.)}]*(b_.))^{(p_.)}*((d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(d*x)^{(m+1)}/(d*n*(c*x^n)^{((m+1)/n)}), \operatorname{Subst}[\operatorname{Int}[E^{((m+1)*x/n)}*(a + b*x)^p, x], x, \operatorname{Log}[c*x^n]], x] /; \operatorname{FreeQ}\{a, b, c, d, m, n, p\}, x]$

#### Rule 2180

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))}/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)], x\_Symbol] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\& !\$UseGamma == True$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2)}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\pi]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x] \&\& \operatorname{PosQ}[b]$

#### Rubi steps



$$\begin{aligned}
\int x^2 \sqrt{\log(ax^n)} dx &= \frac{1}{3} x^3 \sqrt{\log(ax^n)} - \frac{1}{6} n \int \frac{x^2}{\sqrt{\log(ax^n)}} dx \\
&= \frac{1}{3} x^3 \sqrt{\log(ax^n)} - \frac{1}{6} (x^3 (ax^n)^{-3/n}) \text{Subst} \left( \int \frac{e^{\frac{3x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n) \right) \\
&= \frac{1}{3} x^3 \sqrt{\log(ax^n)} - \frac{1}{6} (x^3 (ax^n)^{-3/n}) \text{Subst} \left( \int e^{\frac{3x^2}{n}} dx, x, \sqrt{\log(ax^n)} \right) \\
&= -\frac{1}{6} \sqrt{n} \sqrt{\frac{\pi}{3}} x^3 (ax^n)^{-3/n} \operatorname{erfi} \left( \frac{\sqrt{3} \sqrt{\log(ax^n)}}{\sqrt{n}} \right) + \frac{1}{3} x^3 \sqrt{\log(ax^n)}
\end{aligned}$$

**Mathematica [A]** time = 0.0326051, size = 67, normalized size = 0.93

$$\frac{1}{18} x^3 \left( 6 \sqrt{\log(ax^n)} - \sqrt{3\pi} \sqrt{n} (ax^n)^{-3/n} \operatorname{Erfi} \left( \frac{\sqrt{3} \sqrt{\log(ax^n)}}{\sqrt{n}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Sqrt[Log[a\*x^n]],x]

[Out] (x^3\*(-((Sqrt[n]\*Sqrt[3\*Pi]\*Erfi[(Sqrt[3]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(a\*x^n)^(3/n)) + 6\*Sqrt[Log[a\*x^n]]))/18

**Maple [F]** time = 0.174, size = 0, normalized size = 0.

$$\int x^2 \sqrt{\ln(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*ln(a\*x^n)^(1/2),x)

[Out] int(x^2\*ln(a\*x^n)^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*log(a\*x^n)^(1/2),x, algorithm="maxima")

[Out] integrate(x^2\*sqrt(log(a\*x^n)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(a*x**n)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(log(a*x**n)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*sqrt(log(a*x^n)), x)
```

### 3.116 $\int x\sqrt{\log(ax^n)} dx$

**Optimal.** Leaf size=72

$$\frac{1}{2}x^2\sqrt{\log(ax^n)} - \frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{nx^2(ax^n)^{-2/n}} \operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)$$

[Out]  $-(\operatorname{Sqrt}[n]*\operatorname{Sqrt}[\pi/2]*x^2*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/\operatorname{Sqrt}[n]])/(4*(a*x^n)^{(2/n)}) + (x^2*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/2$

**Rubi [A]** time = 0.0539074, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2305, 2310, 2180, 2204}

$$\frac{1}{2}x^2\sqrt{\log(ax^n)} - \frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{nx^2(ax^n)^{-2/n}} \operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]], x]$

[Out]  $-(\operatorname{Sqrt}[n]*\operatorname{Sqrt}[\pi/2]*x^2*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/\operatorname{Sqrt}[n]])/(4*(a*x^n)^{(2/n)}) + (x^2*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/2$

#### Rule 2305

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]^{(p_.)}*((d_.)*(x_.))^{(m_.)}, x\_Symbol] :> \operatorname{Simp}[(d*x)^{(m+1)}*(a + b*\operatorname{Log}[c*x^n])^p/(d*(m+1)), x] - \operatorname{Dist}[(b*n*p)/(m+1), \operatorname{Int}[(d*x)^m*(a + b*\operatorname{Log}[c*x^n])^{(p-1)}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m, n\}, x$  &&  $\operatorname{NeQ}[m, -1]$  &&  $\operatorname{GtQ}[p, 0]$

#### Rule 2310

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]^{(p_.)}*((d_.)*(x_.))^{(m_.)}, x\_Symbol] :> \operatorname{Dist}[(d*x)^{(m+1)}/(d*n*(c*x^n)^{(m+1)/n}), \operatorname{Subst}[\operatorname{Int}[E^{((m+1)*x)/n}*(a + b*x)^p, x], x, \operatorname{Log}[c*x^n], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m, n, p\}, x$

#### Rule 2180

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_.)))}/\operatorname{Sqrt}[(c_.) + (d_.)*(x_.)], x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x$  &&  $! \$\operatorname{UseGamma} == \operatorname{True}$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^2)}, x\_Symbol] :> \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\pi]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x$  &&  $\operatorname{PosQ}[b]$

#### Rubi steps

$$\begin{aligned}
\int x\sqrt{\log(ax^n)} dx &= \frac{1}{2}x^2\sqrt{\log(ax^n)} - \frac{1}{4}n \int \frac{x}{\sqrt{\log(ax^n)}} dx \\
&= \frac{1}{2}x^2\sqrt{\log(ax^n)} - \frac{1}{4} \left(x^2 (ax^n)^{-2/n}\right) \text{Subst} \left( \int \frac{e^{\frac{2x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n) \right) \\
&= \frac{1}{2}x^2\sqrt{\log(ax^n)} - \frac{1}{2} \left(x^2 (ax^n)^{-2/n}\right) \text{Subst} \left( \int e^{\frac{2x^2}{n}} dx, x, \sqrt{\log(ax^n)} \right) \\
&= -\frac{1}{4}\sqrt{n}\sqrt{\frac{\pi}{2}}x^2(ax^n)^{-2/n} \operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right) + \frac{1}{2}x^2\sqrt{\log(ax^n)}
\end{aligned}$$

**Mathematica [A]** time = 0.031515, size = 67, normalized size = 0.93

$$\frac{1}{8}x^2 \left( 4\sqrt{\log(ax^n)} - \sqrt{2\pi}\sqrt{n} (ax^n)^{-2/n} \operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x\*Sqrt[Log[a\*x^n]], x]

[Out] (x^2\*((Sqrt[n]\*Sqrt[2\*Pi]\*Erfi[(Sqrt[2]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(a\*x^n)^(2/n)) + 4\*Sqrt[Log[a\*x^n]])/8

**Maple [F]** time = 0.177, size = 0, normalized size = 0.

$$\int x\sqrt{\ln(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*ln(a\*x^n)^(1/2), x)

[Out] int(x\*ln(a\*x^n)^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*log(a\*x^n)^(1/2), x, algorithm="maxima")

[Out] integrate(x\*sqrt(log(a\*x^n)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(a*x**n)**(1/2),x)
```

```
[Out] Integral(x*sqrt(log(a*x**n)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(log(a*x^n)), x)
```

### 3.117 $\int \sqrt{\log(ax^n)} dx$

**Optimal.** Leaf size=56

$$x\sqrt{\log(ax^n)} - \frac{1}{2}\sqrt{\pi}\sqrt{n}x(ax^n)^{-1/n} \operatorname{Erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)$$

[Out]  $-(\operatorname{Sqrt}[n]*\operatorname{Sqrt}[\operatorname{Pi}]*x*\operatorname{Erfi}[\operatorname{Sqrt}[\operatorname{Log}[a*x^n]]/\operatorname{Sqrt}[n]])/(2*(a*x^n)^{n^{-1}}) + x*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]]$

**Rubi [A]** time = 0.0284009, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {2296, 2300, 2180, 2204}

$$x\sqrt{\log(ax^n)} - \frac{1}{2}\sqrt{\pi}\sqrt{n}x(ax^n)^{-1/n} \operatorname{Erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sqrt}[\operatorname{Log}[a*x^n]], x]$

[Out]  $-(\operatorname{Sqrt}[n]*\operatorname{Sqrt}[\operatorname{Pi}]*x*\operatorname{Erfi}[\operatorname{Sqrt}[\operatorname{Log}[a*x^n]]/\operatorname{Sqrt}[n]])/(2*(a*x^n)^{n^{-1}}) + x*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]]$

#### Rule 2296

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.)*(x_)^{(n_.)}]*(b_.)]^{(p_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[x*(a + b*\operatorname{Log}[c*x^n])^p, x] - \operatorname{Dist}[b*n*p, \operatorname{Int}[(a + b*\operatorname{Log}[c*x^n])^{(p-1)}, x], x] /;$   
 $\operatorname{FreeQ}\{a, b, c, n\}, x \ \&\& \operatorname{GtQ}[p, 0] \ \&\& \operatorname{IntegerQ}[2*p]$

#### Rule 2300

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.)*(x_)^{(n_.)}]*(b_.)]^{(p_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[x/(n*(c*x^n)^{(1/n)}), \operatorname{Subst}[\operatorname{Int}[E^{(x/n)}*(a + b*x)^p, x], x, \operatorname{Log}[c*x^n]], x] /;$   
 $\operatorname{FreeQ}\{a, b, c, n, p\}, x]$

#### Rule 2180

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)]}, x\_Symbol] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$   
 $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x \ \&\& \operatorname{!}\$UseGamma == True$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2)}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   
 $\operatorname{FreeQ}\{F, a, b, c, d\}, x \ \&\& \operatorname{PosQ}[b]$

#### Rubi steps

$$\begin{aligned}
\int \sqrt{\log(ax^n)} dx &= x\sqrt{\log(ax^n)} - \frac{1}{2}n \int \frac{1}{\sqrt{\log(ax^n)}} dx \\
&= x\sqrt{\log(ax^n)} - \frac{1}{2} \left( x(ax^n)^{-1/n} \right) \text{Subst} \left( \int \frac{e^{\frac{x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n) \right) \\
&= x\sqrt{\log(ax^n)} - \left( x(ax^n)^{-1/n} \right) \text{Subst} \left( \int e^{\frac{x^2}{n}} dx, x, \sqrt{\log(ax^n)} \right) \\
&= -\frac{1}{2}\sqrt{n}\sqrt{\pi}x(ax^n)^{-1/n} \operatorname{erfi} \left( \frac{\sqrt{\log(ax^n)}}{\sqrt{n}} \right) + x\sqrt{\log(ax^n)}
\end{aligned}$$

**Mathematica [A]** time = 0.0157558, size = 56, normalized size = 1.

$$x\sqrt{\log(ax^n)} - \frac{1}{2}\sqrt{\pi}\sqrt{nx}(ax^n)^{-1/n} \operatorname{Erfi} \left( \frac{\sqrt{\log(ax^n)}}{\sqrt{n}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Log[a\*x^n]], x]

[Out] -(Sqrt[n]\*Sqrt[Pi]\*x\*Erfi[Sqrt[Log[a\*x^n]]/Sqrt[n]])/(2\*(a\*x^n)^n^(-1)) + x\*Sqrt[Log[a\*x^n]]

**Maple [F]** time = 0.181, size = 0, normalized size = 0.

$$\int \sqrt{\ln(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(a\*x^n)^(1/2), x)

[Out] int(ln(a\*x^n)^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(log(a\*x^n)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(log(a*x**n)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(log(a*x^n)), x)
```



$$3.118 \quad \int \frac{\sqrt{\log(ax^n)}}{x} dx$$

**Optimal.** Leaf size=17

$$\frac{2 \log^{\frac{3}{2}}(ax^n)}{3n}$$

[Out] (2\*Log[a\*x^n]^(3/2))/(3\*n)

**Rubi [A]** time = 0.0135638, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2302, 30}

$$\frac{2 \log^{\frac{3}{2}}(ax^n)}{3n}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Log[a\*x^n]]/x,x]

[Out] (2\*Log[a\*x^n]^(3/2))/(3\*n)

**Rule 2302**

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] := Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

**Rule 30**

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

**Rubi steps**

$$\begin{aligned} \int \frac{\sqrt{\log(ax^n)}}{x} dx &= \frac{\text{Subst}\left(\int \sqrt{x} dx, x, \log(ax^n)\right)}{n} \\ &= \frac{2 \log^{\frac{3}{2}}(ax^n)}{3n} \end{aligned}$$

**Mathematica [A]** time = 0.0015515, size = 17, normalized size = 1.

$$\frac{2 \log^{\frac{3}{2}}(ax^n)}{3n}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Log[a\*x^n]]/x,x]

[Out] (2\*Log[a\*x^n]^(3/2))/(3\*n)

---

**Maple [A]** time = 0.038, size = 14, normalized size = 0.8

$$\frac{2}{3n} (\ln(ax^n))^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(a*x^n)^(1/2)/x,x)`

[Out] `2/3*ln(a*x^n)^(3/2)/n`

---

**Maxima [A]** time = 1.0185, size = 18, normalized size = 1.06

$$\frac{2 \log(ax^n)^{\frac{3}{2}}}{3n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(a*x^n)^(1/2)/x,x, algorithm="maxima")`

[Out] `2/3*log(a*x^n)^(3/2)/n`

---

**Fricas [A]** time = 1.02915, size = 45, normalized size = 2.65

$$\frac{2(n \log(x) + \log(a))^{\frac{3}{2}}}{3n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(a*x^n)^(1/2)/x,x, algorithm="fricas")`

[Out] `2/3*(n*log(x) + log(a))^(3/2)/n`

---

**Sympy [A]** time = 1.92429, size = 29, normalized size = 1.71

$$-\begin{cases} \sqrt{\log(a)} \log(x) & \text{for } n = 0 \\ \frac{2 \log(ax^n)^{\frac{3}{2}}}{3n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(a*x**n)**(1/2)/x,x)`

[Out] `-Piecewise((-sqrt(log(a))*log(x), Eq(n, 0)), (-2*log(a*x**n)**(3/2)/(3*n), True))`

---

**Giac [A]** time = 1.23235, size = 19, normalized size = 1.12

$$\frac{2(n \log(x) + \log(a))^{\frac{3}{2}}}{3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*x^n)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2/3*(n*log(x) + log(a))^(3/2)/n
```

$$3.119 \quad \int \frac{\sqrt{\log(ax^n)}}{x^2} dx$$

**Optimal.** Leaf size=59

$$\frac{\sqrt{\pi}\sqrt{n}(ax^n)^{\frac{1}{n}} \operatorname{Erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2x} - \frac{\sqrt{\log(ax^n)}}{x}$$

[Out] (Sqrt[n]\*Sqrt[Pi]\*(a\*x^n)^n^(-1)\*Erf[Sqrt[Log[a\*x^n]]/Sqrt[n]])/(2\*x) - Sqrt[Log[a\*x^n]]/x

**Rubi [A]** time = 0.0502454, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2305, 2310, 2180, 2205}

$$\frac{\sqrt{\pi}\sqrt{n}(ax^n)^{\frac{1}{n}} \operatorname{Erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2x} - \frac{\sqrt{\log(ax^n)}}{x}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Log[a\*x^n]]/x^2,x]

[Out] (Sqrt[n]\*Sqrt[Pi]\*(a\*x^n)^n^(-1)\*Erf[Sqrt[Log[a\*x^n]]/Sqrt[n]])/(2\*x) - Sqrt[Log[a\*x^n]]/x

#### Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

#### Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol]
:> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

#### Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

#### Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\log(ax^n)}}{x^2} dx &= -\frac{\sqrt{\log(ax^n)}}{x} + \frac{1}{2}n \int \frac{1}{x^2 \sqrt{\log(ax^n)}} dx \\
&= -\frac{\sqrt{\log(ax^n)}}{x} + \frac{(ax^n)^{\frac{1}{n}} \operatorname{Subst}\left(\int \frac{e^{-\frac{x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{2x} \\
&= -\frac{\sqrt{\log(ax^n)}}{x} + \frac{(ax^n)^{\frac{1}{n}} \operatorname{Subst}\left(\int e^{-\frac{x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{x} \\
&= \frac{\sqrt{n}\sqrt{\pi} (ax^n)^{\frac{1}{n}} \operatorname{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2x} - \frac{\sqrt{\log(ax^n)}}{x}
\end{aligned}$$

**Mathematica [A]** time = 0.0471729, size = 65, normalized size = 1.1

$$-\frac{n (ax^n)^{\frac{1}{n}} \sqrt{\frac{\log(ax^n)}{n}} \operatorname{Gamma}\left(\frac{1}{2}, \frac{\log(ax^n)}{n}\right) + 2 \log(ax^n)}{2x \sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Log[a\*x^n]]/x^2,x]

[Out]  $-(2*\operatorname{Log}[a*x^n] + n*(a*x^n)^n^{(-1)}*\operatorname{Gamma}[1/2, \operatorname{Log}[a*x^n]/n]*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]/n])/(2*x*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])$

**Maple [F]** time = 0.178, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} \sqrt{\ln(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(a\*x^n)^(1/2)/x^2,x)

[Out] int(ln(a\*x^n)^(1/2)/x^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\log(ax^n)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(1/2)/x^2,x, algorithm="maxima")

[Out] integrate(sqrt(log(a\*x^n))/x^2, x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*x^n)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\log(ax^n)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*x**n)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(log(a*x**n))/x**2, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\log(ax^n)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*x^n)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(log(a*x^n))/x^2, x)
```

$$3.120 \quad \int \frac{\sqrt{\log(ax^n)}}{x^3} dx$$

**Optimal.** Leaf size=72

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{n} (ax^n)^{2/n} \operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{4x^2} - \frac{\sqrt{\log(ax^n)}}{2x^2}$$

[Out] (Sqrt[n]\*Sqrt[Pi/2]\*(a\*x^n)^(2/n)\*Erf[(Sqrt[2]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(4\*x^2) - Sqrt[Log[a\*x^n]]/(2\*x^2)

**Rubi [A]** time = 0.0559361, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2305, 2310, 2180, 2205}

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{n} (ax^n)^{2/n} \operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{4x^2} - \frac{\sqrt{\log(ax^n)}}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Log[a\*x^n]]/x^3,x]

[Out] (Sqrt[n]\*Sqrt[Pi/2]\*(a\*x^n)^(2/n)\*Erf[(Sqrt[2]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(4\*x^2) - Sqrt[Log[a\*x^n]]/(2\*x^2)

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*m\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

#### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\log(ax^n)}}{x^3} dx &= -\frac{\sqrt{\log(ax^n)}}{2x^2} + \frac{1}{4}n \int \frac{1}{x^3 \sqrt{\log(ax^n)}} dx \\
&= -\frac{\sqrt{\log(ax^n)}}{2x^2} + \frac{(ax^n)^{2/n} \operatorname{Subst}\left(\int \frac{e^{-\frac{2x}{n}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{4x^2} \\
&= -\frac{\sqrt{\log(ax^n)}}{2x^2} + \frac{(ax^n)^{2/n} \operatorname{Subst}\left(\int e^{-\frac{2x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{2x^2} \\
&= \frac{\sqrt{n} \sqrt{\frac{\pi}{2}} (ax^n)^{2/n} \operatorname{erf}\left(\frac{\sqrt{2} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{4x^2} - \frac{\sqrt{\log(ax^n)}}{2x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0476429, size = 73, normalized size = 1.01

$$\frac{\sqrt{2n} (ax^n)^{2/n} \sqrt{\frac{\log(ax^n)}{n}} \operatorname{Gamma}\left(\frac{1}{2}, \frac{2 \log(ax^n)}{n}\right) + 4 \log(ax^n)}{8x^2 \sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Log[a\*x^n]]/x^3,x]

[Out]  $-(4 \operatorname{Log}[a*x^n] + \operatorname{Sqrt}[2]*n*(a*x^n)^{(2/n)}*\operatorname{Gamma}[1/2, (2*\operatorname{Log}[a*x^n])/n]*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]/n])/(8*x^2*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])$

**Maple [F]** time = 0.184, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} \sqrt{\ln(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(a\*x^n)^(1/2)/x^3,x)

[Out] int(ln(a\*x^n)^(1/2)/x^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\log(ax^n)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(1/2)/x^3,x, algorithm="maxima")

[Out] integrate(sqrt(log(a\*x^n))/x^3, x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*x^n)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\log(ax^n)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*x**n)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(log(a*x**n))/x**3, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\log(ax^n)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*x^n)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(log(a*x^n))/x^3, x)
```

### 3.121 $\int x^3 \log^{\frac{3}{2}}(ax^n) dx$

**Optimal.** Leaf size=82

$$\frac{3}{128} \sqrt{\pi} n^{3/2} x^4 (ax^n)^{-4/n} \operatorname{Erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right) + \frac{1}{4} x^4 \log^{\frac{3}{2}}(ax^n) - \frac{3}{32} n x^4 \sqrt{\log(ax^n)}$$

[Out]  $(3*n^{(3/2)}*\operatorname{Sqrt}[\operatorname{Pi}]*x^4*\operatorname{Erfi}[(2*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/\operatorname{Sqrt}[n]])/(128*(a*x^n)^{(4/n)}) - (3*n*x^4*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/32 + (x^4*\operatorname{Log}[a*x^n]^{(3/2)})/4$

**Rubi [A]** time = 0.0675935, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2305, 2310, 2180, 2204}

$$\frac{3}{128} \sqrt{\pi} n^{3/2} x^4 (ax^n)^{-4/n} \operatorname{Erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right) + \frac{1}{4} x^4 \log^{\frac{3}{2}}(ax^n) - \frac{3}{32} n x^4 \sqrt{\log(ax^n)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^3*\operatorname{Log}[a*x^n]^{(3/2)}, x]$

[Out]  $(3*n^{(3/2)}*\operatorname{Sqrt}[\operatorname{Pi}]*x^4*\operatorname{Erfi}[(2*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/\operatorname{Sqrt}[n]])/(128*(a*x^n)^{(4/n)}) - (3*n*x^4*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/32 + (x^4*\operatorname{Log}[a*x^n]^{(3/2)})/4$

#### Rule 2305

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]^{(p_.)}*((d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(d*x)^{(m+1)}*(a + b*\operatorname{Log}[c*x^n])^p/(d*(m+1)), x] - \operatorname{Dist}[(b*n*p)/(m+1), \operatorname{Int}[(d*x)^m*(a + b*\operatorname{Log}[c*x^n])^{(p-1)}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m, n\}, x \ \&\& \operatorname{NeQ}[m, -1] \ \&\& \operatorname{GtQ}[p, 0]$

#### Rule 2310

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]^{(p_.)}*((d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(d*x)^{(m+1)}/(d*n*(c*x^n)^{((m+1)/n)}), \operatorname{Subst}[\operatorname{Int}[E^{((m+1)*x/n)}*(a + b*x)^p, x], x, \operatorname{Log}[c*x^n]], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m, n, p\}, x]$

#### Rule 2180

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_.)))/\operatorname{Sqrt}[(c_.) + (d_.)*(x_.)]}, x\_Symbol] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x \ \&\& \operatorname{!}\$UseGamma == \operatorname{True}$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^2)}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x \ \&\& \operatorname{PosQ}[b]$

#### Rubi steps

$$\begin{aligned}
\int x^3 \log^{\frac{3}{2}}(ax^n) dx &= \frac{1}{4}x^4 \log^{\frac{3}{2}}(ax^n) - \frac{1}{8}(3n) \int x^3 \sqrt{\log(ax^n)} dx \\
&= -\frac{3}{32}nx^4 \sqrt{\log(ax^n)} + \frac{1}{4}x^4 \log^{\frac{3}{2}}(ax^n) + \frac{1}{64}(3n^2) \int \frac{x^3}{\sqrt{\log(ax^n)}} dx \\
&= -\frac{3}{32}nx^4 \sqrt{\log(ax^n)} + \frac{1}{4}x^4 \log^{\frac{3}{2}}(ax^n) + \frac{1}{64}(3nx^4 (ax^n)^{-4/n}) \text{Subst} \left( \int \frac{e^{\frac{4x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n) \right) \\
&= -\frac{3}{32}nx^4 \sqrt{\log(ax^n)} + \frac{1}{4}x^4 \log^{\frac{3}{2}}(ax^n) + \frac{1}{32}(3nx^4 (ax^n)^{-4/n}) \text{Subst} \left( \int e^{\frac{4x^2}{n}} dx, x, \sqrt{\log(ax^n)} \right) \\
&= \frac{3}{128}n^{3/2} \sqrt{\pi} x^4 (ax^n)^{-4/n} \operatorname{erfi} \left( \frac{2\sqrt{\log(ax^n)}}{\sqrt{n}} \right) - \frac{3}{32}nx^4 \sqrt{\log(ax^n)} + \frac{1}{4}x^4 \log^{\frac{3}{2}}(ax^n)
\end{aligned}$$

**Mathematica [A]** time = 0.0428653, size = 73, normalized size = 0.89

$$\frac{1}{128}x^4 \left( 3\sqrt{\pi}n^{3/2} (ax^n)^{-4/n} \operatorname{Erfi} \left( \frac{2\sqrt{\log(ax^n)}}{\sqrt{n}} \right) + 4\sqrt{\log(ax^n)} (8 \log(ax^n) - 3n) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Log[a\*x^n]^(3/2),x]

[Out] (x^4\*((3\*n^(3/2)\*Sqrt[Pi]\*Erfi[(2\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(a\*x^n)^(4/n) + 4\*Sqrt[Log[a\*x^n]]\*(-3\*n + 8\*Log[a\*x^n]))) / 128

**Maple [F]** time = 0.166, size = 0, normalized size = 0.

$$\int x^3 (\ln(ax^n))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*ln(a\*x^n)^(3/2),x)

[Out] int(x^3\*ln(a\*x^n)^(3/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*log(a\*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate(x^3\*log(a\*x^n)^(3/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*log(a*x^n)^(3/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*ln(a*x**n)**(3/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*log(a*x^n)^(3/2),x, algorithm="giac")`

[Out] `integrate(x^3*log(a*x^n)^(3/2), x)`

### 3.122 $\int x^2 \log^{\frac{3}{2}}(ax^n) dx$

**Optimal.** Leaf size=90

$$\frac{1}{12} \sqrt{\frac{\pi}{3}} n^{3/2} x^3 (ax^n)^{-3/n} \operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right) + \frac{1}{3} x^3 \log^{\frac{3}{2}}(ax^n) - \frac{1}{6} n x^3 \sqrt{\log(ax^n)}$$

[Out]  $(n^{3/2} \sqrt{\pi/3} x^3 \operatorname{Erfi}[\sqrt{3} \sqrt{\log[ax^n]}] / \sqrt{n}) / (12 (ax^n)^{3/n}) - (n x^3 \sqrt{\log[ax^n]} / 6) + (x^3 \log[ax^n]^{3/2}) / 3$

**Rubi [A]** time = 0.0738034, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2305, 2310, 2180, 2204}

$$\frac{1}{12} \sqrt{\frac{\pi}{3}} n^{3/2} x^3 (ax^n)^{-3/n} \operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right) + \frac{1}{3} x^3 \log^{\frac{3}{2}}(ax^n) - \frac{1}{6} n x^3 \sqrt{\log(ax^n)}$$

Antiderivative was successfully verified.

[In]  $\int x^2 \log[ax^n]^{3/2}, x$

[Out]  $(n^{3/2} \sqrt{\pi/3} x^3 \operatorname{Erfi}[\sqrt{3} \sqrt{\log[ax^n]}] / \sqrt{n}) / (12 (ax^n)^{3/n}) - (n x^3 \sqrt{\log[ax^n]} / 6) + (x^3 \log[ax^n]^{3/2}) / 3$

#### Rule 2305

$\operatorname{Int}[(a + \log[(c)(x)^n] (b))^p (d)(x)^m, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(dx)^{m+1} (a + b \log[cx^n])^p / (d(m+1)), x] - \operatorname{Dist}[(b * p) / (m+1), \operatorname{Int}[(dx)^m (a + b \log[cx^n])^{p-1}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m, n\}, x$  &&  $\operatorname{NeQ}[m, -1]$  &&  $\operatorname{GtQ}[p, 0]$

#### Rule 2310

$\operatorname{Int}[(a + \log[(c)(x)^n] (b))^p (d)(x)^m, x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[(dx)^{m+1} / (d * n * (cx^n)^{(m+1)/n}), \operatorname{Subst}[\operatorname{Int}[E^{((m+1)x)/n} (a + bx)^p, x], x, \log[cx^n]], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m, n, p\}, x$

#### Rule 2180

$\operatorname{Int}[(F)^{(g)(e + f)(x)} / \sqrt{(c) + (d)(x)}, x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{g(e - (cf)/d) + (fgx^2)/d}, x], x, \sqrt{c + dx}], x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x$  &&  $! \$\operatorname{UseGamma} == \operatorname{True}$

#### Rule 2204

$\operatorname{Int}[(F)^{(a) + (b)((c) + (d)(x))^2}, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(F^a \sqrt{\pi} \operatorname{Erfi}[(c + dx) \operatorname{Rt}[b \log[F], 2]] / (2 * d \operatorname{Rt}[b \log[F], 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x$  &&  $\operatorname{PosQ}[b]$

#### Rubi steps

$$\begin{aligned}
\int x^2 \log^{\frac{3}{2}}(ax^n) dx &= \frac{1}{3} x^3 \log^{\frac{3}{2}}(ax^n) - \frac{1}{2} n \int x^2 \sqrt{\log(ax^n)} dx \\
&= -\frac{1}{6} n x^3 \sqrt{\log(ax^n)} + \frac{1}{3} x^3 \log^{\frac{3}{2}}(ax^n) + \frac{1}{12} n^2 \int \frac{x^2}{\sqrt{\log(ax^n)}} dx \\
&= -\frac{1}{6} n x^3 \sqrt{\log(ax^n)} + \frac{1}{3} x^3 \log^{\frac{3}{2}}(ax^n) + \frac{1}{12} (n x^3 (ax^n)^{-3/n}) \text{Subst} \left( \int \frac{e^{\frac{3x}{n}}}{\sqrt{x}} dx, x, \log(ax^n) \right) \\
&= -\frac{1}{6} n x^3 \sqrt{\log(ax^n)} + \frac{1}{3} x^3 \log^{\frac{3}{2}}(ax^n) + \frac{1}{6} (n x^3 (ax^n)^{-3/n}) \text{Subst} \left( \int e^{\frac{3x^2}{n}} dx, x, \sqrt{\log(ax^n)} \right) \\
&= \frac{1}{12} n^{3/2} \sqrt{\frac{\pi}{3}} x^3 (ax^n)^{-3/n} \operatorname{erfi} \left( \frac{\sqrt{3} \sqrt{\log(ax^n)}}{\sqrt{n}} \right) - \frac{1}{6} n x^3 \sqrt{\log(ax^n)} + \frac{1}{3} x^3 \log^{\frac{3}{2}}(ax^n)
\end{aligned}$$

**Mathematica [A]** time = 0.0509563, size = 76, normalized size = 0.84

$$\frac{1}{36} x^3 \left( \sqrt{3\pi} n^{3/2} (ax^n)^{-3/n} \operatorname{Erfi} \left( \frac{\sqrt{3} \sqrt{\log(ax^n)}}{\sqrt{n}} \right) - 6(n - 2 \log(ax^n)) \sqrt{\log(ax^n)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Log[a\*x^n]^(3/2),x]

[Out] (x^3\*((n^(3/2)\*Sqrt[3\*Pi]\*Erfi[(Sqrt[3]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(a\*x^n)^(3/n) - 6\*(n - 2\*Log[a\*x^n])\*Sqrt[Log[a\*x^n]]))/36

**Maple [F]** time = 0.171, size = 0, normalized size = 0.

$$\int x^2 (\ln(ax^n))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*ln(a\*x^n)^(3/2),x)

[Out] int(x^2\*ln(a\*x^n)^(3/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*log(a\*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate(x^2\*log(a\*x^n)^(3/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(a*x**n)**(3/2),x)
```

```
[Out] Integral(x**2*log(a*x**n)**(3/2), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*log(a*x^n)^(3/2), x)
```

### 3.123 $\int x \log^{\frac{3}{2}}(ax^n) dx$

**Optimal.** Leaf size=90

$$\frac{3}{16} \sqrt{\frac{\pi}{2}} n^{3/2} x^2 (ax^n)^{-2/n} \operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right) + \frac{1}{2} x^2 \log^{\frac{3}{2}}(ax^n) - \frac{3}{8} n x^2 \sqrt{\log(ax^n)}$$

[Out]  $(3*n^{(3/2)}*\operatorname{Sqrt}[Pi/2]*x^2*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/\operatorname{Sqrt}[n]])/(16*(a*x^n)^{(2/n)}) - (3*n*x^2*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/8 + (x^2*\operatorname{Log}[a*x^n]^{(3/2)})/2$

**Rubi [A]** time = 0.0565114, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2305, 2310, 2180, 2204}

$$\frac{3}{16} \sqrt{\frac{\pi}{2}} n^{3/2} x^2 (ax^n)^{-2/n} \operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right) + \frac{1}{2} x^2 \log^{\frac{3}{2}}(ax^n) - \frac{3}{8} n x^2 \sqrt{\log(ax^n)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x*\operatorname{Log}[a*x^n]^{(3/2)}, x]$

[Out]  $(3*n^{(3/2)}*\operatorname{Sqrt}[Pi/2]*x^2*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/\operatorname{Sqrt}[n]])/(16*(a*x^n)^{(2/n)}) - (3*n*x^2*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/8 + (x^2*\operatorname{Log}[a*x^n]^{(3/2)})/2$

#### Rule 2305

$\operatorname{Int}[(a_. + \operatorname{Log}[c_.*(x_.)^{(n_.)}]*(b_.))^{(p_.)}*((d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(d*x)^{(m+1)}*(a + b*\operatorname{Log}[c*x^n])^p/(d*(m+1)), x] - \operatorname{Dist}[(b*n*p)/(m+1), \operatorname{Int}[(d*x)^m*(a + b*\operatorname{Log}[c*x^n])^{(p-1)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \operatorname{NeQ}[m, -1] \&\& \operatorname{GtQ}[p, 0]$

#### Rule 2310

$\operatorname{Int}[(a_. + \operatorname{Log}[c_.*(x_.)^{(n_.)}]*(b_.))^{(p_.)}*((d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(d*x)^{(m+1)}/(d*n*(c*x^n)^{((m+1)/n)}), \operatorname{Subst}[\operatorname{Int}[E^{((m+1)*x/n)}*(a + b*x)^p, x], x, \operatorname{Log}[c*x^n]], x] /; \operatorname{FreeQ}\{a, b, c, d, m, n, p\}, x]$

#### Rule 2180

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))}/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)], x\_Symbol] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\& !\$UseGamma === \operatorname{True}$

#### Rule 2204

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^2)}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^a*\operatorname{Sqrt}[Pi]*\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]])/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x] \&\& \operatorname{PosQ}[b]$

#### Rubi steps



$$\begin{aligned}
\int x \log^{\frac{3}{2}}(ax^n) dx &= \frac{1}{2}x^2 \log^{\frac{3}{2}}(ax^n) - \frac{1}{4}(3n) \int x \sqrt{\log(ax^n)} dx \\
&= -\frac{3}{8}nx^2 \sqrt{\log(ax^n)} + \frac{1}{2}x^2 \log^{\frac{3}{2}}(ax^n) + \frac{1}{16}(3n^2) \int \frac{x}{\sqrt{\log(ax^n)}} dx \\
&= -\frac{3}{8}nx^2 \sqrt{\log(ax^n)} + \frac{1}{2}x^2 \log^{\frac{3}{2}}(ax^n) + \frac{1}{16}(3nx^2 (ax^n)^{-2/n}) \text{Subst} \left( \int \frac{e^{\frac{2x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n) \right) \\
&= -\frac{3}{8}nx^2 \sqrt{\log(ax^n)} + \frac{1}{2}x^2 \log^{\frac{3}{2}}(ax^n) + \frac{1}{8}(3nx^2 (ax^n)^{-2/n}) \text{Subst} \left( \int e^{\frac{2x^2}{n}} dx, x, \sqrt{\log(ax^n)} \right) \\
&= \frac{3}{16}n^{3/2} \sqrt{\frac{\pi}{2}} x^2 (ax^n)^{-2/n} \operatorname{erfi} \left( \frac{\sqrt{2} \sqrt{\log(ax^n)}}{\sqrt{n}} \right) - \frac{3}{8}nx^2 \sqrt{\log(ax^n)} + \frac{1}{2}x^2 \log^{\frac{3}{2}}(ax^n)
\end{aligned}$$

**Mathematica [A]** time = 0.0574906, size = 79, normalized size = 0.88

$$\frac{1}{32}x^2 \left( 3\sqrt{2\pi}n^{3/2} (ax^n)^{-2/n} \operatorname{Erfi} \left( \frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}} \right) + 4\sqrt{\log(ax^n)} (4 \log(ax^n) - 3n) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x\*Log[a\*x^n]^(3/2),x]

[Out] (x^2\*((3\*n^(3/2)\*Sqrt[2\*Pi]\*Erfi[(Sqrt[2]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(a\*x^n)^(2/n) + 4\*Sqrt[Log[a\*x^n]]\*(-3\*n + 4\*Log[a\*x^n]))) / 32

**Maple [F]** time = 0.173, size = 0, normalized size = 0.

$$\int x (\ln(ax^n))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*ln(a\*x^n)^(3/2),x)

[Out] int(x\*ln(a\*x^n)^(3/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*log(a\*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate(x\*log(a\*x^n)^(3/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(a*x**n)**(3/2),x)
```

```
[Out] Integral(x*log(a*x**n)**(3/2), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*log(a*x^n)^(3/2), x)
```

### 3.124 $\int \log^{\frac{3}{2}}(ax^n) dx$

**Optimal.** Leaf size=72

$$\frac{3}{4}\sqrt{\pi}n^{3/2}x(ax^n)^{-1/n}\operatorname{Erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)+x\log^{\frac{3}{2}}(ax^n)-\frac{3}{2}nx\sqrt{\log(ax^n)}$$

[Out]  $(3*n^{(3/2)}*Sqrt[\text{Pi}]*x*\operatorname{Erfi}[\text{Sqrt}[\text{Log}[a*x^n]]/\text{Sqrt}[n]])/(4*(a*x^n)^n^{(-1)}) - (3*n*x*\text{Sqrt}[\text{Log}[a*x^n]])/2 + x*\text{Log}[a*x^n]^{(3/2)}$

**Rubi [A]** time = 0.0347186, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {2296, 2300, 2180, 2204}

$$\frac{3}{4}\sqrt{\pi}n^{3/2}x(ax^n)^{-1/n}\operatorname{Erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)+x\log^{\frac{3}{2}}(ax^n)-\frac{3}{2}nx\sqrt{\log(ax^n)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Log}[a*x^n]^{(3/2)}, x]$

[Out]  $(3*n^{(3/2)}*Sqrt[\text{Pi}]*x*\operatorname{Erfi}[\text{Sqrt}[\text{Log}[a*x^n]]/\text{Sqrt}[n]])/(4*(a*x^n)^n^{(-1)}) - (3*n*x*\text{Sqrt}[\text{Log}[a*x^n]])/2 + x*\text{Log}[a*x^n]^{(3/2)}$

#### Rule 2296

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]^{(p_.)}, x\_Symbol] :> \text{Simp}[x*(a + b*\text{Log}[c*x^n])^p, x] - \text{Dist}[b*n*p, \text{Int}[(a + b*\text{Log}[c*x^n])^{(p-1)}, x], x] /;$   
 $\text{FreeQ}\{a, b, c, n\}, x \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{IntegerQ}[2*p]$

#### Rule 2300

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]^{(p_.)}, x\_Symbol] :> \text{Dist}[x/(n*(c*x^n)^{(1/n)}), \text{Subst}[\text{Int}[E^{(x/n)}*(a + b*x)^p, x], x, \text{Log}[c*x^n]], x] /;$   
 $\text{FreeQ}\{a, b, c, n, p\}, x]$

#### Rule 2180

$\text{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_.)))/\text{Sqrt}[(c_.) + (d_.)*(x_.)]}, x\_Symbol] :> \text{Dist}[2/d, \text{Subst}[\text{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \text{Sqrt}[c + d*x]], x] /;$   
 $\text{FreeQ}\{F, c, d, e, f, g\}, x \ \&\& \ !\$UseGamma == True$

#### Rule 2204

$\text{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^2)}, x\_Symbol] :> \text{Simp}[(F^a*\text{Sqrt}[\text{Pi}]*\operatorname{Erfi}[(c + d*x)*\text{Rt}[b*\text{Log}[F], 2]])/(2*d*\text{Rt}[b*\text{Log}[F], 2]), x] /;$   
 $\text{FreeQ}\{F, a, b, c, d\}, x \ \&\& \ \text{PosQ}[b]$

#### Rubi steps

$$\begin{aligned}
\int \log^{\frac{3}{2}}(ax^n) dx &= x \log^{\frac{3}{2}}(ax^n) - \frac{1}{2}(3n) \int \sqrt{\log(ax^n)} dx \\
&= -\frac{3}{2}nx\sqrt{\log(ax^n)} + x \log^{\frac{3}{2}}(ax^n) + \frac{1}{4}(3n^2) \int \frac{1}{\sqrt{\log(ax^n)}} dx \\
&= -\frac{3}{2}nx\sqrt{\log(ax^n)} + x \log^{\frac{3}{2}}(ax^n) + \frac{1}{4}(3nx(ax^n)^{-1/n}) \text{Subst} \left( \int \frac{e^{\frac{x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n) \right) \\
&= -\frac{3}{2}nx\sqrt{\log(ax^n)} + x \log^{\frac{3}{2}}(ax^n) + \frac{1}{2}(3nx(ax^n)^{-1/n}) \text{Subst} \left( \int e^{\frac{x^2}{n}} dx, x, \sqrt{\log(ax^n)} \right) \\
&= \frac{3}{4}n^{3/2}\sqrt{\pi}x(ax^n)^{-1/n} \operatorname{erfi} \left( \frac{\sqrt{\log(ax^n)}}{\sqrt{n}} \right) - \frac{3}{2}nx\sqrt{\log(ax^n)} + x \log^{\frac{3}{2}}(ax^n)
\end{aligned}$$

**Mathematica [A]** time = 0.0380405, size = 72, normalized size = 1.

$$\frac{3}{4}\sqrt{\pi}n^{3/2}x(ax^n)^{-1/n} \operatorname{Erfi} \left( \frac{\sqrt{\log(ax^n)}}{\sqrt{n}} \right) + x \log^{\frac{3}{2}}(ax^n) - \frac{3}{2}nx\sqrt{\log(ax^n)}$$

Antiderivative was successfully verified.

[In] Integrate[Log[a\*x^n]^(3/2), x]

[Out] (3\*n^(3/2)\*Sqrt[Pi]\*x\*Erfi[Sqrt[Log[a\*x^n]]/Sqrt[n]])/(4\*(a\*x^n)^n^(-1)) - (3\*n\*x\*Sqrt[Log[a\*x^n]])/2 + x\*Log[a\*x^n]^(3/2)

**Maple [F]** time = 0.17, size = 0, normalized size = 0.

$$\int (\ln(ax^n))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(a\*x^n)^(3/2), x)

[Out] int(ln(a\*x^n)^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(3/2), x, algorithm="maxima")

[Out] integrate(log(a\*x^n)^(3/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a*x**n)**(3/2),x)
```

```
[Out] Integral(log(a*x**n)**(3/2), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(log(a*x^n)^(3/2), x)
```

$$3.125 \quad \int \frac{\log^{\frac{3}{2}}(ax^n)}{x} dx$$

**Optimal.** Leaf size=17

$$\frac{2 \log^{\frac{5}{2}}(ax^n)}{5n}$$

[Out] (2\*Log[a\*x^n]^(5/2))/(5\*n)

**Rubi [A]** time = 0.0136927, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2302, 30}

$$\frac{2 \log^{\frac{5}{2}}(ax^n)}{5n}$$

Antiderivative was successfully verified.

[In] Int[Log[a\*x^n]^(3/2)/x,x]

[Out] (2\*Log[a\*x^n]^(5/2))/(5\*n)

#### Rule 2302

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] :> Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{\log^{\frac{3}{2}}(ax^n)}{x} dx &= \frac{\text{Subst}\left(\int x^{3/2} dx, x, \log(ax^n)\right)}{n} \\ &= \frac{2 \log^{\frac{5}{2}}(ax^n)}{5n} \end{aligned}$$

**Mathematica [A]** time = 0.0017068, size = 17, normalized size = 1.

$$\frac{2 \log^{\frac{5}{2}}(ax^n)}{5n}$$

Antiderivative was successfully verified.

[In] Integrate[Log[a\*x^n]^(3/2)/x,x]

[Out] (2\*Log[a\*x^n]^(5/2))/(5\*n)

---

**Maple [A]** time = 0.039, size = 14, normalized size = 0.8

$$\frac{2}{5n} (\ln(ax^n))^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(a\*x^n)^(3/2)/x,x)

[Out] 2/5\*ln(a\*x^n)^(5/2)/n

---

**Maxima [A]** time = 1.09763, size = 18, normalized size = 1.06

$$\frac{2 \log(ax^n)^{\frac{5}{2}}}{5n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(3/2)/x,x, algorithm="maxima")

[Out] 2/5\*log(a\*x^n)^(5/2)/n

---

**Fricas [B]** time = 0.993865, size = 104, normalized size = 6.12

$$\frac{2(n^2 \log(x)^2 + 2n \log(a) \log(x) + \log(a)^2) \sqrt{n \log(x) + \log(a)}}{5n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(3/2)/x,x, algorithm="fricas")

[Out] 2/5\*(n^2\*log(x)^2 + 2\*n\*log(a)\*log(x) + log(a)^2)\*sqrt(n\*log(x) + log(a))/n

---

**Sympy [A]** time = 37.1645, size = 75, normalized size = 4.41

$$\begin{cases} \frac{2n\sqrt{n \log(x) + \log(a)} \log(x)^2}{5} + \frac{4\sqrt{n \log(x) + \log(a)} \log(a) \log(x)}{5} + \frac{2\sqrt{n \log(x) + \log(a)} \log(a)^2}{5n} & \text{for } n \neq 0 \\ \log(a)^{\frac{3}{2}} \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(a\*x\*\*n)\*\*(3/2)/x,x)

[Out] Piecewise((2\*n\*sqrt(n\*log(x) + log(a))\*log(x)\*\*2/5 + 4\*sqrt(n\*log(x) + log(a))\*log(a)\*log(x)/5 + 2\*sqrt(n\*log(x) + log(a))\*log(a)\*\*2/(5\*n), Ne(n, 0)), (log(a)\*\*(3/2)\*log(x), True))

---

**Giac [A]** time = 1.33377, size = 19, normalized size = 1.12

$$\frac{2(n \log(x) + \log(a))^{\frac{5}{2}}}{5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a*x^n)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 2/5*(n*log(x) + log(a))^(5/2)/n
```



$$3.126 \quad \int \frac{\log^2(ax^n)}{x^2} dx$$

**Optimal.** Leaf size=77

$$\frac{3\sqrt{\pi}n^{3/2}(ax^n)^{\frac{1}{n}}\operatorname{Erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{4x} - \frac{\log^2(ax^n)}{x} - \frac{3n\sqrt{\log(ax^n)}}{2x}$$

[Out] (3\*n^(3/2)\*Sqrt[Pi]\*(a\*x^n)^n^(-1)\*Erf[Sqrt[Log[a\*x^n]]/Sqrt[n]])/(4\*x) - (3\*n\*Sqrt[Log[a\*x^n]])/(2\*x) - Log[a\*x^n]^(3/2)/x

**Rubi [A]** time = 0.0665744, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2305, 2310, 2180, 2205}

$$\frac{3\sqrt{\pi}n^{3/2}(ax^n)^{\frac{1}{n}}\operatorname{Erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{4x} - \frac{\log^2(ax^n)}{x} - \frac{3n\sqrt{\log(ax^n)}}{2x}$$

Antiderivative was successfully verified.

[In] Int[Log[a\*x^n]^(3/2)/x^2,x]

[Out] (3\*n^(3/2)\*Sqrt[Pi]\*(a\*x^n)^n^(-1)\*Erf[Sqrt[Log[a\*x^n]]/Sqrt[n]])/(4\*x) - (3\*n\*Sqrt[Log[a\*x^n]])/(2\*x) - Log[a\*x^n]^(3/2)/x

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

#### Rubi steps

$$\begin{aligned}
\int \frac{\log^{\frac{3}{2}}(ax^n)}{x^2} dx &= -\frac{\log^{\frac{3}{2}}(ax^n)}{x} + \frac{1}{2}(3n) \int \frac{\sqrt{\log(ax^n)}}{x^2} dx \\
&= -\frac{3n\sqrt{\log(ax^n)}}{2x} - \frac{\log^{\frac{3}{2}}(ax^n)}{x} + \frac{1}{4}(3n^2) \int \frac{1}{x^2\sqrt{\log(ax^n)}} dx \\
&= -\frac{3n\sqrt{\log(ax^n)}}{2x} - \frac{\log^{\frac{3}{2}}(ax^n)}{x} + \frac{(3n(ax^n)^{\frac{1}{n}}) \text{Subst}\left(\int \frac{e^{-\frac{x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{4x} \\
&= -\frac{3n\sqrt{\log(ax^n)}}{2x} - \frac{\log^{\frac{3}{2}}(ax^n)}{x} + \frac{(3n(ax^n)^{\frac{1}{n}}) \text{Subst}\left(\int e^{-\frac{x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{2x} \\
&= \frac{3n^{3/2}\sqrt{\pi}(ax^n)^{\frac{1}{n}} \text{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{4x} - \frac{3n\sqrt{\log(ax^n)}}{2x} - \frac{\log^{\frac{3}{2}}(ax^n)}{x}
\end{aligned}$$

**Mathematica [A]** time = 0.0562425, size = 79, normalized size = 1.03

$$\frac{3n^2(ax^n)^{\frac{1}{n}} \sqrt{\frac{\log(ax^n)}{n}} \text{Gamma}\left(\frac{1}{2}, \frac{\log(ax^n)}{n}\right) + 4\log^2(ax^n) + 6n\log(ax^n)}{4x\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Integrate[Log[a\*x^n]^(3/2)/x^2,x]

[Out] -(6\*n\*Log[a\*x^n] + 4\*Log[a\*x^n]^2 + 3\*n^2\*(a\*x^n)^n^(-1)\*Gamma[1/2, Log[a\*x^n]/n]\*Sqrt[Log[a\*x^n]/n])/(4\*x\*Sqrt[Log[a\*x^n]])

**Maple [F]** time = 0.169, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (\ln(ax^n))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(a\*x^n)^(3/2)/x^2,x)

[Out] int(ln(a\*x^n)^(3/2)/x^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(ax^n)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(3/2)/x^2,x, algorithm="maxima")

[Out] integrate(log(a\*x^n)^(3/2)/x^2, x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(3/2)/x^2,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(ax^n)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(a\*x\*\*n)\*\*(3/2)/x\*\*2,x)

[Out] Integral(log(a\*x\*\*n)\*\*(3/2)/x\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(ax^n)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(3/2)/x^2,x, algorithm="giac")

[Out] integrate(log(a\*x^n)^(3/2)/x^2, x)

$$3.127 \quad \int \frac{\log^3(ax^n)}{x^3} dx$$

**Optimal.** Leaf size=90

$$\frac{3\sqrt{\frac{\pi}{2}}n^{3/2}(ax^n)^{2/n}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{16x^2} - \frac{\log^3(ax^n)}{2x^2} - \frac{3n\sqrt{\log(ax^n)}}{8x^2}$$

[Out] (3\*n^(3/2)\*Sqrt[Pi/2]\*(a\*x^n)^(2/n)\*Erf[(Sqrt[2]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(16\*x^2) - (3\*n\*Sqrt[Log[a\*x^n]])/(8\*x^2) - Log[a\*x^n]^(3/2)/(2\*x^2)

**Rubi [A]** time = 0.0725493, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2305, 2310, 2180, 2205}

$$\frac{3\sqrt{\frac{\pi}{2}}n^{3/2}(ax^n)^{2/n}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{16x^2} - \frac{\log^3(ax^n)}{2x^2} - \frac{3n\sqrt{\log(ax^n)}}{8x^2}$$

Antiderivative was successfully verified.

[In] Int[Log[a\*x^n]^(3/2)/x^3,x]

[Out] (3\*n^(3/2)\*Sqrt[Pi/2]\*(a\*x^n)^(2/n)\*Erf[(Sqrt[2]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(16\*x^2) - (3\*n\*Sqrt[Log[a\*x^n]])/(8\*x^2) - Log[a\*x^n]^(3/2)/(2\*x^2)

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

#### Rubi steps

$$\begin{aligned}
\int \frac{\log^{\frac{3}{2}}(ax^n)}{x^3} dx &= -\frac{\log^{\frac{3}{2}}(ax^n)}{2x^2} + \frac{1}{4}(3n) \int \frac{\sqrt{\log(ax^n)}}{x^3} dx \\
&= -\frac{3n\sqrt{\log(ax^n)}}{8x^2} - \frac{\log^{\frac{3}{2}}(ax^n)}{2x^2} + \frac{1}{16}(3n^2) \int \frac{1}{x^3\sqrt{\log(ax^n)}} dx \\
&= -\frac{3n\sqrt{\log(ax^n)}}{8x^2} - \frac{\log^{\frac{3}{2}}(ax^n)}{2x^2} + \frac{(3n(ax^n)^{2/n}) \operatorname{Subst}\left(\int \frac{e^{-\frac{2x}{n}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{16x^2} \\
&= -\frac{3n\sqrt{\log(ax^n)}}{8x^2} - \frac{\log^{\frac{3}{2}}(ax^n)}{2x^2} + \frac{(3n(ax^n)^{2/n}) \operatorname{Subst}\left(\int e^{-\frac{2x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{8x^2} \\
&= \frac{3n^{3/2} \sqrt{\frac{\pi}{2}} (ax^n)^{2/n} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{16x^2} - \frac{3n\sqrt{\log(ax^n)}}{8x^2} - \frac{\log^{\frac{3}{2}}(ax^n)}{2x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0638241, size = 88, normalized size = 0.98

$$\frac{3\sqrt{2}n^2(ax^n)^{2/n}\sqrt{\frac{\log(ax^n)}{n}}\operatorname{Gamma}\left(\frac{1}{2}, \frac{2\log(ax^n)}{n}\right) + 4\log(ax^n)(4\log(ax^n) + 3n)}{32x^2\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Integrate[Log[a\*x^n]^(3/2)/x^3,x]

[Out]  $-(3\sqrt{2}n^2(ax^n)^{2/n}\sqrt{\frac{\log(ax^n)}{n}}\operatorname{Gamma}\left[\frac{1}{2}, \frac{2\log(ax^n)}{n}\right]\sqrt{\log(ax^n)} + 4\log(ax^n)(3n + 4\log(ax^n)))/(32x^2\sqrt{\log(ax^n)})$

**Maple [F]** time = 0.174, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} (\ln(ax^n))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(a\*x^n)^(3/2)/x^3,x)

[Out] int(ln(a\*x^n)^(3/2)/x^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(ax^n)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(3/2)/x^3,x, algorithm="maxima")

[Out] integrate(log(a\*x^n)^(3/2)/x^3, x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(3/2)/x^3,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(ax^n)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(a\*x\*\*n)\*\*(3/2)/x\*\*3,x)

[Out] Integral(log(a\*x\*\*n)\*\*(3/2)/x\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(ax^n)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(a\*x^n)^(3/2)/x^3,x, algorithm="giac")

[Out] integrate(log(a\*x^n)^(3/2)/x^3, x)

$$3.128 \quad \int \frac{x^3}{\sqrt{\log(ax^n)}} dx$$

Optimal. Leaf size=46

$$\frac{\sqrt{\pi} x^4 (ax^n)^{-4/n} \operatorname{Erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2\sqrt{n}}$$

[Out] (Sqrt[Pi]\*x^4\*Erfi[(2\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(2\*Sqrt[n]\*(a\*x^n)^(4/n))

**Rubi [A]** time = 0.0352387, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {2310, 2180, 2204}

$$\frac{\sqrt{\pi} x^4 (ax^n)^{-4/n} \operatorname{Erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2\sqrt{n}}$$

Antiderivative was successfully verified.

[In] Int[x^3/Sqrt[Log[a\*x^n]],x]

[Out] (Sqrt[Pi]\*x^4\*Erfi[(2\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(2\*Sqrt[n]\*(a\*x^n)^(4/n))

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x)/n\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned} \int \frac{x^3}{\sqrt{\log(ax^n)}} dx &= \frac{(x^4 (ax^n)^{-4/n}) \operatorname{Subst}\left(\int \frac{e^{\frac{4x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n} \\ &= \frac{(2x^4 (ax^n)^{-4/n}) \operatorname{Subst}\left(\int e^{\frac{4x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{n} \\ &= \frac{\sqrt{\pi} x^4 (ax^n)^{-4/n} \operatorname{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2\sqrt{n}} \end{aligned}$$

**Mathematica [A]** time = 0.0054898, size = 46, normalized size = 1.

$$\frac{\sqrt{\pi}x^4 (ax^n)^{-4/n} \operatorname{Erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2\sqrt{n}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Sqrt[Log[a\*x^n]],x]

[Out] (Sqrt[Pi]\*x^4\*Erfi[(2\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(2\*Sqrt[n]\*(a\*x^n)^(4/n))

**Maple [F]** time = 0.178, size = 0, normalized size = 0.

$$\int x^3 \frac{1}{\sqrt{\ln(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/ln(a\*x^n)^(1/2),x)

[Out] int(x^3/ln(a\*x^n)^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(a\*x^n)^(1/2),x, algorithm="maxima")

[Out] integrate(x^3/sqrt(log(a\*x^n)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(a\*x^n)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(x**3/ln(a*x**n)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(log(a*x**n)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/log(a*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(log(a*x^n)), x)
```

$$3.129 \quad \int \frac{x^2}{\sqrt{\log(ax^n)}} dx$$

**Optimal.** Leaf size=51

$$\frac{\sqrt{\frac{\pi}{3}} x^3 (ax^n)^{-3/n} \operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

[Out] (Sqrt[Pi/3]\*x^3\*Erfi[(Sqrt[3]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(Sqrt[n]\*(a\*x^n)^(3/n))

**Rubi [A]** time = 0.0360221, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {2310, 2180, 2204}

$$\frac{\sqrt{\frac{\pi}{3}} x^3 (ax^n)^{-3/n} \operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[Log[a\*x^n]], x]

[Out] (Sqrt[Pi/3]\*x^3\*Erfi[(Sqrt[3]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(Sqrt[n]\*(a\*x^n)^(3/n))

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sqrt{\log(ax^n)}} dx &= \frac{(x^3 (ax^n)^{-3/n}) \operatorname{Subst}\left(\int \frac{e^{\frac{3x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n} \\ &= \frac{(2x^3 (ax^n)^{-3/n}) \operatorname{Subst}\left(\int e^{\frac{3x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{n} \\ &= \frac{\sqrt{\frac{\pi}{3}} x^3 (ax^n)^{-3/n} \operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}} \end{aligned}$$

**Mathematica [A]** time = 0.0093325, size = 51, normalized size = 1.

$$\frac{\sqrt{\frac{\pi}{3}} x^3 (ax^n)^{-3/n} \operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[Log[a\*x^n]], x]

[Out] (Sqrt[Pi/3]\*x^3\*Erfi[(Sqrt[3]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(Sqrt[n]\*(a\*x^n)^(3/n))

**Maple [F]** time = 0.169, size = 0, normalized size = 0.

$$\int x^2 \frac{1}{\sqrt{\ln(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/ln(a\*x^n)^(1/2), x)

[Out] int(x^2/ln(a\*x^n)^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/log(a\*x^n)^(1/2), x, algorithm="maxima")

[Out] integrate(x^2/sqrt(log(a\*x^n)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(a*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/ln(a*x**n)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(log(a*x**n)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/log(a*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(log(a*x^n)), x)
```

$$3.130 \quad \int \frac{x}{\sqrt{\log(ax^n)}} dx$$

**Optimal.** Leaf size=51

$$\frac{\sqrt{\frac{\pi}{2}} x^2 (ax^n)^{-2/n} \operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

[Out] (Sqrt [Pi/2] \*x^2\*Erfi [(Sqrt [2]\*Sqrt [Log [a\*x^n]])/Sqrt [n]])/(Sqrt [n]\*(a\*x^n)^(2/n))

**Rubi [A]** time = 0.0326668, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {2310, 2180, 2204}

$$\frac{\sqrt{\frac{\pi}{2}} x^2 (ax^n)^{-2/n} \operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

Antiderivative was successfully verified.

[In] Int [x/Sqrt [Log [a\*x^n]], x]

[Out] (Sqrt [Pi/2] \*x^2\*Erfi [(Sqrt [2]\*Sqrt [Log [a\*x^n]])/Sqrt [n]])/(Sqrt [n]\*(a\*x^n)^(2/n))

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt [Pi]\*Erfi[(c + d\*x)\*Rt [b\*Log [F], 2]])/(2\*d\*Rt [b\*Log [F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{\log(ax^n)}} dx &= \frac{(x^2 (ax^n)^{-2/n}) \operatorname{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n} \\ &= \frac{(2x^2 (ax^n)^{-2/n}) \operatorname{Subst}\left(\int e^{\frac{2x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{n} \\ &= \frac{\sqrt{\frac{\pi}{2}} x^2 (ax^n)^{-2/n} \operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}} \end{aligned}$$

**Mathematica [A]** time = 0.0089993, size = 51, normalized size = 1.

$$\frac{\sqrt{\frac{\pi}{2}}x^2(ax^n)^{-2/n}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[Log[a\*x^n]],x]

[Out] (Sqrt[Pi/2]\*x^2\*Erfi[(Sqrt[2]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(Sqrt[n]\*(a\*x^n)^(2/n))

**Maple [F]** time = 0.181, size = 0, normalized size = 0.

$$\int x \frac{1}{\sqrt{\ln(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/ln(a\*x^n)^(1/2),x)

[Out] int(x/ln(a\*x^n)^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(a\*x^n)^(1/2),x, algorithm="maxima")

[Out] integrate(x/sqrt(log(a\*x^n)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(a\*x^n)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/ln(a\*x\*\*n)\*\*(1/2),x)

[Out] Integral(x/sqrt(log(a\*x\*\*n)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(a\*x^n)^(1/2),x, algorithm="giac")

[Out] integrate(x/sqrt(log(a\*x^n)), x)

### 3.131 $\int \frac{1}{\sqrt{\log(ax^n)}} dx$

**Optimal.** Leaf size=40

$$\frac{\sqrt{\pi}x(ax^n)^{-1/n} \operatorname{Erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

[Out] (Sqrt[Pi]\*x\*Erfi[Sqrt[Log[a\*x^n]]/Sqrt[n]])/(Sqrt[n]\*(a\*x^n)^n^(-1))

**Rubi [A]** time = 0.0222031, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {2300, 2180, 2204}

$$\frac{\sqrt{\pi}x(ax^n)^{-1/n} \operatorname{Erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[Log[a\*x^n]],x]

[Out] (Sqrt[Pi]\*x\*Erfi[Sqrt[Log[a\*x^n]]/Sqrt[n]])/(Sqrt[n]\*(a\*x^n)^n^(-1))

#### Rule 2300

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^p, x\_Symbol] :> Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[E^(x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{\log(ax^n)}} dx &= \frac{(x(ax^n)^{-1/n}) \operatorname{Subst}\left(\int \frac{e^{\frac{x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n} \\ &= \frac{(2x(ax^n)^{-1/n}) \operatorname{Subst}\left(\int e^{\frac{x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{n} \\ &= \frac{\sqrt{\pi}x(ax^n)^{-1/n} \operatorname{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}} \end{aligned}$$



**Mathematica [A]** time = 0.0037399, size = 40, normalized size = 1.

$$\frac{\sqrt{\pi}x(ax^n)^{-1/n} \operatorname{Erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[Log[a\*x^n]],x]

[Out] (Sqrt[Pi]\*x\*Erfi[Sqrt[Log[a\*x^n]]/Sqrt[n]])/(Sqrt[n]\*(a\*x^n)^n^(-1))

**Maple [F]** time = 0.178, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\ln(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/ln(a\*x^n)^(1/2),x)

[Out] int(1/ln(a\*x^n)^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(a\*x^n)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(log(a\*x^n)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(a\*x^n)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(a*x**n)**(1/2),x)
```

```
[Out] Integral(1/sqrt(log(a*x**n)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(a*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(log(a*x^n)), x)
```

$$3.132 \quad \int \frac{1}{x\sqrt{\log(ax^n)}} dx$$

**Optimal.** Leaf size=15

$$\frac{2\sqrt{\log(ax^n)}}{n}$$

[Out] (2\*Sqrt[Log[a\*x^n]])/n

**Rubi [A]** time = 0.013232, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2302, 30}

$$\frac{2\sqrt{\log(ax^n)}}{n}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Sqrt[Log[a\*x^n]]), x]

[Out] (2\*Sqrt[Log[a\*x^n]])/n

**Rule 2302**

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] := Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

**Rule 30**

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

**Rubi steps**

$$\begin{aligned} \int \frac{1}{x\sqrt{\log(ax^n)}} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n} \\ &= \frac{2\sqrt{\log(ax^n)}}{n} \end{aligned}$$

**Mathematica [A]** time = 0.0013985, size = 15, normalized size = 1.

$$\frac{2\sqrt{\log(ax^n)}}{n}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Sqrt[Log[a\*x^n]]), x]

[Out] (2\*Sqrt[Log[a\*x^n]])/n

**Maple [A]** time = 0.039, size = 14, normalized size = 0.9

$$2 \frac{\sqrt{\ln(ax^n)}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/ln(a\*x^n)^(1/2),x)

[Out] 2\*ln(a\*x^n)^(1/2)/n

---

**Maxima [A]** time = 1.1009, size = 18, normalized size = 1.2

$$2 \frac{\sqrt{\log(ax^n)}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(a\*x^n)^(1/2),x, algorithm="maxima")

[Out] 2\*sqrt(log(a\*x^n))/n

---

**Fricas [A]** time = 0.943296, size = 39, normalized size = 2.6

$$2 \frac{\sqrt{n \log(x) + \log(a)}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(a\*x^n)^(1/2),x, algorithm="fricas")

[Out] 2\*sqrt(n\*log(x) + log(a))/n

---

**Sympy [A]** time = 3.26988, size = 24, normalized size = 1.6

$$\begin{cases} \frac{2\sqrt{n \log(x) + \log(a)}}{n} & \text{for } n \neq 0 \\ \frac{\log(x)}{\sqrt{\log(a)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/ln(a\*x\*\*n)\*\*(1/2),x)

[Out] Piecewise((2\*sqrt(n\*log(x) + log(a))/n, Ne(n, 0)), (log(x)/sqrt(log(a)), True))

---

**Giac [A]** time = 1.28396, size = 19, normalized size = 1.27

$$2 \frac{\sqrt{n \log(x) + \log(a)}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(a*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(n*log(x) + log(a))/n
```

$$3.133 \quad \int \frac{1}{x^2 \sqrt{\log(ax^n)}} dx$$

**Optimal.** Leaf size=40

$$\frac{\sqrt{\pi} (ax^n)^{\frac{1}{n}} \operatorname{Erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{nx}}$$

[Out] (Sqrt[Pi]\*(a\*x^n)^n^(-1)\*Erf[Sqrt[Log[a\*x^n]]/Sqrt[n]])/(Sqrt[n]\*x)

**Rubi [A]** time = 0.0327074, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {2310, 2180, 2205}

$$\frac{\sqrt{\pi} (ax^n)^{\frac{1}{n}} \operatorname{Erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{nx}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*Sqrt[Log[a\*x^n]]), x]

[Out] (Sqrt[Pi]\*(a\*x^n)^n^(-1)\*Erf[Sqrt[Log[a\*x^n]]/Sqrt[n]])/(Sqrt[n]\*x)

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^p]\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

#### Rubi steps

$$\begin{aligned} \int \frac{1}{x^2 \sqrt{\log(ax^n)}} dx &= \frac{(ax^n)^{\frac{1}{n}} \operatorname{Subst}\left(\int \frac{e^{-\frac{x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{nx} \\ &= \frac{\left(2(ax^n)^{\frac{1}{n}}\right) \operatorname{Subst}\left(\int e^{-\frac{x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{nx} \\ &= \frac{\sqrt{\pi} (ax^n)^{\frac{1}{n}} \operatorname{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{nx}} \end{aligned}$$

**Mathematica [A]** time = 0.0342237, size = 52, normalized size = 1.3

$$\frac{(ax^n)^{\frac{1}{n}} \sqrt{\frac{\log(ax^n)}{n}} \text{Gamma}\left(\frac{1}{2}, \frac{\log(ax^n)}{n}\right)}{x \sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*Sqrt[Log[a\*x^n]]), x]

[Out] -(((a\*x^n)^n^(-1)\*Gamma[1/2, Log[a\*x^n]/n]\*Sqrt[Log[a\*x^n]/n])/(x\*Sqrt[Log[a\*x^n]]))

**Maple [F]** time = 0.171, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt{\ln(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/ln(a\*x^n)^(1/2), x)

[Out] int(1/x^2/ln(a\*x^n)^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(a\*x^n)^(1/2), x, algorithm="maxima")

[Out] integrate(1/(x^2\*sqrt(log(a\*x^n))), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(a\*x^n)^(1/2), x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/ln(a\*x\*\*n)\*\*(1/2),x)

[Out] Integral(1/(x\*\*2\*sqrt(log(a\*x\*\*n))), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(a\*x^n)^(1/2),x, algorithm="giac")

[Out] integrate(1/(x^2\*sqrt(log(a\*x^n))), x)



$$3.134 \quad \int \frac{1}{x^3 \sqrt{\log(ax^n)}} dx$$

**Optimal.** Leaf size=51

$$\frac{\sqrt{\frac{\pi}{2}} (ax^n)^{2/n} \operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{nx^2}}$$

[Out] (Sqrt[Pi/2]\*(a\*x^n)^(2/n)\*Erf[(Sqrt[2]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(Sqrt[n]\*x^2)

**Rubi [A]** time = 0.038955, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {2310, 2180, 2205}

$$\frac{\sqrt{\frac{\pi}{2}} (ax^n)^{2/n} \operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{nx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*Sqrt[Log[a\*x^n]]), x]

[Out] (Sqrt[Pi/2]\*(a\*x^n)^(2/n)\*Erf[(Sqrt[2]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(Sqrt[n]\*x^2)

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

#### Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

#### Rubi steps

$$\begin{aligned} \int \frac{1}{x^3 \sqrt{\log(ax^n)}} dx &= \frac{(ax^n)^{2/n} \operatorname{Subst}\left(\int \frac{e^{-\frac{2x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{nx^2} \\ &= \frac{(2(ax^n)^{2/n}) \operatorname{Subst}\left(\int e^{-\frac{2x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{nx^2} \\ &= \frac{\sqrt{\frac{\pi}{2}} (ax^n)^{2/n} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{nx^2}} \end{aligned}$$

**Mathematica [A]** time = 0.0345246, size = 60, normalized size = 1.18

$$\frac{(ax^n)^{2/n} \sqrt{\frac{\log(ax^n)}{n}} \Gamma\left(\frac{1}{2}, \frac{2\log(ax^n)}{n}\right)}{\sqrt{2}x^2\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*Sqrt[Log[a\*x^n]]), x]

[Out] -(((a\*x^n)^(2/n)\*Gamma[1/2, (2\*Log[a\*x^n])/n]\*Sqrt[Log[a\*x^n]/n])/(Sqrt[2]\*x^2\*Sqrt[Log[a\*x^n]]))

**Maple [F]** time = 0.174, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} \frac{1}{\sqrt{\ln(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/ln(a\*x^n)^(1/2), x)

[Out] int(1/x^3/ln(a\*x^n)^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(a\*x^n)^(1/2), x, algorithm="maxima")

[Out] integrate(1/(x^3\*sqrt(log(a\*x^n))), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(a\*x^n)^(1/2), x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/ln(a\*x\*\*n)\*\*(1/2), x)

[Out] Integral(1/(x\*\*3\*sqrt(log(a\*x\*\*n))), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(a\*x^n)^(1/2), x, algorithm="giac")

[Out] integrate(1/(x^3\*sqrt(log(a\*x^n))), x)

$$3.135 \quad \int \frac{x^3}{\log^2(ax^n)} dx$$

**Optimal.** Leaf size=63

$$\frac{4\sqrt{\pi}x^4(ax^n)^{-4/n} \operatorname{Erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^4}{n\sqrt{\log(ax^n)}}$$

[Out] (4\*Sqrt[Pi]\*x^4\*Erfi[(2\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(n^(3/2)\*(a\*x^n)^(4/n)) - (2\*x^4)/(n\*Sqrt[Log[a\*x^n]])

**Rubi [A]** time = 0.052964, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2306, 2310, 2180, 2204}

$$\frac{4\sqrt{\pi}x^4(ax^n)^{-4/n} \operatorname{Erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^4}{n\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Int[x^3/Log[a\*x^n]^(3/2), x]

[Out] (4\*Sqrt[Pi]\*x^4\*Erfi[(2\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(n^(3/2)\*(a\*x^n)^(4/n)) - (2\*x^4)/(n\*Sqrt[Log[a\*x^n]])

#### Rule 2306

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^(p + 1))/(b*d*n*(p + 1)), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

#### Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_)^(m_.), x_Symbol]
:> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)
/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

#### Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol]
:> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

#### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol]
:> Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\log^{\frac{3}{2}}(ax^n)} dx &= -\frac{2x^4}{n\sqrt{\log(ax^n)}} + \frac{8 \int \frac{x^3}{\sqrt{\log(ax^n)}} dx}{n} \\
&= -\frac{2x^4}{n\sqrt{\log(ax^n)}} + \frac{(8x^4 (ax^n)^{-4/n}) \text{Subst}\left(\int \frac{e^{\frac{4x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n^2} \\
&= -\frac{2x^4}{n\sqrt{\log(ax^n)}} + \frac{(16x^4 (ax^n)^{-4/n}) \text{Subst}\left(\int e^{\frac{4x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{n^2} \\
&= \frac{4\sqrt{\pi}x^4 (ax^n)^{-4/n} \operatorname{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^4}{n\sqrt{\log(ax^n)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0451144, size = 73, normalized size = 1.16

$$-\frac{2x^4 (ax^n)^{-4/n} \left( (ax^n)^{4/n} - 2\sqrt{-\frac{\log(ax^n)}{n}} \operatorname{Gamma}\left(\frac{1}{2}, -\frac{4\log(ax^n)}{n}\right) \right)}{n\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Log[a\*x^n]^(3/2), x]

[Out] (-2\*x^4\*((a\*x^n)^(4/n) - 2\*Gamma[1/2, (-4\*Log[a\*x^n])/n]\*Sqrt[-(Log[a\*x^n]/n)])/(n\*(a\*x^n)^(4/n)\*Sqrt[Log[a\*x^n]])

**Maple [F]** time = 0.189, size = 0, normalized size = 0.

$$\int x^3 (\ln(ax^n))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/ln(a\*x^n)^(3/2), x)

[Out] int(x^3/ln(a\*x^n)^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(a\*x^n)^(3/2), x, algorithm="maxima")

[Out] integrate(x^3/log(a\*x^n)^(3/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(a\*x^n)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/ln(a\*x\*\*n)\*\*(3/2),x)

[Out] Integral(x\*\*3/log(a\*x\*\*n)\*\*(3/2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(a\*x^n)^(3/2),x, algorithm="giac")

[Out] integrate(x^3/log(a\*x^n)^(3/2), x)

$$3.136 \quad \int \frac{x^2}{\log^2(ax^n)} dx$$

**Optimal.** Leaf size=69

$$\frac{2\sqrt{3}\pi x^3 (ax^n)^{-3/n} \operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^3}{n\sqrt{\log(ax^n)}}$$

[Out] (2\*Sqrt[3\*Pi]\*x^3\*Erfi[(Sqrt[3]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(n^(3/2)\*(a\*x^n)^(3/n)) - (2\*x^3)/(n\*Sqrt[Log[a\*x^n]])

**Rubi [A]** time = 0.0577886, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2306, 2310, 2180, 2204}

$$\frac{2\sqrt{3}\pi x^3 (ax^n)^{-3/n} \operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^3}{n\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Int[x^2/Log[a\*x^n]^(3/2),x]

[Out] (2\*Sqrt[3\*Pi]\*x^3\*Erfi[(Sqrt[3]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(n^(3/2)\*(a\*x^n)^(3/n)) - (2\*x^3)/(n\*Sqrt[Log[a\*x^n]])

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^ (p\_)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^ (p\_)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\log^{\frac{3}{2}}(ax^n)} dx &= -\frac{2x^3}{n\sqrt{\log(ax^n)}} + \frac{6 \int \frac{x^2}{\sqrt{\log(ax^n)}} dx}{n} \\
&= -\frac{2x^3}{n\sqrt{\log(ax^n)}} + \frac{(6x^3 (ax^n)^{-3/n}) \text{Subst} \left( \int \frac{e^{\frac{3x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n) \right)}{n^2} \\
&= -\frac{2x^3}{n\sqrt{\log(ax^n)}} + \frac{(12x^3 (ax^n)^{-3/n}) \text{Subst} \left( \int e^{\frac{3x^2}{n}} dx, x, \sqrt{\log(ax^n)} \right)}{n^2} \\
&= \frac{2\sqrt{3}\pi x^3 (ax^n)^{-3/n} \operatorname{erfi} \left( \frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}} \right)}{n^{3/2}} - \frac{2x^3}{n\sqrt{\log(ax^n)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0487053, size = 78, normalized size = 1.13

$$-\frac{2x^3 (ax^n)^{-3/n} \left( (ax^n)^{3/n} - \sqrt{3} \sqrt{-\frac{\log(ax^n)}{n}} \operatorname{Gamma} \left( \frac{1}{2}, -\frac{3 \log(ax^n)}{n} \right) \right)}{n\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Log[a\*x^n]^(3/2), x]

[Out] (-2\*x^3\*((a\*x^n)^(3/n) - Sqrt[3]\*Gamma[1/2, (-3\*Log[a\*x^n])/n]\*Sqrt[-(Log[a\*x^n]/n)]))/(n\*(a\*x^n)^(3/n)\*Sqrt[Log[a\*x^n]])

**Maple [F]** time = 0.175, size = 0, normalized size = 0.

$$\int x^2 (\ln(ax^n))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/ln(a\*x^n)^(3/2), x)

[Out] int(x^2/ln(a\*x^n)^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/log(a\*x^n)^(3/2), x, algorithm="maxima")

[Out] integrate(x^2/log(a\*x^n)^(3/2), x)



**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/log(a\*x^n)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/ln(a\*x\*\*n)\*\*(3/2),x)

[Out] Integral(x\*\*2/log(a\*x\*\*n)\*\*(3/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/log(a\*x^n)^(3/2),x, algorithm="giac")

[Out] integrate(x^2/log(a\*x^n)^(3/2), x)

$$3.137 \quad \int \frac{x}{3 \log^2(ax^n)} dx$$

**Optimal.** Leaf size=69

$$\frac{2\sqrt{2\pi}x^2(ax^n)^{-2/n} \operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^2}{n\sqrt{\log(ax^n)}}$$

[Out] (2\*Sqrt[2\*Pi]\*x^2\*Erfi[(Sqrt[2]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(n^(3/2)\*(a\*x^n)^(2/n)) - (2\*x^2)/(n\*Sqrt[Log[a\*x^n]])

**Rubi [A]** time = 0.0488579, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2306, 2310, 2180, 2204}

$$\frac{2\sqrt{2\pi}x^2(ax^n)^{-2/n} \operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^2}{n\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Int[x/Log[a\*x^n]^(3/2), x]

[Out] (2\*Sqrt[2\*Pi]\*x^2\*Erfi[(Sqrt[2]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(n^(3/2)\*(a\*x^n)^(2/n)) - (2\*x^2)/(n\*Sqrt[Log[a\*x^n]])

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned}
\int \frac{x}{\log^{\frac{3}{2}}(ax^n)} dx &= -\frac{2x^2}{n\sqrt{\log(ax^n)}} + \frac{4 \int \frac{x}{\sqrt{\log(ax^n)}} dx}{n} \\
&= -\frac{2x^2}{n\sqrt{\log(ax^n)}} + \frac{(4x^2 (ax^n)^{-2/n}) \text{Subst}\left(\int \frac{2x}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n^2} \\
&= -\frac{2x^2}{n\sqrt{\log(ax^n)}} + \frac{(8x^2 (ax^n)^{-2/n}) \text{Subst}\left(\int e^{\frac{2x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{n^2} \\
&= \frac{2\sqrt{2}\pi x^2 (ax^n)^{-2/n} \operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^2}{n\sqrt{\log(ax^n)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0463273, size = 78, normalized size = 1.13

$$-\frac{2x^2 (ax^n)^{-2/n} \left( (ax^n)^{2/n} - \sqrt{2} \sqrt{-\frac{\log(ax^n)}{n}} \operatorname{Gamma}\left(\frac{1}{2}, -\frac{2\log(ax^n)}{n}\right) \right)}{n\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Integrate[x/Log[a\*x^n]^(3/2), x]

[Out]  $(-2*x^2*((a*x^n)^(2/n) - \text{Sqrt}[2]*\text{Gamma}[1/2, (-2*\text{Log}[a*x^n])/n]*\text{Sqrt}[-(\text{Log}[a*x^n]/n)]))/ (n*(a*x^n)^(2/n)*\text{Sqrt}[\text{Log}[a*x^n]])$

**Maple [F]** time = 0.171, size = 0, normalized size = 0.

$$\int x (\ln(ax^n))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/ln(a\*x^n)^(3/2), x)

[Out] int(x/ln(a\*x^n)^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(a\*x^n)^(3/2), x, algorithm="maxima")

[Out] integrate(x/log(a\*x^n)^(3/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(a\*x^n)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/ln(a\*x\*\*n)\*\*(3/2),x)

[Out] Integral(x/log(a\*x\*\*n)\*\*(3/2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(a\*x^n)^(3/2),x, algorithm="giac")

[Out] integrate(x/log(a\*x^n)^(3/2), x)

$$3.138 \quad \int \frac{1}{\log^{\frac{3}{2}}(ax^n)} dx$$

**Optimal.** Leaf size=58

$$\frac{2\sqrt{\pi}x(ax^n)^{-1/n} \operatorname{Erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x}{n\sqrt{\log(ax^n)}}$$

[Out] (2\*Sqrt[Pi]\*x\*Erfi[Sqrt[Log[a\*x^n]]/Sqrt[n]])/(n^(3/2)\*(a\*x^n)^n^(-1)) - (2\*x)/(n\*Sqrt[Log[a\*x^n]])

**Rubi [A]** time = 0.0285107, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {2297, 2300, 2180, 2204}

$$\frac{2\sqrt{\pi}x(ax^n)^{-1/n} \operatorname{Erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x}{n\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Int[Log[a\*x^n]^(-3/2), x]

[Out] (2\*Sqrt[Pi]\*x\*Erfi[Sqrt[Log[a\*x^n]]/Sqrt[n]])/(n^(3/2)\*(a\*x^n)^n^(-1)) - (2\*x)/(n\*Sqrt[Log[a\*x^n]])

#### Rule 2297

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*n\*(p + 1)), x] - Dist[1/(b\*n\*(p + 1)), Int[(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && IntegerQ[2\*p]

#### Rule 2300

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_), x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[E^(x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]]/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{\log^{\frac{3}{2}}(ax^n)} dx &= -\frac{2x}{n\sqrt{\log(ax^n)}} + \frac{2 \int \frac{1}{\sqrt{\log(ax^n)}} dx}{n} \\
&= -\frac{2x}{n\sqrt{\log(ax^n)}} + \frac{(2x(ax^n)^{-1/n}) \text{Subst}\left(\int \frac{e^{\frac{x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n^2} \\
&= -\frac{2x}{n\sqrt{\log(ax^n)}} + \frac{(4x(ax^n)^{-1/n}) \text{Subst}\left(\int e^{\frac{x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{n^2} \\
&= \frac{2\sqrt{\pi}x(ax^n)^{-1/n} \operatorname{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x}{n\sqrt{\log(ax^n)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0343871, size = 69, normalized size = 1.19

$$-\frac{2x(ax^n)^{-1/n} \left( (ax^n)^{\frac{1}{n}} - \sqrt{-\frac{\log(ax^n)}{n}} \operatorname{Gamma}\left(\frac{1}{2}, -\frac{\log(ax^n)}{n}\right) \right)}{n\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Integrate[Log[a\*x^n]^(-3/2), x]

[Out] (-2\*x\*((a\*x^n)^n^(-1) - Gamma[1/2, -(Log[a\*x^n]/n)]\*Sqrt[-(Log[a\*x^n]/n)])) / (n\*(a\*x^n)^n^(-1)\*Sqrt[Log[a\*x^n]])

**Maple [F]** time = 0.175, size = 0, normalized size = 0.

$$\int (\ln(ax^n))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/ln(a\*x^n)^(3/2), x)

[Out] int(1/ln(a\*x^n)^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(a\*x^n)^(3/2), x, algorithm="maxima")

[Out] integrate(log(a\*x^n)^(-3/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(a\*x^n)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/ln(a\*x\*\*n)\*\*(3/2),x)

[Out] Integral(log(a\*x\*\*n)\*\*(-3/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(a\*x^n)^(3/2),x, algorithm="giac")

[Out] integrate(log(a\*x^n)^(-3/2), x)

$$3.139 \quad \int \frac{1}{x \log^{\frac{3}{2}}(ax^n)} dx$$

**Optimal.** Leaf size=15

$$-\frac{2}{n\sqrt{\log(ax^n)}}$$

[Out] -2/(n\*Sqrt[Log[a\*x^n]])

**Rubi [A]** time = 0.0140202, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2302, 30}

$$-\frac{2}{n\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Log[a\*x^n]^(3/2)),x]

[Out] -2/(n\*Sqrt[Log[a\*x^n]])

#### Rule 2302

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] := Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{1}{x \log^{\frac{3}{2}}(ax^n)} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^{3/2}} dx, x, \log(ax^n)\right)}{n} \\ &= -\frac{2}{n\sqrt{\log(ax^n)}} \end{aligned}$$

**Mathematica [A]** time = 0.0018364, size = 15, normalized size = 1.

$$-\frac{2}{n\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Log[a\*x^n]^(3/2)),x]

[Out] -2/(n\*Sqrt[Log[a\*x^n]])



---

**Maple [A]** time = 0.039, size = 14, normalized size = 0.9

$$-2 \frac{1}{n \sqrt{\ln(ax^n)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/ln(a\*x^n)^(3/2),x)

[Out] -2/n/ln(a\*x^n)^(1/2)

---

**Maxima [A]** time = 1.13858, size = 18, normalized size = 1.2

$$-\frac{2}{n \sqrt{\log(ax^n)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(a\*x^n)^(3/2),x, algorithm="maxima")

[Out] -2/(n\*sqrt(log(a\*x^n)))

---

**Fricas [A]** time = 0.94426, size = 70, normalized size = 4.67

$$\frac{2 \sqrt{n \log(x) + \log(a)}}{n^2 \log(x) + n \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(a\*x^n)^(3/2),x, algorithm="fricas")

[Out] -2\*sqrt(n\*log(x) + log(a))/(n^2\*log(x) + n\*log(a))

---

**Sympy [A]** time = 165.01, size = 48, normalized size = 3.2

$$\begin{cases} \infty \log(x) & \text{for } (a = 1 \vee a = e^{-n \log(x)}) \wedge (a = e^{-n \log(x)} \vee n = 0) \\ \frac{\log(x)}{\log(a)^3} & \text{for } n = 0 \\ -\frac{2}{n \sqrt{n \log(x) + \log(a)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/ln(a\*x\*\*n)\*\*(3/2),x)

[Out] Piecewise((zoo\*log(x), (Eq(a, 1) | Eq(a, exp(-n\*log(x)))) & (Eq(n, 0) | Eq(a, exp(-n\*log(x))))), (log(x)/log(a)\*\*(3/2), Eq(n, 0)), (-2/(n\*sqrt(n\*log(x) + log(a))), True))

---

**Giac [A]** time = 1.19797, size = 19, normalized size = 1.27

$$-\frac{2}{\sqrt{n \log(x) + \log(a)n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(a*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] -2/(sqrt(n*log(x) + log(a))*n)
```

$$3.140 \quad \int \frac{1}{x^2 \log^{\frac{3}{2}}(ax^n)} dx$$

**Optimal.** Leaf size=60

$$-\frac{2\sqrt{\pi}(ax^n)^{\frac{1}{n}} \operatorname{Erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}x} - \frac{2}{nx\sqrt{\log(ax^n)}}$$

[Out]  $(-2*\operatorname{Sqrt}[\operatorname{Pi}]*(a*x^n)^n^{-1}*\operatorname{Erf}[\operatorname{Sqrt}[\operatorname{Log}[a*x^n]]/\operatorname{Sqrt}[n]])/(n^{(3/2)*x}) - 2/(n*x*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])$

**Rubi [A]** time = 0.0501952, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2306, 2310, 2180, 2205}

$$-\frac{2\sqrt{\pi}(ax^n)^{\frac{1}{n}} \operatorname{Erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}x} - \frac{2}{nx\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[1/(x^2*\operatorname{Log}[a*x^n]^{(3/2)}), x]$

[Out]  $(-2*\operatorname{Sqrt}[\operatorname{Pi}]*(a*x^n)^n^{-1}*\operatorname{Erf}[\operatorname{Sqrt}[\operatorname{Log}[a*x^n]]/\operatorname{Sqrt}[n]])/(n^{(3/2)*x}) - 2/(n*x*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])$

#### Rule 2306

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.)*(x_)^{(n_.)}]*(b_.)]^{(p_.)}*((d_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(d*x)^{(m+1)}*(a + b*\operatorname{Log}[c*x^n])^{(p+1)} / (b*d*n*(p+1)), x] - \operatorname{Dist}[(m+1)/(b*n*(p+1)), \operatorname{Int}[(d*x)^m*(a + b*\operatorname{Log}[c*x^n])^{(p+1)}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m, n\}, x \ \&\& \ \operatorname{NeQ}[m, -1] \ \&\& \ \operatorname{LtQ}[p, -1]$

#### Rule 2310

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.)*(x_)^{(n_.)}]*(b_.)]^{(p_.)}*((d_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(d*x)^{(m+1)} / (d*n*(c*x^n)^{((m+1)/n)}), \operatorname{Subst}[\operatorname{Int}[E^{((m+1)*x/n)}*(a + b*x)^p, x], x, \operatorname{Log}[c*x^n]], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m, n, p\}, x]$

#### Rule 2180

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))}/\operatorname{Sqrt}[(c_.) + (d_.)*(x_)], x\_Symbol] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x \ \&\& \ \operatorname{!}\$UseGamma == \operatorname{True}$

#### Rule 2205

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2})}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(c + d*x)*\operatorname{Rt}[-(b*\operatorname{Log}[F]), 2]]) / (2*d*\operatorname{Rt}[-(b*\operatorname{Log}[F]), 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x \ \&\& \ \operatorname{NegQ}[b]$

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 \log^{\frac{3}{2}}(ax^n)} dx &= -\frac{2}{nx\sqrt{\log(ax^n)}} - \frac{2 \int \frac{1}{x^2 \sqrt{\log(ax^n)}} dx}{n} \\
&= -\frac{2}{nx\sqrt{\log(ax^n)}} - \frac{\left(2(ax^n)^{\frac{1}{n}}\right) \text{Subst}\left(\int \frac{e^{-\frac{x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n^2 x} \\
&= -\frac{2}{nx\sqrt{\log(ax^n)}} - \frac{\left(4(ax^n)^{\frac{1}{n}}\right) \text{Subst}\left(\int e^{-\frac{x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{n^2 x} \\
&= -\frac{2\sqrt{\pi}(ax^n)^{\frac{1}{n}} \text{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}x} - \frac{2}{nx\sqrt{\log(ax^n)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0490753, size = 58, normalized size = 0.97

$$\frac{2 \left( (ax^n)^{\frac{1}{n}} \sqrt{\frac{\log(ax^n)}{n}} \text{Gamma}\left(\frac{1}{2}, \frac{\log(ax^n)}{n}\right) - 1 \right)}{nx\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*Log[a\*x^n]^(3/2)),x]

[Out] (2\*(-1 + (a\*x^n)^n^(-1)\*Gamma[1/2, Log[a\*x^n]/n]\*Sqrt[Log[a\*x^n]/n]))/(n\*x\*Sqrt[Log[a\*x^n]])

**Maple [F]** time = 0.173, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (\ln(ax^n))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/ln(a\*x^n)^(3/2),x)

[Out] int(1/x^2/ln(a\*x^n)^(3/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(a\*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate(1/(x^2\*log(a\*x^n)^(3/2)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(a\*x^n)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/ln(a\*x\*\*n)\*\*(3/2),x)

[Out] Integral(1/(x\*\*2\*log(a\*x\*\*n)\*\*(3/2)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(a\*x^n)^(3/2),x, algorithm="giac")

[Out] integrate(1/(x^2\*log(a\*x^n)^(3/2)), x)

$$3.141 \quad \int \frac{1}{x^3 \log^2(ax^n)} dx$$

**Optimal.** Leaf size=69

$$-\frac{2\sqrt{2\pi}(ax^n)^{2/n} \operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}x^2} - \frac{2}{nx^2\sqrt{\log(ax^n)}}$$

[Out]  $(-2*\operatorname{Sqrt}[2*\operatorname{Pi}]*(a*x^n)^{(2/n)}*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/\operatorname{Sqrt}[n]])/(n^{(3/2)}*x^2) - 2/(n*x^2*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])$

**Rubi [A]** time = 0.0571192, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2306, 2310, 2180, 2205}

$$-\frac{2\sqrt{2\pi}(ax^n)^{2/n} \operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}x^2} - \frac{2}{nx^2\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[1/(x^3*\operatorname{Log}[a*x^n]^{(3/2)}), x]$

[Out]  $(-2*\operatorname{Sqrt}[2*\operatorname{Pi}]*(a*x^n)^{(2/n)}*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])/\operatorname{Sqrt}[n]])/(n^{(3/2)}*x^2) - 2/(n*x^2*\operatorname{Sqrt}[\operatorname{Log}[a*x^n]])$

#### Rule 2306

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]^{(p_.)}*((d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(d*x)^{(m+1)}*(a + b*\operatorname{Log}[c*x^n])^{(p+1)} / (b*d*n*(p+1)), x] - \operatorname{Dist}[(m+1)/(b*n*(p+1)), \operatorname{Int}[(d*x)^m*(a + b*\operatorname{Log}[c*x^n])^{(p+1)}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m, n\}, x \ \&\& \operatorname{NeQ}[m, -1] \ \&\& \operatorname{LtQ}[p, -1]$

#### Rule 2310

$\operatorname{Int}[(a_.) + \operatorname{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]^{(p_.)}*((d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(d*x)^{(m+1)} / (d*n*(c*x^n)^{(m+1)/n}), \operatorname{Subst}[\operatorname{Int}[E^{((m+1)*x/n)}*(a + b*x)^p, x], x, \operatorname{Log}[c*x^n]], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m, n, p\}, x]$

#### Rule 2180

$\operatorname{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_.)))}/\operatorname{Sqrt}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x \ \&\& \operatorname{!}\$UseGamma == \operatorname{True}$

#### Rule 2205

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^2)}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(c + d*x)*\operatorname{Rt}[-(b*\operatorname{Log}[F]), 2]])/(2*d*\operatorname{Rt}[-(b*\operatorname{Log}[F]), 2]), x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d\}, x \ \&\& \operatorname{NegQ}[b]$

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 \log^{\frac{3}{2}}(ax^n)} dx &= -\frac{2}{nx^2 \sqrt{\log(ax^n)}} - \frac{4 \int \frac{1}{x^3 \sqrt{\log(ax^n)}} dx}{n} \\
&= -\frac{2}{nx^2 \sqrt{\log(ax^n)}} - \frac{(4(ax^n)^{2/n}) \text{Subst}\left(\int \frac{e^{-\frac{2x}{n}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n^2 x^2} \\
&= -\frac{2}{nx^2 \sqrt{\log(ax^n)}} - \frac{(8(ax^n)^{2/n}) \text{Subst}\left(\int e^{-\frac{2x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{n^2 x^2} \\
&= -\frac{2\sqrt{2\pi} (ax^n)^{2/n} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2} x^2} - \frac{2}{nx^2 \sqrt{\log(ax^n)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0468122, size = 66, normalized size = 0.96

$$\frac{2\left(\sqrt{2}(ax^n)^{2/n} \sqrt{\frac{\log(ax^n)}{n}} \operatorname{Gamma}\left(\frac{1}{2}, \frac{2\log(ax^n)}{n}\right) - 1\right)}{nx^2 \sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*Log[a\*x^n]^(3/2)),x]

[Out] (2\*(-1 + Sqrt[2]\*(a\*x^n)^(2/n)\*Gamma[1/2, (2\*Log[a\*x^n])/n]\*Sqrt[Log[a\*x^n]/n]))/(n\*x^2\*Sqrt[Log[a\*x^n]])

**Maple [F]** time = 0.168, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} (\ln(ax^n))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/ln(a\*x^n)^(3/2),x)

[Out] int(1/x^3/ln(a\*x^n)^(3/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(a\*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate(1/(x^3\*log(a\*x^n)^(3/2)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(a\*x^n)^(3/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/ln(a\*x\*\*n)\*\*(3/2),x)

[Out] Integral(1/(x\*\*3\*log(a\*x\*\*n)\*\*(3/2)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(a\*x^n)^(3/2),x, algorithm="giac")

[Out] integrate(1/(x^3\*log(a\*x^n)^(3/2)), x)



$$3.142 \quad \int \frac{x^3}{5 \log^2(ax^n)} dx$$

**Optimal.** Leaf size=87

$$\frac{32\sqrt{\pi}x^4 (ax^n)^{-4/n} \operatorname{Erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{16x^4}{3n^2\sqrt{\log(ax^n)}} - \frac{2x^4}{3n \log^{\frac{3}{2}}(ax^n)}$$

[Out] (32\*sqrt(Pi)\*x^4\*Erfi[(2\*sqrt(Log[a\*x^n])/sqrt(n))]/(3\*n^(5/2)\*(a\*x^n)^(4/n)) - (2\*x^4)/(3\*n\*Log[a\*x^n]^(3/2)) - (16\*x^4)/(3\*n^2\*sqrt(Log[a\*x^n]))

**Rubi [A]** time = 0.072671, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2306, 2310, 2180, 2204}

$$\frac{32\sqrt{\pi}x^4 (ax^n)^{-4/n} \operatorname{Erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{16x^4}{3n^2\sqrt{\log(ax^n)}} - \frac{2x^4}{3n \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In] Int[x^3/Log[a\*x^n]^(5/2), x]

[Out] (32\*sqrt(Pi)\*x^4\*Erfi[(2\*sqrt(Log[a\*x^n])/sqrt(n))]/(3\*n^(5/2)\*(a\*x^n)^(4/n)) - (2\*x^4)/(3\*n\*Log[a\*x^n]^(3/2)) - (16\*x^4)/(3\*n^2\*sqrt(Log[a\*x^n]))

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*sqrt(Pi)\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]]/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\log^{\frac{5}{2}}(ax^n)} dx &= -\frac{2x^4}{3n \log^{\frac{3}{2}}(ax^n)} + \frac{8 \int \frac{x^3}{\log^2(ax^n)} dx}{3n} \\
&= -\frac{2x^4}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{16x^4}{3n^2 \sqrt{\log(ax^n)}} + \frac{64 \int \frac{x^3}{\sqrt{\log(ax^n)}} dx}{3n^2} \\
&= -\frac{2x^4}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{16x^4}{3n^2 \sqrt{\log(ax^n)}} + \frac{(64x^4 (ax^n)^{-4/n}) \text{Subst} \left( \int \frac{e^{\frac{4x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n) \right)}{3n^3} \\
&= -\frac{2x^4}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{16x^4}{3n^2 \sqrt{\log(ax^n)}} + \frac{(128x^4 (ax^n)^{-4/n}) \text{Subst} \left( \int e^{\frac{4x^2}{n}} dx, x, \sqrt{\log(ax^n)} \right)}{3n^3} \\
&= \frac{32\sqrt{\pi}x^4 (ax^n)^{-4/n} \operatorname{erfi} \left( \frac{2\sqrt{\log(ax^n)}}{\sqrt{n}} \right)}{3n^{5/2}} - \frac{2x^4}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{16x^4}{3n^2 \sqrt{\log(ax^n)}}
\end{aligned}$$

**Mathematica [A]** time = 0.059149, size = 87, normalized size = 1.

$$\frac{2x^4 (ax^n)^{-4/n} \left( 16n \left( -\frac{\log(ax^n)}{n} \right)^{3/2} \operatorname{Gamma} \left( \frac{1}{2}, -\frac{4\log(ax^n)}{n} \right) + (ax^n)^{4/n} (8 \log(ax^n) + n) \right)}{3n^2 \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Log[a\*x^n]^(5/2),x]

[Out] (-2\*x^4\*(16\*n\*Gamma[1/2, (-4\*Log[a\*x^n])/n]\*(-Log[a\*x^n]/n))^(3/2) + (a\*x^n)^(4/n)\*(n + 8\*Log[a\*x^n]))/(3\*n^2\*(a\*x^n)^(4/n)\*Log[a\*x^n]^(3/2))

**Maple [F]** time = 0.17, size = 0, normalized size = 0.

$$\int x^3 (\ln(ax^n))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/ln(a\*x^n)^(5/2),x)

[Out] int(x^3/ln(a\*x^n)^(5/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/log(a\*x^n)^(5/2),x, algorithm="maxima")

[Out] integrate(x<sup>3</sup>/log(a\*x<sup>n</sup>)<sup>(5/2)</sup>, x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>3</sup>/log(a\*x<sup>n</sup>)<sup>(5/2)</sup>,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/ln(a\*x\*\*n)\*\*(5/2),x)

[Out] Integral(x\*\*3/log(a\*x\*\*n)\*\*(5/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>3</sup>/log(a\*x<sup>n</sup>)<sup>(5/2)</sup>,x, algorithm="giac")

[Out] integrate(x<sup>3</sup>/log(a\*x<sup>n</sup>)<sup>(5/2)</sup>, x)

$$3.143 \quad \int \frac{x^2}{5 \log^2(ax^n)} dx$$

**Optimal.** Leaf size=89

$$\frac{4\sqrt{3}\pi x^3 (ax^n)^{-3/n} \operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{5/2}} - \frac{4x^3}{n^2\sqrt{\log(ax^n)}} - \frac{2x^3}{3n \log^{\frac{3}{2}}(ax^n)}$$

[Out] (4\*Sqrt[3\*Pi]\*x^3\*Erfi[(Sqrt[3]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(n^(5/2)\*(a\*x^n)^(3/n)) - (2\*x^3)/(3\*n\*Log[a\*x^n]^(3/2)) - (4\*x^3)/(n^2\*Sqrt[Log[a\*x^n]])

**Rubi [A]** time = 0.0730742, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2306, 2310, 2180, 2204}

$$\frac{4\sqrt{3}\pi x^3 (ax^n)^{-3/n} \operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{5/2}} - \frac{4x^3}{n^2\sqrt{\log(ax^n)}} - \frac{2x^3}{3n \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In] Int[x^2/Log[a\*x^n]^(5/2), x]

[Out] (4\*Sqrt[3\*Pi]\*x^3\*Erfi[(Sqrt[3]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(n^(5/2)\*(a\*x^n)^(3/n)) - (2\*x^3)/(3\*n\*Log[a\*x^n]^(3/2)) - (4\*x^3)/(n^2\*Sqrt[Log[a\*x^n]])

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\log^{\frac{5}{2}}(ax^n)} dx &= -\frac{2x^3}{3n \log^{\frac{3}{2}}(ax^n)} + \frac{2 \int \frac{x^2}{\log^{\frac{3}{2}}(ax^n)} dx}{n} \\
&= -\frac{2x^3}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{4x^3}{n^2 \sqrt{\log(ax^n)}} + \frac{12 \int \frac{x^2}{\sqrt{\log(ax^n)}} dx}{n^2} \\
&= -\frac{2x^3}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{4x^3}{n^2 \sqrt{\log(ax^n)}} + \frac{(12x^3 (ax^n)^{-3/n}) \text{Subst}\left(\int \frac{e^{\frac{3x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n^3} \\
&= -\frac{2x^3}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{4x^3}{n^2 \sqrt{\log(ax^n)}} + \frac{(24x^3 (ax^n)^{-3/n}) \text{Subst}\left(\int e^{\frac{3x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{n^3} \\
&= \frac{4\sqrt{3}\pi x^3 (ax^n)^{-3/n} \operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{5/2}} - \frac{2x^3}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{4x^3}{n^2 \sqrt{\log(ax^n)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0636208, size = 92, normalized size = 1.03

$$\frac{2x^3 (ax^n)^{-3/n} \left(6\sqrt{3}n \left(-\frac{\log(ax^n)}{n}\right)^{3/2} \Gamma\left(\frac{1}{2}, -\frac{3\log(ax^n)}{n}\right) + (ax^n)^{3/n} (6\log(ax^n) + n)\right)}{3n^2 \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Log[a\*x^n]^(5/2), x]

[Out] (-2\*x^3\*(6\*Sqrt[3]\*n\*Gamma[1/2, (-3\*Log[a\*x^n])/n])\*(-(Log[a\*x^n]/n))^(3/2) + (a\*x^n)^(3/n)\*(n + 6\*Log[a\*x^n]))/(3\*n^2\*(a\*x^n)^(3/n)\*Log[a\*x^n]^(3/2))

**Maple [F]** time = 0.168, size = 0, normalized size = 0.

$$\int x^2 (\ln(ax^n))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/ln(a\*x^n)^(5/2), x)

[Out] int(x^2/ln(a\*x^n)^(5/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/log(a\*x^n)^(5/2), x, algorithm="maxima")

[Out] integrate(x<sup>2</sup>/log(a\*x<sup>n</sup>)<sup>(5/2)</sup>, x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>2</sup>/log(a\*x<sup>n</sup>)<sup>(5/2)</sup>,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/ln(a\*x\*\*n)\*\*(5/2),x)

[Out] Integral(x\*\*2/log(a\*x\*\*n)\*\*(5/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>2</sup>/log(a\*x<sup>n</sup>)<sup>(5/2)</sup>,x, algorithm="giac")

[Out] integrate(x<sup>2</sup>/log(a\*x<sup>n</sup>)<sup>(5/2)</sup>, x)

$$3.144 \quad \int \frac{x}{\log^2(ax^n)} dx$$

**Optimal.** Leaf size=93

$$\frac{8\sqrt{2\pi}x^2(ax^n)^{-2/n} \operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{8x^2}{3n^2\sqrt{\log(ax^n)}} - \frac{2x^2}{3n\log^{\frac{3}{2}}(ax^n)}$$

[Out] (8\*Sqrt[2\*Pi]\*x^2\*Erfi[(Sqrt[2]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(3\*n^(5/2)\*(a\*x^n)^(2/n)) - (2\*x^2)/(3\*n\*Log[a\*x^n]^(3/2)) - (8\*x^2)/(3\*n^2\*Sqrt[Log[a\*x^n]])

**Rubi [A]** time = 0.0579577, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2306, 2310, 2180, 2204}

$$\frac{8\sqrt{2\pi}x^2(ax^n)^{-2/n} \operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{8x^2}{3n^2\sqrt{\log(ax^n)}} - \frac{2x^2}{3n\log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In] Int[x/Log[a\*x^n]^(5/2),x]

[Out] (8\*Sqrt[2\*Pi]\*x^2\*Erfi[(Sqrt[2]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(3\*n^(5/2)\*(a\*x^n)^(2/n)) - (2\*x^2)/(3\*n\*Log[a\*x^n]^(3/2)) - (8\*x^2)/(3\*n^2\*Sqrt[Log[a\*x^n]])

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned}
\int \frac{x}{\log^{\frac{5}{2}}(ax^n)} dx &= -\frac{2x^2}{3n \log^{\frac{3}{2}}(ax^n)} + \frac{4 \int \frac{x}{\log^{\frac{3}{2}}(ax^n)} dx}{3n} \\
&= -\frac{2x^2}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{8x^2}{3n^2 \sqrt{\log(ax^n)}} + \frac{16 \int \frac{x}{\sqrt{\log(ax^n)}} dx}{3n^2} \\
&= -\frac{2x^2}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{8x^2}{3n^2 \sqrt{\log(ax^n)}} + \frac{(16x^2 (ax^n)^{-2/n}) \text{Subst} \left( \int \frac{e^{\frac{2x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n) \right)}{3n^3} \\
&= -\frac{2x^2}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{8x^2}{3n^2 \sqrt{\log(ax^n)}} + \frac{(32x^2 (ax^n)^{-2/n}) \text{Subst} \left( \int e^{\frac{2x^2}{n}} dx, x, \sqrt{\log(ax^n)} \right)}{3n^3} \\
&= \frac{8\sqrt{2\pi}x^2 (ax^n)^{-2/n} \operatorname{erfi} \left( \frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}} \right)}{3n^{5/2}} - \frac{2x^2}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{8x^2}{3n^2 \sqrt{\log(ax^n)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0636767, size = 92, normalized size = 0.99

$$\frac{2x^2 (ax^n)^{-2/n} \left( 4\sqrt{2}n \left( -\frac{\log(ax^n)}{n} \right)^{3/2} \Gamma \left( \frac{1}{2}, -\frac{2\log(ax^n)}{n} \right) + (ax^n)^{2/n} (4\log(ax^n) + n) \right)}{3n^2 \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In] Integrate[x/Log[a\*x^n]^(5/2),x]

[Out] (-2\*x^2\*(4\*Sqrt[2]\*n\*Gamma[1/2, (-2\*Log[a\*x^n])/n]\*(-Log[a\*x^n]/n))^(3/2) + (a\*x^n)^(2/n)\*(n + 4\*Log[a\*x^n]))/(3\*n^2\*(a\*x^n)^(2/n)\*Log[a\*x^n]^(3/2))

**Maple [F]** time = 0.168, size = 0, normalized size = 0.

$$\int x (\ln(ax^n))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/ln(a\*x^n)^(5/2),x)

[Out] int(x/ln(a\*x^n)^(5/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(a\*x^n)^(5/2),x, algorithm="maxima")



[Out] integrate(x/log(a\*x^n)^(5/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(a\*x^n)^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/ln(a\*x\*\*n)\*\*(5/2),x)

[Out] Integral(x/log(a\*x\*\*n)\*\*(5/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/log(a\*x^n)^(5/2),x, algorithm="giac")

[Out] integrate(x/log(a\*x^n)^(5/2), x)

$$3.145 \quad \int \frac{1}{\log^{\frac{5}{2}}(ax^n)} dx$$

**Optimal.** Leaf size=80

$$\frac{4\sqrt{\pi}x(ax^n)^{-1/n} \operatorname{Erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{4x}{3n^2\sqrt{\log(ax^n)}} - \frac{2x}{3n \log^{\frac{3}{2}}(ax^n)}$$

[Out] (4\*Sqrt[Pi]\*x\*Erfi[Sqrt[Log[a\*x^n]]/Sqrt[n]])/(3\*n^(5/2)\*(a\*x^n)^n^(-1)) - (2\*x)/(3\*n\*Log[a\*x^n]^(3/2)) - (4\*x)/(3\*n^2\*Sqrt[Log[a\*x^n]])

**Rubi [A]** time = 0.0372524, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {2297, 2300, 2180, 2204}

$$\frac{4\sqrt{\pi}x(ax^n)^{-1/n} \operatorname{Erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{4x}{3n^2\sqrt{\log(ax^n)}} - \frac{2x}{3n \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In] Int[Log[a\*x^n]^(-5/2), x]

[Out] (4\*Sqrt[Pi]\*x\*Erfi[Sqrt[Log[a\*x^n]]/Sqrt[n]])/(3\*n^(5/2)\*(a\*x^n)^n^(-1)) - (2\*x)/(3\*n\*Log[a\*x^n]^(3/2)) - (4\*x)/(3\*n^2\*Sqrt[Log[a\*x^n]])

#### Rule 2297

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_), x\_Symbol] :> Simp[(x\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*n\*(p + 1)), x] - Dist[1/(b\*n\*(p + 1)), Int[(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && IntegerQ[2\*p]

#### Rule 2300

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_), x\_Symbol] :> Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[E^(x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2)), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{\log^{\frac{5}{2}}(ax^n)} dx &= -\frac{2x}{3n \log^{\frac{3}{2}}(ax^n)} + \frac{2 \int \frac{1}{\log^{\frac{3}{2}}(ax^n)} dx}{3n} \\
&= -\frac{2x}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{4x}{3n^2 \sqrt{\log(ax^n)}} + \frac{4 \int \frac{1}{\sqrt{\log(ax^n)}} dx}{3n^2} \\
&= -\frac{2x}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{4x}{3n^2 \sqrt{\log(ax^n)}} + \frac{(4x(ax^n)^{-1/n}) \text{Subst}\left(\int \frac{e^{\frac{x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{3n^3} \\
&= -\frac{2x}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{4x}{3n^2 \sqrt{\log(ax^n)}} + \frac{(8x(ax^n)^{-1/n}) \text{Subst}\left(\int e^{\frac{x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{3n^3} \\
&= \frac{4\sqrt{\pi}x(ax^n)^{-1/n} \operatorname{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{2x}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{4x}{3n^2 \sqrt{\log(ax^n)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0482157, size = 83, normalized size = 1.04

$$-\frac{2x(ax^n)^{-1/n} \left(2n \left(-\frac{\log(ax^n)}{n}\right)^{3/2} \operatorname{Gamma}\left(\frac{1}{2}, -\frac{\log(ax^n)}{n}\right) + (ax^n)^{\frac{1}{n}} (2 \log(ax^n) + n)\right)}{3n^2 \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In] Integrate[Log[a\*x^n]^(-5/2), x]

[Out] (-2\*x\*(2\*n\*Gamma[1/2, -(Log[a\*x^n]/n)]\*(-(Log[a\*x^n]/n))^(3/2) + (a\*x^n)^n\*(-1)\*(n + 2\*Log[a\*x^n]))/(3\*n^2\*(a\*x^n)^n\*(-1)\*Log[a\*x^n]^(3/2))

**Maple [F]** time = 0.174, size = 0, normalized size = 0.

$$\int (\ln(ax^n))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/ln(a\*x^n)^(5/2), x)

[Out] int(1/ln(a\*x^n)^(5/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(a\*x^n)^(5/2), x, algorithm="maxima")

[Out] integrate(log(a\*x^n)^(-5/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(a\*x^n)^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/ln(a\*x\*\*n)\*\*(5/2),x)

[Out] Integral(log(a\*x\*\*n)\*\*(-5/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/log(a\*x^n)^(5/2),x, algorithm="giac")

[Out] integrate(log(a\*x^n)^(-5/2), x)

$$3.146 \quad \int \frac{1}{x \log^{\frac{5}{2}}(ax^n)} dx$$

**Optimal.** Leaf size=17

$$-\frac{2}{3n \log^{\frac{3}{2}}(ax^n)}$$

[Out]  $-2/(3*n*\text{Log}[a*x^n]^{(3/2)})$

**Rubi [A]** time = 0.0139537, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2302, 30}

$$-\frac{2}{3n \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x*\text{Log}[a*x^n]^{(5/2)}), x]$

[Out]  $-2/(3*n*\text{Log}[a*x^n]^{(3/2)})$

#### Rule 2302

$\text{Int}[(a_. + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.))^{(p_.)}/(x_.), x\_Symbol] \text{ :> Dist}[1/(b*n), \text{Subst}[\text{Int}[x^p, x], x, a + b*\text{Log}[c*x^n]], x] \text{ /; FreeQ}\{a, b, c, n, p\}, x]$

#### Rule 30

$\text{Int}[(x_)^{(m_.)}, x\_Symbol] \text{ :> Simp}[x^{(m+1)}/(m+1), x] \text{ /; FreeQ}[m, x] \ \&\& \ \text{NeQ}[m, -1]$

#### Rubi steps

$$\begin{aligned} \int \frac{1}{x \log^{\frac{5}{2}}(ax^n)} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^{5/2}} dx, x, \log(ax^n)\right)}{n} \\ &= -\frac{2}{3n \log^{\frac{3}{2}}(ax^n)} \end{aligned}$$

**Mathematica [A]** time = 0.001796, size = 17, normalized size = 1.

$$-\frac{2}{3n \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[1/(x*\text{Log}[a*x^n]^{(5/2)}), x]$

[Out]  $-2/(3*n*\text{Log}[a*x^n]^{(3/2)})$

---

**Maple [A]** time = 0.039, size = 14, normalized size = 0.8

$$-\frac{2}{3n} (\ln(ax^n))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/ln(a*x^n)^(5/2),x)`

[Out]  $-2/3/n/\ln(a*x^n)^{(3/2)}$

---

**Maxima [A]** time = 0.979796, size = 18, normalized size = 1.06

$$-\frac{2}{3n \log(ax^n)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/log(a*x^n)^(5/2),x, algorithm="maxima")`

[Out]  $-2/3/(n*\log(a*x^n)^{(3/2)})$

---

**Fricas [B]** time = 0.984485, size = 108, normalized size = 6.35

$$\frac{2\sqrt{n\log(x) + \log(a)}}{3(n^3\log(x)^2 + 2n^2\log(a)\log(x) + n\log(a)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/log(a*x^n)^(5/2),x, algorithm="fricas")`

[Out]  $-2/3*\text{sqrt}(n*\log(x) + \log(a))/(n^3*\log(x)^2 + 2*n^2*\log(a)*\log(x) + n*\log(a)^2)$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/ln(a*x**n)**(5/2),x)`

[Out] Timed out

---

**Giac [A]** time = 1.27013, size = 19, normalized size = 1.12

$$-\frac{2}{3(n \log(x) + \log(a))^{\frac{3}{2}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/log(a\*x^n)^(5/2),x, algorithm="giac")

[Out] -2/3/((n\*log(x) + log(a))^(3/2)\*n)

$$3.147 \quad \int \frac{1}{x^2 \log^2(ax^n)} dx$$

**Optimal.** Leaf size=84

$$\frac{4\sqrt{\pi}(ax^n)^{\frac{1}{n}} \operatorname{Erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}x} + \frac{4}{3n^2x\sqrt{\log(ax^n)}} - \frac{2}{3nx \log^{\frac{3}{2}}(ax^n)}$$

[Out]  $(4*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{a}*x^n)^n^{-1}*\operatorname{Erf}[\operatorname{Sqrt}[\operatorname{Log}[\operatorname{a}*x^n]]/\operatorname{Sqrt}[n]])/(3*n^{(5/2)*x}) - 2/(3*n*x*\operatorname{Log}[\operatorname{a}*x^n]^{(3/2)}) + 4/(3*n^2*x*\operatorname{Sqrt}[\operatorname{Log}[\operatorname{a}*x^n]])$

**Rubi [A]** time = 0.0688181, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2306, 2310, 2180, 2205}

$$\frac{4\sqrt{\pi}(ax^n)^{\frac{1}{n}} \operatorname{Erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}x} + \frac{4}{3n^2x\sqrt{\log(ax^n)}} - \frac{2}{3nx \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[1/(x^2*\operatorname{Log}[\operatorname{a}*x^n]^{(5/2)}), x]$

[Out]  $(4*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{a}*x^n)^n^{-1}*\operatorname{Erf}[\operatorname{Sqrt}[\operatorname{Log}[\operatorname{a}*x^n]]/\operatorname{Sqrt}[n]])/(3*n^{(5/2)*x}) - 2/(3*n*x*\operatorname{Log}[\operatorname{a}*x^n]^{(3/2)}) + 4/(3*n^2*x*\operatorname{Sqrt}[\operatorname{Log}[\operatorname{a}*x^n]])$

#### Rule 2306

$\operatorname{Int}[(\operatorname{a}_.) + \operatorname{Log}[(\operatorname{c}_.)*(x_)^{(n_.)}]*(\operatorname{b}_.)^{(p_.)}*((\operatorname{d}_.)*(x_))^{(m_.)}], x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{d}*x)^{(m+1)}*(\operatorname{a} + \operatorname{b}*\operatorname{Log}[\operatorname{c}*x^n])^{(p+1)}]/(\operatorname{b}*d*n*(p+1)), x] - \operatorname{Dist}[(m+1)/(\operatorname{b}*n*(p+1)), \operatorname{Int}[(\operatorname{d}*x)^m*(\operatorname{a} + \operatorname{b}*\operatorname{Log}[\operatorname{c}*x^n])^{(p+1)}, x], x] /; \operatorname{FreeQ}\{\operatorname{a}, \operatorname{b}, \operatorname{c}, \operatorname{d}, \operatorname{m}, \operatorname{n}\}, x] \&\& \operatorname{NeQ}[m, -1] \&\& \operatorname{LtQ}[p, -1]$

#### Rule 2310

$\operatorname{Int}[(\operatorname{a}_.) + \operatorname{Log}[(\operatorname{c}_.)*(x_)^{(n_.)}]*(\operatorname{b}_.)^{(p_.)}*((\operatorname{d}_.)*(x_))^{(m_.)}], x\_Symbol] \rightarrow \operatorname{Dist}[(\operatorname{d}*x)^{(m+1)}/(\operatorname{d}*n*(\operatorname{c}*x^n)^{((m+1)/n)}), \operatorname{Subst}[\operatorname{Int}[E^{((m+1)*x/n)}*(\operatorname{a} + \operatorname{b}*x)^p, x], x, \operatorname{Log}[\operatorname{c}*x^n]], x] /; \operatorname{FreeQ}\{\operatorname{a}, \operatorname{b}, \operatorname{c}, \operatorname{d}, \operatorname{m}, \operatorname{n}, \operatorname{p}\}, x]$

#### Rule 2180

$\operatorname{Int}[(F_)^{(g_.)}*((e_.) + (f_.)*(x_))/\operatorname{Sqrt}[(\operatorname{c}_.) + (\operatorname{d}_.)*(x_)], x\_Symbol] \rightarrow \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g*(e - (c*f)/d) + (f*g*x^2)/d)}, x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\& !\$UseGamma === \operatorname{True}$

#### Rule 2205

$\operatorname{Int}[(F_)^{((\operatorname{a}_.) + (\operatorname{b}_.)*((\operatorname{c}_.) + (\operatorname{d}_.)*(x_))^{2}))}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{c} + \operatorname{d}*x)*\operatorname{Rt}[-(\operatorname{b}*\operatorname{Log}[F]), 2]])/(2*d*\operatorname{Rt}[-(\operatorname{b}*\operatorname{Log}[F]), 2]), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x] \&\& \operatorname{NegQ}[b]$

#### Rubi steps



$$\begin{aligned}
\int \frac{1}{x^2 \log^{\frac{5}{2}}(ax^n)} dx &= -\frac{2}{3nx \log^{\frac{3}{2}}(ax^n)} - \frac{2 \int \frac{1}{x^2 \log^{\frac{3}{2}}(ax^n)} dx}{3n} \\
&= -\frac{2}{3nx \log^{\frac{3}{2}}(ax^n)} + \frac{4}{3n^2 x \sqrt{\log(ax^n)}} + \frac{4 \int \frac{1}{x^2 \sqrt{\log(ax^n)}} dx}{3n^2} \\
&= -\frac{2}{3nx \log^{\frac{3}{2}}(ax^n)} + \frac{4}{3n^2 x \sqrt{\log(ax^n)}} + \frac{\left(4(ax^n)^{\frac{1}{n}}\right) \text{Subst}\left(\int \frac{e^{-\frac{x}{n}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{3n^3 x} \\
&= -\frac{2}{3nx \log^{\frac{3}{2}}(ax^n)} + \frac{4}{3n^2 x \sqrt{\log(ax^n)}} + \frac{\left(8(ax^n)^{\frac{1}{n}}\right) \text{Subst}\left(\int e^{-\frac{x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{3n^3 x} \\
&= \frac{4\sqrt{\pi}(ax^n)^{\frac{1}{n}} \text{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2} x} - \frac{2}{3nx \log^{\frac{3}{2}}(ax^n)} + \frac{4}{3n^2 x \sqrt{\log(ax^n)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0542735, size = 70, normalized size = 0.83

$$\frac{2 \left( 2n (ax^n)^{\frac{1}{n}} \left( \frac{\log(ax^n)}{n} \right)^{3/2} \text{Gamma}\left(\frac{1}{2}, \frac{\log(ax^n)}{n}\right) - 2 \log(ax^n) + n \right)}{3n^2 x \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*Log[a\*x^n]^(5/2)),x]

[Out] (-2\*(n - 2\*Log[a\*x^n] + 2\*n\*(a\*x^n)^n^(-1)\*Gamma[1/2, Log[a\*x^n]/n]\*(Log[a\*x^n]/n)^(3/2)))/(3\*n^2\*x\*Log[a\*x^n]^(3/2))

**Maple [F]** time = 0.182, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (\ln(ax^n))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/ln(a\*x^n)^(5/2),x)

[Out] int(1/x^2/ln(a\*x^n)^(5/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(a\*x^n)^(5/2),x, algorithm="maxima")

[Out] integrate(1/(x^2\*log(a\*x^n)^(5/2)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(a\*x^n)^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/ln(a\*x\*\*n)\*\*(5/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/log(a\*x^n)^(5/2),x, algorithm="giac")

[Out] integrate(1/(x^2\*log(a\*x^n)^(5/2)), x)

$$3.148 \quad \int \frac{1}{x^3 \log^{\frac{5}{2}}(ax^n)} dx$$

**Optimal.** Leaf size=93

$$\frac{8\sqrt{2\pi} (ax^n)^{2/n} \operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}x^2} + \frac{8}{3n^2x^2\sqrt{\log(ax^n)}} - \frac{2}{3nx^2 \log^{\frac{3}{2}}(ax^n)}$$

[Out] (8\*Sqrt[2\*Pi]\*(a\*x^n)^(2/n)\*Erf[(Sqrt[2]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(3\*n^(5/2)\*x^2) - 2/(3\*n\*x^2\*Log[a\*x^n]^(3/2)) + 8/(3\*n^2\*x^2\*Sqrt[Log[a\*x^n]])

**Rubi [A]** time = 0.0755038, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2306, 2310, 2180, 2205}

$$\frac{8\sqrt{2\pi} (ax^n)^{2/n} \operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}x^2} + \frac{8}{3n^2x^2\sqrt{\log(ax^n)}} - \frac{2}{3nx^2 \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*Log[a\*x^n]^(5/2)),x]

[Out] (8\*Sqrt[2\*Pi]\*(a\*x^n)^(2/n)\*Erf[(Sqrt[2]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(3\*n^(5/2)\*x^2) - 2/(3\*n\*x^2\*Log[a\*x^n]^(3/2)) + 8/(3\*n^2\*x^2\*Sqrt[Log[a\*x^n]])

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^(m + 1)/n), Subst[Int[E^((m + 1)\*x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 \log^{\frac{5}{2}}(ax^n)} dx &= -\frac{2}{3nx^2 \log^{\frac{3}{2}}(ax^n)} - \frac{4 \int \frac{1}{x^3 \log^{\frac{3}{2}}(ax^n)} dx}{3n} \\
&= -\frac{2}{3nx^2 \log^{\frac{3}{2}}(ax^n)} + \frac{8}{3n^2 x^2 \sqrt{\log(ax^n)}} + \frac{16 \int \frac{1}{x^3 \sqrt{\log(ax^n)}} dx}{3n^2} \\
&= -\frac{2}{3nx^2 \log^{\frac{3}{2}}(ax^n)} + \frac{8}{3n^2 x^2 \sqrt{\log(ax^n)}} + \frac{(16 (ax^n)^{2/n}) \text{Subst} \left( \int \frac{e^{-\frac{2x}{n}}}{\sqrt{x}} dx, x, \log(ax^n) \right)}{3n^3 x^2} \\
&= -\frac{2}{3nx^2 \log^{\frac{3}{2}}(ax^n)} + \frac{8}{3n^2 x^2 \sqrt{\log(ax^n)}} + \frac{(32 (ax^n)^{2/n}) \text{Subst} \left( \int e^{-\frac{2x^2}{n}} dx, x, \sqrt{\log(ax^n)} \right)}{3n^3 x^2} \\
&= \frac{8\sqrt{2\pi} (ax^n)^{2/n} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2} x^2} - \frac{2}{3nx^2 \log^{\frac{3}{2}}(ax^n)} + \frac{8}{3n^2 x^2 \sqrt{\log(ax^n)}}
\end{aligned}$$

**Mathematica [A]** time = 0.052848, size = 78, normalized size = 0.84

$$\frac{2 \left( 4\sqrt{2}n (ax^n)^{2/n} \left( \frac{\log(ax^n)}{n} \right)^{3/2} \Gamma\left(\frac{1}{2}, \frac{2\log(ax^n)}{n}\right) - 4\log(ax^n) + n \right)}{3n^2 x^2 \log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*Log[a\*x^n]^(5/2)),x]

[Out] (-2\*(n - 4\*Log[a\*x^n] + 4\*Sqrt[2]\*n\*(a\*x^n)^(2/n)\*Gamma[1/2, (2\*Log[a\*x^n])/n]\*(Log[a\*x^n]/n)^(3/2)))/(3\*n^2\*x^2\*Log[a\*x^n]^(3/2))

**Maple [F]** time = 0.178, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} (\ln(ax^n))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/ln(a\*x^n)^(5/2),x)

[Out] int(1/x^3/ln(a\*x^n)^(5/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(a\*x^n)^(5/2),x, algorithm="maxima")

[Out] integrate(1/(x^3\*log(a\*x^n)^(5/2)), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(a\*x^n)^(5/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/ln(a\*x\*\*n)\*\*(5/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/log(a\*x^n)^(5/2),x, algorithm="giac")

[Out] integrate(1/(x^3\*log(a\*x^n)^(5/2)), x)

$$3.149 \quad \int (dx)^m \left( a + \frac{a(1+m) \log(cx^n)}{n} \right) dx$$

**Optimal.** Leaf size=21

$$\frac{a(dx)^{m+1} \log(cx^n)}{dn}$$

[Out] (a\*(d\*x)^(1 + m)\*Log[c\*x^n])/(d\*n)

**Rubi [A]** time = 0.0204304, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$ , Rules used = {2303}

$$\frac{a(dx)^{m+1} \log(cx^n)}{dn}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m\*(a + (a\*(1 + m)\*Log[c\*x^n])/n), x]

[Out] (a\*(d\*x)^(1 + m)\*Log[c\*x^n])/(d\*n)

**Rule 2303**

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[(b\*(d\*x)^(m + 1)\*Log[c\*x^n])/(d\*(m + 1)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && EqQ[a\*(m + 1) - b\*n, 0]

**Rubi steps**

$$\int (dx)^m \left( a + \frac{a(1+m) \log(cx^n)}{n} \right) dx = \frac{a(dx)^{1+m} \log(cx^n)}{dn}$$

**Mathematica [A]** time = 0.0113577, size = 17, normalized size = 0.81

$$\frac{ax(dx)^m \log(cx^n)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m\*(a + (a\*(1 + m)\*Log[c\*x^n])/n), x]

[Out] (a\*x\*(d\*x)^m\*Log[c\*x^n])/n

**Maple [C]** time = 0.117, size = 260, normalized size = 12.4

$$\frac{ax \ln(x^n)}{n} e^{\frac{m(-i \operatorname{csgn}(ix))^3 \pi + i \operatorname{csgn}(ix)^2 \operatorname{csgn}(id) \pi + i \operatorname{csgn}(ix)^2 \operatorname{csgn}(ix) \pi - i \pi \operatorname{csgn}(ix) \operatorname{csgn}(id) \operatorname{csgn}(ix) + 2 \ln(x) + 2 \ln(d)}{2}} + \frac{a \left( i \pi \operatorname{csgn}(ix^n) (\operatorname{csgn}(icx^n))^2 \right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+a*(1+m)*ln(c*x^n)/n),x)
```

```
[Out] a/n*x*exp(1/2*m*(-I*csgn(I*d*x)^3*Pi+I*csgn(I*d*x)^2*csgn(I*d)*Pi+I*csgn(I*d*x)^2*csgn(I*x)*Pi-I*csgn(I*d*x)*csgn(I*d)*csgn(I*x)*Pi+2*ln(x)+2*ln(d)))*ln(x^n)+1/2*a*(I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-I*Pi*csgn(I*c*x^n)^3+I*Pi*csgn(I*c*x^n)^2*csgn(I*c)+2*ln(c))*x/n*exp(1/2*m*(-I*csgn(I*d*x)^3*Pi+I*csgn(I*d*x)^2*csgn(I*d)*Pi+I*csgn(I*d*x)^2*csgn(I*x)*Pi-I*csgn(I*d*x)*csgn(I*d)*csgn(I*x)*Pi+2*ln(x)+2*ln(d)))
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+a*(1+m)*log(c*x^n)/n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 1.00632, size = 74, normalized size = 3.52

$$\frac{(anx \log(x) + ax \log(c))e^{(m \log(d) + m \log(x))}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+a*(1+m)*log(c*x^n)/n),x, algorithm="fricas")
```

```
[Out] (a*n*x*log(x) + a*x*log(c))*e^(m*log(d) + m*log(x))/n
```

**Sympy [A]** time = 1.35617, size = 27, normalized size = 1.29

$$ad^mxx^m \log(x) + \frac{ad^mxx^m \log(c)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+a*(1+m)*ln(c*x**n)/n),x)
```

```
[Out] a*d**m*x*x**m*log(x) + a*d**m*x*x**m*log(c)/n
```

**Giac [B]** time = 1.3511, size = 289, normalized size = 13.76

$$\frac{ad^2 \frac{1}{d} mxx^m |d|^{2m} \log(c)}{(d^2m + d^2)n} + \frac{ad^2 \frac{1}{d} xx^m |d|^{2m}}{d^2m + d^2} + \frac{ad^2 \frac{1}{d} xx^m |d|^{2m} \log(c)}{(d^2m + d^2)n} + \frac{ad^m m^2 xx^m \log(x)}{m^2 + 2m + 1} + \frac{2ad^m mxx^m \log(x)}{m^2 + 2m + 1} - \frac{a}{m^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+a*(1+m)*log(c*x^n)/n),x, algorithm="giac")
```

```
[Out] a*d^2*(1/d)^m*m*x*x^m*abs(d)^(2*m)*log(c)/((d^2*m + d^2)*n) + a*d^2*(1/d)^m
*x*x^m*abs(d)^(2*m)/(d^2*m + d^2) + a*d^2*(1/d)^m*x*x^m*abs(d)^(2*m)*log(c)
/((d^2*m + d^2)*n) + a*d^m*m^2*x*x^m*log(x)/(m^2 + 2*m + 1) + 2*a*d^m*m*x*x
^m*log(x)/(m^2 + 2*m + 1) - a*d^m*m*x*x^m/(m^2 + 2*m + 1) + a*d^m*x*x^m*log
(x)/(m^2 + 2*m + 1) - a*d^m*x*x^m/(m^2 + 2*m + 1)
```



### 3.150 $\int (dx)^m (a + b \log(cx^n))^3 dx$

**Optimal.** Leaf size=116

$$\frac{6b^2n^2(dx)^{m+1}(a+b\log(cx^n))}{d(m+1)^3} + \frac{(dx)^{m+1}(a+b\log(cx^n))^3}{d(m+1)} - \frac{3bn(dx)^{m+1}(a+b\log(cx^n))^2}{d(m+1)^2} - \frac{6b^3n^3(dx)^{m+1}}{d(m+1)^4}$$

[Out]  $(-6*b^3*n^3*(d*x)^(1+m))/(d*(1+m)^4) + (6*b^2*n^2*(d*x)^(1+m)*(a+b*\text{Log}[c*x^n]))/(d*(1+m)^3) - (3*b*n*(d*x)^(1+m)*(a+b*\text{Log}[c*x^n])^2)/(d*(1+m)^2) + ((d*x)^(1+m)*(a+b*\text{Log}[c*x^n])^3)/(d*(1+m))$

**Rubi [A]** time = 0.0905633, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2305, 2304}

$$\frac{6b^2n^2(dx)^{m+1}(a+b\log(cx^n))}{d(m+1)^3} + \frac{(dx)^{m+1}(a+b\log(cx^n))^3}{d(m+1)} - \frac{3bn(dx)^{m+1}(a+b\log(cx^n))^2}{d(m+1)^2} - \frac{6b^3n^3(dx)^{m+1}}{d(m+1)^4}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^3, x]

[Out]  $(-6*b^3*n^3*(d*x)^(1+m))/(d*(1+m)^4) + (6*b^2*n^2*(d*x)^(1+m)*(a+b*\text{Log}[c*x^n]))/(d*(1+m)^3) - (3*b*n*(d*x)^(1+m)*(a+b*\text{Log}[c*x^n])^2)/(d*(1+m)^2) + ((d*x)^(1+m)*(a+b*\text{Log}[c*x^n])^3)/(d*(1+m))$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m+1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m+1)), x] - Dist[(b\*n\*p)/(m+1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p-1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m+1)\*(a + b\*Log[c\*x^n]))/(d\*(m+1)), x] - Simp[(b\*n\*(d\*x)^(m+1))/(d\*(m+1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int (dx)^m (a + b \log(cx^n))^3 dx &= \frac{(dx)^{1+m} (a + b \log(cx^n))^3}{d(1+m)} - \frac{(3bn) \int (dx)^m (a + b \log(cx^n))^2 dx}{1+m} \\ &= -\frac{3bn(dx)^{1+m} (a + b \log(cx^n))^2}{d(1+m)^2} + \frac{(dx)^{1+m} (a + b \log(cx^n))^3}{d(1+m)} + \frac{(6b^2n^2) \int (dx)^m (a + b \log(cx^n)) dx}{(1+m)^2} \\ &= -\frac{6b^3n^3(dx)^{1+m}}{d(1+m)^4} + \frac{6b^2n^2(dx)^{1+m} (a + b \log(cx^n))}{d(1+m)^3} - \frac{3bn(dx)^{1+m} (a + b \log(cx^n))^2}{d(1+m)^2} + \frac{(a + b \log(cx^n))^3 (dx)^{1+m}}{d(1+m)} \end{aligned}$$

**Mathematica [A]** time = 0.0453227, size = 76, normalized size = 0.66

$$\frac{x(dx)^m \left( (a + b \log(cx^n))^3 - \frac{3bn((m+1)^2(a+b\log(cx^n))^2 + 2bn(bn-(m+1)(a+b\log(cx^n))))}{(m+1)^3} \right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*Log[c*x^n])^3,x]
```

```
[Out] (x*(d*x)^m*((a + b*Log[c*x^n])^3 - (3*b*n*((1 + m)^2*(a + b*Log[c*x^n])^2 + 2*b*n*(b*n - (1 + m)*(a + b*Log[c*x^n])))))/(1 + m)^3)/(1 + m)
```

**Maple [C]** time = 0.698, size = 9684, normalized size = 83.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*ln(c*x^n))^3,x)
```

```
[Out] result too large to display
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [B]** time = 1.05821, size = 1242, normalized size = 10.71

$$\left( (b^3 m^3 + 3 b^3 m^2 + 3 b^3 m + b^3) n^3 x \log(x)^3 + (b^3 m^3 + 3 b^3 m^2 + 3 b^3 m + b^3) x \log(c)^3 + 3 (ab^2 m^3 + 3 ab^2 m^2 + 3 ab^2 m + ab^2) n^3 x \log(x)^2 + \dots \right) e^{(m \log(d) + m \log(x))} / (m^4 + 4 m^3 + 6 m^2 + 4 m + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 3*b^3*m^2 + 3*b^3*m + b^3)*n^3*x*log(x)^3 + (b^3*m^3 + 3*b^3*m^2 + 3*b^3*m + b^3)*x*log(c)^3 + 3*(a*b^2*m^3 + 3*a*b^2*m^2 + 3*a*b^2*m + a*b^2 - (b^3*m^2 + 2*b^3*m + b^3)*n)*x*log(c)^2 + 3*(a^2*b*m^3 + 3*a^2*b*m^2 + 3*a^2*b*m + a^2*b + 2*(b^3*m + b^3)*n^2 - 2*(a*b^2*m^2 + 2*a*b^2*m + a*b^2)*n)*x*log(c) + 3*((b^3*m^3 + 3*b^3*m^2 + 3*b^3*m + b^3)*n^2*x*log(c) - ((b^3*m^2 + 2*b^3*m + b^3)*n^3 - (a*b^2*m^3 + 3*a*b^2*m^2 + 3*a*b^2*m + a*b^2)*n^2)*x)*log(x)^2 + (a^3*m^3 - 6*b^3*n^3 + 3*a^3*m^2 + 3*a^3*m + a^3 + 6*(a*b^2*m + a*b^2)*n^2 - 3*(a^2*b*m^2 + 2*a^2*b*m + a^2*b)*n)*x + 3*((b^3*m^3 + 3*b^3*m^2 + 3*b^3*m + b^3)*n*x*log(c)^2 - 2*((b^3*m^2 + 2*b^3*m + b^3)*n^2 - (a*b^2*m^3 + 3*a*b^2*m^2 + 3*a*b^2*m + a*b^2)*n)*x*log(c) + (2*(b^3*m + b^3)*n^3 - 2*(a*b^2*m^2 + 2*a*b^2*m + a*b^2)*n^2 + (a^2*b*m^3 + 3*a^2*b*m^2 + 3*a^2*b*m + a^2*b)*n)*x)*log(x))*e^(m*log(d) + m*log(x))/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1)
```

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*(a+b\*ln(c\*x\*\*n))\*\*3,x)

[Out] Exception raised: TypeError

**Giac [B]** time = 1.93021, size = 1530, normalized size = 13.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out] 
$$\begin{aligned} & b^3 d^m m^3 n^3 x^m \log(x)^3 / (m^4 + 4m^3 + 6m^2 + 4m + 1) + 3b^3 d^m m^2 n^3 x^m \log(x)^3 / (m^4 + 4m^3 + 6m^2 + 4m + 1) - 3b^3 d^m m^2 n^3 x^m \log(x)^2 / (m^4 + 4m^3 + 6m^2 + 4m + 1) + 3b^3 d^m m^2 n^2 x^m \log(c) \log(x)^2 / (m^3 + 3m^2 + 3m + 1) + 3b^3 d^m m n^3 x^m \log(x)^3 / (m^4 + 4m^3 + 6m^2 + 4m + 1) + 3a^2 b^2 d^m m^2 n^2 x^m \log(x)^2 / (m^3 + 3m^2 + 3m + 1) - 6b^3 d^m m n^3 x^m \log(x)^2 / (m^4 + 4m^3 + 6m^2 + 4m + 1) + 6b^3 d^m m n^2 x^m \log(c) \log(x)^2 / (m^3 + 3m^2 + 3m + 1) + b^3 d^m n^3 x^m \log(x)^3 / (m^4 + 4m^3 + 6m^2 + 4m + 1) + 6b^3 d^m m n^3 x^m \log(x) / (m^4 + 4m^3 + 6m^2 + 4m + 1) - 6b^3 d^m m n^2 x^m \log(c) \log(x) / (m^3 + 3m^2 + 3m + 1) + 3b^3 d^m m n x^m \log(c)^2 \log(x) / (m^2 + 2m + 1) + 6a^2 b^2 d^m m n^2 x^m \log(x)^2 / (m^3 + 3m^2 + 3m + 1) - 3b^3 d^m n^3 x^m \log(x)^2 / (m^4 + 4m^3 + 6m^2 + 4m + 1) + 3b^3 d^m n^2 x^m \log(c) \log(x)^2 / (m^3 + 3m^2 + 3m + 1) - 6a^2 b^2 d^m m n^2 x^m \log(x) / (m^3 + 3m^2 + 3m + 1) + 6b^3 d^m m n^3 x^m \log(x) / (m^4 + 4m^3 + 6m^2 + 4m + 1) + 6a^2 b^2 d^m m n x^m \log(c) \log(x) / (m^2 + 2m + 1) - 6b^3 d^m n^2 x^m \log(c) \log(x) / (m^3 + 3m^2 + 3m + 1) + 3b^3 d^m n x^m \log(c)^2 \log(x) / (m^2 + 2m + 1) + 3a^2 b^2 d^m n^2 x^m \log(x)^2 / (m^3 + 3m^2 + 3m + 1) - 6b^3 d^m n^3 x^m \log(x) / (m^4 + 4m^3 + 6m^2 + 4m + 1) + 6b^3 d^m n^2 x^m \log(c) \log(x) / (m^3 + 3m^2 + 3m + 1) - 3b^3 d^m n x^m \log(c)^2 / (m^2 + 2m + 1) + 3a^2 b^2 d^m m n x^m \log(x) / (m^2 + 2m + 1) - 6a^2 b^2 d^m n^2 x^m \log(x) / (m^3 + 3m^2 + 3m + 1) + 6a^2 b^2 d^m n x^m \log(c) \log(x) / (m^2 + 2m + 1) + 6a^2 b^2 d^m m n^2 x^m \log(x) / (m^3 + 3m^2 + 3m + 1) - 6a^2 b^2 d^m n x^m \log(c) / (m^2 + 2m + 1) + (d*x)^m b^3 x \log(c)^3 / (m + 1) + 3a^2 b^2 d^m n x^m \log(x) / (m^2 + 2m + 1) - 3a^2 b^2 d^m n x^m \log(c) / (m^2 + 2m + 1) + 3(d*x)^m a^2 b^2 x \log(c)^2 / (m + 1) + 3(d*x)^m a^2 b x \log(c) / (m + 1) + (d*x)^m a^3 x / (m + 1) \end{aligned}$$

### 3.151 $\int (dx)^m (a + b \log(cx^n))^2 dx$

**Optimal.** Leaf size=81

$$\frac{(dx)^{m+1} (a + b \log(cx^n))^2}{d(m+1)} - \frac{2bn(dx)^{m+1} (a + b \log(cx^n))}{d(m+1)^2} + \frac{2b^2n^2(dx)^{m+1}}{d(m+1)^3}$$

[Out]  $(2*b^2*n^2*(d*x)^(1+m))/(d*(1+m)^3) - (2*b*n*(d*x)^(1+m)*(a + b*Log[c*x^n]))/(d*(1+m)^2) + ((d*x)^(1+m)*(a + b*Log[c*x^n])^2)/(d*(1+m))$

**Rubi [A]** time = 0.0461994, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2305, 2304}

$$\frac{(dx)^{m+1} (a + b \log(cx^n))^2}{d(m+1)} - \frac{2bn(dx)^{m+1} (a + b \log(cx^n))}{d(m+1)^2} + \frac{2b^2n^2(dx)^{m+1}}{d(m+1)^3}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^2,x]

[Out]  $(2*b^2*n^2*(d*x)^(1+m))/(d*(1+m)^3) - (2*b*n*(d*x)^(1+m)*(a + b*Log[c*x^n]))/(d*(1+m)^2) + ((d*x)^(1+m)*(a + b*Log[c*x^n])^2)/(d*(1+m))$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m+1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m+1)), x] - Dist[(b\*n\*p)/(m+1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p-1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m+1)\*(a + b\*Log[c\*x^n]))/(d\*(m+1)), x] - Simp[(b\*n\*(d\*x)^(m+1))/(d\*(m+1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int (dx)^m (a + b \log(cx^n))^2 dx &= \frac{(dx)^{1+m} (a + b \log(cx^n))^2}{d(1+m)} - \frac{(2bn) \int (dx)^m (a + b \log(cx^n)) dx}{1+m} \\ &= \frac{2b^2n^2(dx)^{1+m}}{d(1+m)^3} - \frac{2bn(dx)^{1+m} (a + b \log(cx^n))}{d(1+m)^2} + \frac{(dx)^{1+m} (a + b \log(cx^n))^2}{d(1+m)} \end{aligned}$$

**Mathematica [A]** time = 0.0319102, size = 76, normalized size = 0.94

$$\frac{x(dx)^m (a^2(m+1)^2 + 2b(m+1)(am + a - bn) \log(cx^n) - 2ab(m+1)n + b^2(m+1)^2 \log^2(cx^n) + 2b^2n^2)}{(m+1)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m\*(a + b\*Log[c\*x^n])^2,x]

```
[Out] (x*(d*x)^m*(a^2*(1+m)^2 - 2*a*b*(1+m)*n + 2*b^2*n^2 + 2*b*(1+m)*(a +
a*m - b*n)*Log[c*x^n] + b^2*(1+m)^2*Log[c*x^n]^2))/(1+m)^3
```

**Maple [C]** time = 0.197, size = 2126, normalized size = 26.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*ln(c*x^n))^2,x)
```

```
[Out] b^2/(1+m)*x*exp(1/2*m*(-I*csgn(I*d*x)^3*Pi+I*csgn(I*d*x)^2*csgn(I*d)*Pi+I*cs
sgn(I*d*x)^2*csgn(I*x)*Pi-I*csgn(I*d*x)*csgn(I*d)*csgn(I*x)*Pi+2*ln(x)+2*ln
(d))*ln(x^n)^2-b*(-I*Pi*b*csgn(I*x^n)*csgn(I*c*x^n)^2*m+I*Pi*b*csgn(I*x^n)
*csgn(I*c*x^n)*csgn(I*c)*m+I*Pi*b*csgn(I*c*x^n)^3*m-I*Pi*b*csgn(I*c*x^n)^2*
csgn(I*c)*m-I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*
x^n)*csgn(I*c)+I*b*Pi*csgn(I*c*x^n)^3-I*b*Pi*csgn(I*c*x^n)^2*csgn(I*c)-2*b*
ln(c)*m-2*b*ln(c)-2*a*m+2*b*n-2*a)/(1+m)^2*x*exp(1/2*m*(-I*csgn(I*d*x)^3*Pi
+I*csgn(I*d*x)^2*csgn(I*d)*Pi+I*csgn(I*d*x)^2*csgn(I*x)*Pi-I*csgn(I*d*x)*cs
gn(I*d)*csgn(I*x)*Pi+2*ln(x)+2*ln(d))*ln(x^n)+1/4*(-8*a*b*m*n+4*I*Pi*b^2*m
*n*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-4*I*Pi*ln(c)*b^2*m^2*csgn(I*x^n)*csg
n(I*c*x^n)*csgn(I*c)+4*I*ln(c)*Pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2+4*ln(c)^2
*b^2-Pi^2*b^2*csgn(I*c*x^n)^4*csgn(I*c)^2-Pi^2*b^2*m^2*csgn(I*c*x^n)^6-2*Pi
^2*b^2*m*csgn(I*c*x^n)^6-8*a*b*n+4*a^2*m^2+8*b^2*n^2+8*a^2*m+4*a^2+8*ln(c)^
2*b^2*m+4*ln(c)^2*b^2*m^2+2*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^
2+2*Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-Pi^2*b^2*csgn(I*x^n)^2
*csgn(I*c*x^n)^2*csgn(I*c)^2-4*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*
c)+4*I*ln(c)*Pi*b^2*csgn(I*c*x^n)^2*csgn(I*c)+4*I*Pi*a*b*csgn(I*x^n)*csgn(I
*c*x^n)^2+4*I*Pi*a*b*csgn(I*c*x^n)^2*csgn(I*c)-4*I*Pi*a*b*csgn(I*c*x^n)^3-4
*I*ln(c)*Pi*b^2*csgn(I*c*x^n)^3-4*I*ln(c)*Pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)*
csgn(I*c)-4*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-Pi^2*b^2*csgn(I*c*
x^n)^6+8*ln(c)*a*b-8*ln(c)*b^2*n+4*I*Pi*a*b*m^2*csgn(I*x^n)*csgn(I*c*x^n)^2
+4*I*Pi*a*b*m^2*csgn(I*c*x^n)^2*csgn(I*c)-4*I*Pi*b^2*m*n*csgn(I*x^n)*csgn(I
*c*x^n)^2-4*I*Pi*b^2*m*n*csgn(I*c*x^n)^2*csgn(I*c)-2*Pi^2*b^2*m*csgn(I*x^n)
^2*csgn(I*c*x^n)^4+8*ln(c)*a*b*m^2+16*ln(c)*a*b*m-8*ln(c)*b^2*m*n+2*Pi^2*b^
2*csgn(I*c*x^n)^5*csgn(I*c)+2*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^5-Pi^2*b^2
*csgn(I*x^n)^2*csgn(I*c*x^n)^4-8*I*Pi*a*b*m*csgn(I*x^n)*csgn(I*c*x^n)*csgn(
I*c)-8*I*Pi*ln(c)*b^2*m*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-4*I*Pi*a*b*m^2*
csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+2*Pi^2*b^2*m^2*csgn(I*x^n)*csgn(I*c*x^n)
^5+4*Pi^2*b^2*m*csgn(I*x^n)*csgn(I*c*x^n)^5+2*Pi^2*b^2*m^2*csgn(I*c*x^n)^5
*csgn(I*c)+4*Pi^2*b^2*m*csgn(I*c*x^n)^5*csgn(I*c)-Pi^2*b^2*m^2*csgn(I*c*x^n)
^4*csgn(I*c)^2-2*Pi^2*b^2*m*csgn(I*c*x^n)^4*csgn(I*c)^2-Pi^2*b^2*m^2*csgn(
I*x^n)^2*csgn(I*c*x^n)^4-8*Pi^2*b^2*m*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)
+2*Pi^2*b^2*m^2*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2+4*Pi^2*b^2*m*csgn(I
*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2+4*I*Pi*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)*cs
gn(I*c)+8*I*Pi*ln(c)*b^2*m*csgn(I*c*x^n)^2*csgn(I*c)+8*I*Pi*ln(c)*b^2*m*cs
gn(I*x^n)*csgn(I*c*x^n)^2+4*I*Pi*b^2*n*csgn(I*c*x^n)^3+4*I*Pi*ln(c)*b^2*m^2*
csgn(I*x^n)*csgn(I*c*x^n)^2-8*I*Pi*a*b*m*csgn(I*c*x^n)^3+2*Pi^2*b^2*m^2*cs
gn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)+4*Pi^2*b^2*m*csgn(I*x^n)^2*csgn(I*c*x^
n)^3*csgn(I*c)-Pi^2*b^2*m^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2-2*Pi^
2*b^2*m*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2-4*Pi^2*b^2*m^2*csgn(I*x^
n)*csgn(I*c*x^n)^4*csgn(I*c)-4*I*Pi*ln(c)*b^2*m^2*csgn(I*c*x^n)^3-8*I*Pi*ln(
c)*b^2*m*csgn(I*c*x^n)^3-4*I*Pi*a*b*m^2*csgn(I*c*x^n)^3+4*I*Pi*b^2*m*n*cs
gn(I*c*x^n)^3+8*I*Pi*a*b*m*csgn(I*x^n)*csgn(I*c*x^n)^2+8*I*Pi*a*b*m*csgn(I*c*
x^n)^2*csgn(I*c)+4*I*Pi*ln(c)*b^2*m^2*csgn(I*c*x^n)^2*csgn(I*c)-4*I*Pi*b^2*
n*csgn(I*x^n)*csgn(I*c*x^n)^2-4*I*Pi*b^2*n*csgn(I*c*x^n)^2*csgn(I*c))/(1+m)
^3*x*exp(1/2*m*(-I*csgn(I*d*x)^3*Pi+I*csgn(I*d*x)^2*csgn(I*d)*Pi+I*csgn(I*d
```

$*x)^2 * \text{csgn}(I*x) * \text{Pi} - I * \text{csgn}(I*d*x) * \text{csgn}(I*d) * \text{csgn}(I*x) * \text{Pi} + 2 * \ln(x) + 2 * \ln(d))$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 1.04582, size = 478, normalized size = 5.9

$$\frac{\left( (b^2 m^2 + 2 b^2 m + b^2) n^2 x \log(x)^2 + (b^2 m^2 + 2 b^2 m + b^2) x \log(c)^2 + 2 (ab m^2 + 2 ab m + ab - (b^2 m + b^2) n) x \log(c) + (a^2 m^2 + 2 a^2 m + a^2 - (b^2 m + b^2) n) x \log(x) \right) e^{(m \log(d) + m \log(x))}}{m^3 + 3 m^2 + 3 m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out] 
$$\frac{\left( (b^2 m^2 + 2 b^2 m + b^2) n^2 x \log(x)^2 + (b^2 m^2 + 2 b^2 m + b^2) x \log(c)^2 + 2 (a^2 m^2 + 2 a^2 m + a^2 - (b^2 m + b^2) n) x \log(x) + 2 (ab m^2 + 2 ab m + ab - (b^2 m + b^2) n) x \log(c) - ((b^2 m + b^2) n^2 - (a b m^2 + 2 a b m + a b) n) x \log(c) \right) e^{(m \log(d) + m \log(x))}}{m^3 + 3 m^2 + 3 m + 1}$$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Exception raised: TypeError

**Giac [B]** time = 1.30953, size = 543, normalized size = 6.7

$$\frac{b^2 d^m m^2 n^2 x x^m \log(x)^2}{m^3 + 3 m^2 + 3 m + 1} + \frac{2 b^2 d^m m n^2 x x^m \log(x)^2}{m^3 + 3 m^2 + 3 m + 1} - \frac{2 b^2 d^m m n^2 x x^m \log(x)}{m^3 + 3 m^2 + 3 m + 1} + \frac{2 b^2 d^m m n x x^m \log(c) \log(x)}{m^2 + 2 m + 1} + \frac{b^2 d^m n^2 x x^m}{m^3 + 3 m^2 + 3 m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] 
$$b^2 d^m m^2 n^2 x x^m \log(x)^2 / (m^3 + 3 m^2 + 3 m + 1) + 2 b^2 d^m m n^2 x x^m \log(x)^2 / (m^3 + 3 m^2 + 3 m + 1) - 2 b^2 d^m m n^2 x x^m \log(x) / (m^3 + 3 m^2 + 3 m + 1) + 2 b^2 d^m m n x x^m \log(c) \log(x) / (m^2 + 2 m + 1) + b^2 d^m n^2 x x^m / (m^3 + 3 m^2 + 3 m + 1)$$

$$\begin{aligned}
& d^m n^2 x^m \log(x)^2 / (m^3 + 3m^2 + 3m + 1) + 2ab d^m m n x^m \log(x) / (m^2 + 2m + 1) - 2b^2 d^m n^2 x^m \log(x) / (m^3 + 3m^2 + 3m + 1) + 2b^2 d^m n x^m \log(c) \log(x) / (m^2 + 2m + 1) + 2b^2 d^m n^2 x^m / (m^3 + 3m^2 + 3m + 1) - 2b^2 d^m n x^m \log(c) / (m^2 + 2m + 1) + 2ab d^m n x^m \log(x) / (m^2 + 2m + 1) - 2ab d^m n x^m / (m^2 + 2m + 1) + (dx)^m b^2 x \log(c)^2 / (m + 1) + 2(dx)^m a b x \log(c) / (m + 1) + (dx)^m a^2 x / (m + 1)
\end{aligned}$$

### 3.152 $\int (dx)^m (a + b \log(cx^n)) dx$

**Optimal.** Leaf size=46

$$\frac{(dx)^{m+1} (a + b \log(cx^n))}{d(m+1)} - \frac{bn(dx)^{m+1}}{d(m+1)^2}$$

[Out]  $-\frac{(b*n*(d*x)^{(1+m)})/(d*(1+m)^2)}{d*(1+m)} + \frac{((d*x)^{(1+m})*(a + b*Log[c*x^n]))}{d*(1+m)}$

**Rubi [A]** time = 0.0151003, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$ , Rules used = {2304}

$$\frac{(dx)^{m+1} (a + b \log(cx^n))}{d(m+1)} - \frac{bn(dx)^{m+1}}{d(m+1)^2}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m\*(a + b\*Log[c\*x^n]),x]

[Out]  $-\frac{(b*n*(d*x)^{(1+m)})/(d*(1+m)^2)}{d*(1+m)} + \frac{((d*x)^{(1+m})*(a + b*Log[c*x^n]))}{d*(1+m)}$

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m+1)\*(a + b\*Log[c\*x^n]))/(d\*(m+1)), x] - Simp[(b\*n\*(d\*x)^(m+1))/(d\*(m+1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\int (dx)^m (a + b \log(cx^n)) dx = -\frac{bn(dx)^{1+m}}{d(1+m)^2} + \frac{(dx)^{1+m} (a + b \log(cx^n))}{d(1+m)}$$

**Mathematica [A]** time = 0.0117765, size = 32, normalized size = 0.7

$$\frac{x(dx)^m (am + a + b(m+1) \log(cx^n) - bn)}{(m+1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m\*(a + b\*Log[c\*x^n]),x]

[Out]  $(x*(d*x)^m*(a + a*m - b*n + b*(1+m)*Log[c*x^n]))/(1+m)^2$

**Maple [C]** time = 0.103, size = 371, normalized size = 8.1

$$\frac{bx \ln(x^n)}{1+m} e^{\frac{m(-i \operatorname{csgn}(id x))^3 \pi + i \operatorname{csgn}(id x)^2 \operatorname{csgn}(id) \pi + i \operatorname{csgn}(id x)^2 \operatorname{csgn}(ix) \pi - i \operatorname{csgn}(id x) \operatorname{csgn}(id) \operatorname{csgn}(ix) \pi + 2 \ln(x) + 2 \ln(d)}{2}} - \frac{(-i \pi b \operatorname{csgn}(ix^n) (\operatorname{csgn}(icx^n)))}{2}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^m*(a+b*ln(c*x^n)),x)`

[Out] 
$$\frac{b}{(1+m)} x \exp\left(\frac{1}{2} m (-i \operatorname{csgn}(I d x)^3 \pi + i \operatorname{csgn}(I d x)^2 \operatorname{csgn}(I d) \pi + i \operatorname{csgn}(I d x)^2 \operatorname{csgn}(I x) \pi - i \operatorname{csgn}(I d x) \operatorname{csgn}(I d) \operatorname{csgn}(I x) \pi + 2 \ln(x) + 2 \ln(d))\right) \ln(x^n) - \frac{1}{2} (-i \pi b \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n)^{2m} + i \pi b \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n) \operatorname{csgn}(I c)^m + i \pi b \operatorname{csgn}(I c x^n)^{3m} - i \pi b \operatorname{csgn}(I c x^n)^2 \operatorname{csgn}(I c)^m - i b \pi \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n)^2 + i b \pi \operatorname{csgn}(I x^n) \operatorname{csgn}(I c x^n) \operatorname{csgn}(I c) + i b \pi \operatorname{csgn}(I c x^n)^3 - i b \pi \operatorname{csgn}(I c x^n)^2 \operatorname{csgn}(I c) - 2 b \ln(c)^m - 2 b \ln(c) - 2 a m + 2 b n - 2 a) / (1+m)^2 x \exp\left(\frac{1}{2} m (-i \operatorname{csgn}(I d x)^3 \pi + i \operatorname{csgn}(I d x)^2 \operatorname{csgn}(I d) \pi + i \operatorname{csgn}(I d x)^2 \operatorname{csgn}(I x) \pi - i \operatorname{csgn}(I d x) \operatorname{csgn}(I d) \operatorname{csgn}(I x) \pi + 2 \ln(x) + 2 \ln(d))\right)$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^m*(a+b*log(c*x^n)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.03528, size = 142, normalized size = 3.09

$$\frac{(bm + b)nx \log(x) + (bm + b)x \log(c) + (am - bn + a)x e^{(m \log(d) + m \log(x))}}{m^2 + 2m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^m*(a+b*log(c*x^n)),x, algorithm="fricas")`

[Out] 
$$\frac{(b^m + b)^n x \log(x) + (b^m + b) x \log(c) + (a^m - b^n + a) x e^{(m \log(d) + m \log(x))}}{m^2 + 2m + 1}$$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)**m*(a+b*ln(c*x**n)),x)`

[Out] Exception raised: TypeError

**Giac [B]** time = 1.28532, size = 128, normalized size = 2.78

$$\frac{bd^m m n x x^m \log(x)}{m^2 + 2m + 1} + \frac{bd^m n x x^m \log(x)}{m^2 + 2m + 1} - \frac{bd^m n x x^m}{m^2 + 2m + 1} + \frac{(dx)^m b x \log(c)}{m + 1} + \frac{(dx)^m a x}{m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] b*d^m*m*n*x*x^m*log(x)/(m^2 + 2*m + 1) + b*d^m*n*x*x^m*log(x)/(m^2 + 2*m + 1) - b*d^m*n*x*x^m/(m^2 + 2*m + 1) + (d*x)^m*b*x*log(c)/(m + 1) + (d*x)^m*a*x/(m + 1)
```

$$3.153 \quad \int \frac{(dx)^m}{a+b \log(cx^n)} dx$$

**Optimal.** Leaf size=66

$$\frac{(dx)^{m+1} e^{-\frac{a(m+1)}{bn}} (cx^n)^{-\frac{m+1}{n}} \operatorname{Ei}\left(\frac{(m+1)(a+b \log(cx^n))}{bn}\right)}{bdn}$$

[Out] ((d\*x)^(1 + m)\*ExpIntegralEi[((1 + m)\*(a + b\*Log[c\*x^n])/(b\*n))]/(b\*d\*E^((a\*(1 + m))/(b\*n))\*n\*(c\*x^n)^((1 + m)/n))

**Rubi [A]** time = 0.0736044, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2310, 2178}

$$\frac{(dx)^{m+1} e^{-\frac{a(m+1)}{bn}} (cx^n)^{-\frac{m+1}{n}} \operatorname{Ei}\left(\frac{(m+1)(a+b \log(cx^n))}{bn}\right)}{bdn}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m/(a + b\*Log[c\*x^n]), x]

[Out] ((d\*x)^(1 + m)\*ExpIntegralEi[((1 + m)\*(a + b\*Log[c\*x^n])/(b\*n))]/(b\*d\*E^((a\*(1 + m))/(b\*n))\*n\*(c\*x^n)^((1 + m)/n))

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2178

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(F^(g\*(e - (c\*f)/d))\*ExpIntegralEi[(f\*g\*(c + d\*x)\*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

#### Rubi steps

$$\begin{aligned} \int \frac{(dx)^m}{a+b \log(cx^n)} dx &= \frac{\left((dx)^{1+m} (cx^n)^{-\frac{1+m}{n}}\right) \operatorname{Subst}\left(\int \frac{e^{\frac{(1+m)x}{a+bx}}}{a+bx} dx, x, \log(cx^n)\right)}{dn} \\ &= \frac{e^{-\frac{a(1+m)}{bn}} (dx)^{1+m} (cx^n)^{-\frac{1+m}{n}} \operatorname{Ei}\left(\frac{(1+m)(a+b \log(cx^n))}{bn}\right)}{bdn} \end{aligned}$$

**Mathematica [A]** time = 0.105898, size = 67, normalized size = 1.02

$$\frac{x^{-m} (dx)^m \exp\left(-\frac{(m+1)(a+b(\log(cx^n)-n \log(x)))}{bn}\right) \operatorname{Ei}\left(\frac{(m+1)(a+b \log(cx^n))}{bn}\right)}{bn}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m/(a + b\*Log[c\*x^n]),x]

[Out] ((d\*x)^m\*ExpIntegralEi[((1 + m)\*(a + b\*Log[c\*x^n]))/(b\*n)]/(b\*E^(((1 + m)\*(a + b\*(-(n\*Log[x]) + Log[c\*x^n])))/(b\*n))\*n\*x^m)

**Maple [F]** time = 0.245, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a + b \ln(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(a+b\*ln(c\*x^n)),x)

[Out] int((d\*x)^m/(a+b\*ln(c\*x^n)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b \log(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*log(c\*x^n)),x, algorithm="maxima")

[Out] integrate((d\*x)^m/(b\*log(c\*x^n) + a), x)

**Fricas [A]** time = 0.989485, size = 163, normalized size = 2.47

$$\frac{\text{Ei}\left(\frac{(bm+b)n \log(x) + am + (bm+b) \log(c) + a}{bn}\right) e^{\left(\frac{bmn \log(d) - am - (bm+b) \log(c) - a}{bn}\right)}}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*log(c\*x^n)),x, algorithm="fricas")

[Out] Ei(((b\*m + b)\*n\*log(x) + a\*m + (b\*m + b)\*log(c) + a)/(b\*n))\*e^((b\*m\*n\*log(d) - a\*m - (b\*m + b)\*log(c) - a)/(b\*n))/(b\*n)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a + b \log(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m/(a+b\*ln(c\*x\*\*n)),x)

```
[Out] Integral((d*x)**m/(a + b*log(c*x**n)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b \log(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*log(c*x^n) + a), x)
```

$$3.154 \quad \int \frac{(dx)^m}{(a+b \log(cx^n))^2} dx$$

**Optimal.** Leaf size=100

$$\frac{(m+1)(dx)^{m+1} e^{-\frac{a(m+1)}{bn}} (cx^n)^{-\frac{m+1}{n}} \operatorname{Ei}\left(\frac{(m+1)(a+b \log(cx^n))}{bn}\right)}{b^2 d n^2} - \frac{(dx)^{m+1}}{bdn (a+b \log(cx^n))}$$

[Out]  $((1+m)*(d*x)^{(1+m)*\operatorname{ExpIntegralEi}[(1+m)*(a+b*\operatorname{Log}[c*x^n])]/(b*n)]/(b^2*d*E^{((a*(1+m))/(b*n))*n^2*(c*x^n)^{(1+m)/n}} - (d*x)^{(1+m)/(b*d*n*(a+b*\operatorname{Log}[c*x^n])}))$

**Rubi [A]** time = 0.0931316, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2306, 2310, 2178}

$$\frac{(m+1)(dx)^{m+1} e^{-\frac{a(m+1)}{bn}} (cx^n)^{-\frac{m+1}{n}} \operatorname{Ei}\left(\frac{(m+1)(a+b \log(cx^n))}{bn}\right)}{b^2 d n^2} - \frac{(dx)^{m+1}}{bdn (a+b \log(cx^n))}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(d*x)^m/(a+b*\operatorname{Log}[c*x^n])^2, x]$

[Out]  $((1+m)*(d*x)^{(1+m)*\operatorname{ExpIntegralEi}[(1+m)*(a+b*\operatorname{Log}[c*x^n])]/(b*n)]/(b^2*d*E^{((a*(1+m))/(b*n))*n^2*(c*x^n)^{(1+m)/n}} - (d*x)^{(1+m)/(b*d*n*(a+b*\operatorname{Log}[c*x^n])}))$

#### Rule 2306

$\operatorname{Int}[(a_. + \operatorname{Log}[c_.*(x_.)^{n_.}]*b_.)^{(p_.)*((d_.)*(x_.)^{m_.}), x\_Symbol] \rightarrow \operatorname{Simp}[(d*x)^{(m+1)*(a+b*\operatorname{Log}[c*x^n])^{(p+1)/(b*d*n*(p+1))}, x] - \operatorname{Dist}[(m+1)/(b*n*(p+1)), \operatorname{Int}[(d*x)^m*(a+b*\operatorname{Log}[c*x^n])^{(p+1)}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m, n\}, x\} \&\& \operatorname{NeQ}[m, -1] \&\& \operatorname{LtQ}[p, -1]$

#### Rule 2310

$\operatorname{Int}[(a_. + \operatorname{Log}[c_.*(x_.)^{n_.}]*b_.)^{(p_.)*((d_.)*(x_.)^{m_.}), x\_Symbol] \rightarrow \operatorname{Dist}[(d*x)^{(m+1)/(d*n*(c*x^n)^{(m+1)/n})}, \operatorname{Subst}[\operatorname{Int}[E^{((m+1)*x/n)*(a+b*x)^p}, x], x, \operatorname{Log}[c*x^n]], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m, n, p\}, x\}$

#### Rule 2178

$\operatorname{Int}[(F_.)^{((g_.)*((e_.) + (f_.)*(x_.)))/((c_.) + (d_.)*(x_.))}, x\_Symbol] \rightarrow \operatorname{Simp}[(F^{(g*(e - (c*f)/d))*\operatorname{ExpIntegralEi}[(f*g*(c + d*x)*\operatorname{Log}[F])/d]}/d, x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x\} \&\& !\$UseGamma == \operatorname{True}$

#### Rubi steps

$$\begin{aligned} \int \frac{(dx)^m}{(a + b \log(cx^n))^2} dx &= -\frac{(dx)^{1+m}}{bdn(a + b \log(cx^n))} + \frac{(1+m) \int \frac{(dx)^m}{a+b \log(cx^n)} dx}{bn} \\ &= -\frac{(dx)^{1+m}}{bdn(a + b \log(cx^n))} + \frac{\left( (1+m)(dx)^{1+m} (cx^n)^{-\frac{1+m}{n}} \right) \text{Subst} \left( \int \frac{e^{\frac{(1+m)x}{a+bx}}}{a+bx} dx, x, \log(cx^n) \right)}{bdn^2} \\ &= \frac{e^{-\frac{a(1+m)}{bn}} (1+m)(dx)^{1+m} (cx^n)^{-\frac{1+m}{n}} \text{Ei} \left( \frac{(1+m)(a+b \log(cx^n))}{bn} \right)}{b^2dn^2} - \frac{(dx)^{1+m}}{bdn(a + b \log(cx^n))} \end{aligned}$$

**Mathematica [A]** time = 0.233248, size = 89, normalized size = 0.89

$$\frac{(dx)^m \left( (m+1)x^{-m} \exp \left( -\frac{(m+1)(a+b \log(cx^n)-bn \log(x))}{bn} \right) \text{Ei} \left( \frac{(m+1)(a+b \log(cx^n))}{bn} \right) - \frac{bnx}{a+b \log(cx^n)} \right)}{b^2n^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m/(a + b\*Log[c\*x^n])^2,x]

[Out] ((d\*x)^m\*((1+m)\*ExpIntegralEi[((1+m)\*(a + b\*Log[c\*x^n]))/(b\*n)])/(E^(((1+m)\*(a - b\*n\*Log[x] + b\*Log[c\*x^n]))/(b\*n))\*x^m) - (b\*n\*x)/(a + b\*Log[c\*x^n]))/(b^2\*n^2)

**Maple [F]** time = 1.303, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(a + b \ln(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(a+b\*ln(c\*x^n))^2,x)

[Out] int((d\*x)^m/(a+b\*ln(c\*x^n))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$d^m(m+1) \int \frac{x^m}{b^2n \log(c) + b^2n \log(x^n) + abn} dx - \frac{d^m x x^m}{b^2n \log(c) + b^2n \log(x^n) + abn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*log(c\*x^n))^2,x, algorithm="maxima")

[Out] d^m\*(m + 1)\*integrate(x^m/(b^2\*n\*log(c) + b^2\*n\*log(x^n) + a\*b\*n), x) - d^m\*x\*x^m/(b^2\*n\*log(c) + b^2\*n\*log(x^n) + a\*b\*n)

**Fricas [A]** time = 1.00347, size = 332, normalized size = 3.32

$$\frac{bnxe^{(m \log(d)+m \log(x))} - ((bm + b)n \log(x) + am + (bm + b) \log(c) + a) \text{Ei} \left( \frac{(bm+b)n \log(x)+am+(bm+b) \log(c)+a}{bn} \right) e^{\left( \frac{bmn \log(d)-an}{bn} \right)}}{b^3n^3 \log(x) + b^3n^2 \log(c) + ab^2n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*log(c\*x^n))^2,x, algorithm="fricas")

[Out]  $-(b*n*x*e^{(m*\log(d) + m*\log(x))} - ((b*m + b)*n*\log(x) + a*m + (b*m + b)*\log(c) + a)*Ei(((b*m + b)*n*\log(x) + a*m + (b*m + b)*\log(c) + a)/(b*n)))*e^{(b*m*n*\log(d) - a*m - (b*m + b)*\log(c) - a)/(b*n))}/(b^3*n^3*\log(x) + b^3*n^2*\log(c) + a*b^2*n^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(a + b \log(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m/(a+b\*ln(c\*x\*\*n))\*\*2,x)

[Out] Integral((d\*x)\*\*m/(a + b\*log(c\*x\*\*n))\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(b \log(cx^n) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*log(c\*x^n))^2,x, algorithm="giac")

[Out] integrate((d\*x)^m/(b\*log(c\*x^n) + a)^2, x)



$$3.155 \quad \int \frac{(dx)^m}{(a+b \log(cx^n))^3} dx$$

**Optimal.** Leaf size=142

$$\frac{(m+1)^2(dx)^{m+1}e^{-\frac{a(m+1)}{bn}}(cx^n)^{-\frac{m+1}{n}}\operatorname{Ei}\left(\frac{(m+1)(a+b \log(cx^n))}{bn}\right)}{2b^3dn^3} - \frac{(m+1)(dx)^{m+1}}{2b^2dn^2(a+b \log(cx^n))} - \frac{(dx)^{m+1}}{2bdn(a+b \log(cx^n))^2}$$

[Out]  $((1+m)^2(dx)^{(1+m)}\operatorname{ExpIntegralEi}[\frac{(1+m)(a+b \operatorname{Log}[c*x^n])}{(b*n)}]) / (2*b^3*d*E^{\frac{a*(1+m)}{(b*n)}*n^3*(c*x^n)^{\frac{(1+m)}{n}}} - (dx)^{(1+m)} / (2*b*d*n*(a+b \operatorname{Log}[c*x^n])^2) - ((1+m)*(dx)^{(1+m)}) / (2*b^2*d*n^2*(a+b \operatorname{Log}[c*x^n]))$

**Rubi [A]** time = 0.13917, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2306, 2310, 2178}

$$\frac{(m+1)^2(dx)^{m+1}e^{-\frac{a(m+1)}{bn}}(cx^n)^{-\frac{m+1}{n}}\operatorname{Ei}\left(\frac{(m+1)(a+b \log(cx^n))}{bn}\right)}{2b^3dn^3} - \frac{(m+1)(dx)^{m+1}}{2b^2dn^2(a+b \log(cx^n))} - \frac{(dx)^{m+1}}{2bdn(a+b \log(cx^n))^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(dx)^m/(a+b \operatorname{Log}[c*x^n])^3, x]$

[Out]  $((1+m)^2(dx)^{(1+m)}\operatorname{ExpIntegralEi}[\frac{(1+m)(a+b \operatorname{Log}[c*x^n])}{(b*n)}]) / (2*b^3*d*E^{\frac{a*(1+m)}{(b*n)}*n^3*(c*x^n)^{\frac{(1+m)}{n}}} - (dx)^{(1+m)} / (2*b*d*n*(a+b \operatorname{Log}[c*x^n])^2) - ((1+m)*(dx)^{(1+m)}) / (2*b^2*d*n^2*(a+b \operatorname{Log}[c*x^n]))$

### Rule 2306

$\operatorname{Int}[(a + \operatorname{Log}[c(x)^n])^p(b)^q((d)(x))^m, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(dx)^{m+1}(a+b \operatorname{Log}[c*x^n])^{p+1}/(b*d*n*(p+1)), x] - \operatorname{Dist}[(m+1)/(b*n*(p+1)), \operatorname{Int}[(dx)^m(a+b \operatorname{Log}[c*x^n])^{p+1}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m, n\}, x \ \&\& \operatorname{NeQ}[m, -1] \ \&\& \operatorname{LtQ}[p, -1]$

### Rule 2310

$\operatorname{Int}[(a + \operatorname{Log}[c(x)^n])^p(b)^q((d)(x))^m, x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[(dx)^{m+1}/(d*n*(c*x^n)^{\frac{m+1}{n}}), \operatorname{Subst}[\operatorname{Int}[E^{((m+1)*x/n)}(a+b*x)^p, x], x, \operatorname{Log}[c*x^n]], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m, n, p\}, x]$

### Rule 2178

$\operatorname{Int}[(F)^g((g)(e) + (f)(x))/((c) + (d)(x)), x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(F^{g*(e-(c*f)/d)}\operatorname{ExpIntegralEi}[(f*g*(c+d*x)*\operatorname{Log}[F])/d])/d, x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x \ \&\& \operatorname{!}\$UseGamma == \operatorname{True}$

### Rubi steps

$$\begin{aligned}
\int \frac{(dx)^m}{(a + b \log(cx^n))^3} dx &= -\frac{(dx)^{1+m}}{2bdn(a + b \log(cx^n))^2} + \frac{(1+m) \int \frac{(dx)^m}{(a+b \log(cx^n))^2} dx}{2bn} \\
&= -\frac{(dx)^{1+m}}{2bdn(a + b \log(cx^n))^2} - \frac{(1+m)(dx)^{1+m}}{2b^2dn^2(a + b \log(cx^n))} + \frac{(1+m)^2 \int \frac{(dx)^m}{a+b \log(cx^n)} dx}{2b^2n^2} \\
&= -\frac{(dx)^{1+m}}{2bdn(a + b \log(cx^n))^2} - \frac{(1+m)(dx)^{1+m}}{2b^2dn^2(a + b \log(cx^n))} + \frac{\left((1+m)^2(dx)^{1+m}(cx^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int \frac{e^{-\frac{a(1+m)}{bn}}}{a+b \log(x)} dx\right)}{2b^2dn^3} \\
&= \frac{e^{-\frac{a(1+m)}{bn}}(1+m)^2(dx)^{1+m}(cx^n)^{-\frac{1+m}{n}} \text{Ei}\left(\frac{(1+m)(a+b \log(cx^n))}{bn}\right)}{2b^3dn^3} - \frac{(dx)^{1+m}}{2bdn(a + b \log(cx^n))^2} - \frac{(1+m)(dx)^{1+m}}{2b^2dn^2(a + b \log(cx^n))}
\end{aligned}$$

**Mathematica [A]** time = 0.357028, size = 113, normalized size = 0.8

$$\frac{(dx)^m \left( (m+1)^2 x^{-m} \exp\left(-\frac{(m+1)(a+b \log(cx^n)-bn \log(x))}{bn}\right) \text{Ei}\left(\frac{(m+1)(a+b \log(cx^n))}{bn}\right) - \frac{bnx(am+a+b(m+1) \log(cx^n)+bn)}{(a+b \log(cx^n))^2} \right)}{2b^3n^3}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m/(a + b\*Log[c\*x^n])^3,x]

[Out] ((d\*x)^m\*(((1+m)^2\*ExpIntegralEi[((1+m)\*(a + b\*Log[c\*x^n]))/(b\*n)])/(E^(-((1+m)\*(a - b\*n\*Log[x] + b\*Log[c\*x^n]))/(b\*n))\*x^m) - (b\*n\*x\*(a + a\*m + b\*n + b\*(1+m)\*Log[c\*x^n]))/(a + b\*Log[c\*x^n])^2))/(2\*b^3\*n^3)

**Maple [F]** time = 1.312, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(a + b \ln(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(a+b\*ln(c\*x^n))^3,x)

[Out] int((d\*x)^m/(a+b\*ln(c\*x^n))^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$(m^2 + 2m + 1)d^m \int \frac{x^m}{2(b^3n^2 \log(c) + b^3n^2 \log(x^n) + ab^2n^2)} dx - \frac{bd^m(m+1)xx^m \log(x^n) + (ad^m(m+1) + d^m(m+1) \log(c) + d^m n)b}{2(b^4n^2 \log(c)^2 + b^4n^2 \log(x^n)^2 + 2ab^3n^2 \log(c) + a^2b^3n^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*log(c\*x^n))^3,x, algorithm="maxima")

[Out] (m^2 + 2\*m + 1)\*d^m\*integrate(1/2\*x^m/(b^3\*n^2\*log(c) + b^3\*n^2\*log(x^n) + a\*b^2\*n^2), x) - 1/2\*(b\*d^m\*(m + 1)\*x\*x^m\*log(x^n) + (a\*d^m\*(m + 1) + (d^m\*(m + 1)\*log(c) + d^m\*n)\*b)\*x\*x^m)/(b^4\*n^2\*log(c)^2 + b^4\*n^2\*log(x^n)^2 +

$$2ab^3n^2\log(c) + a^2b^2n^2 + 2(b^4n^2\log(c) + ab^3n^2)\log(x^n)$$

**Fricas [B]** time = 1.03328, size = 765, normalized size = 5.39

$$\frac{\left((b^2m^2 + 2b^2m + b^2)n^2 \log(x)^2 + a^2m^2 + 2a^2m + (b^2m^2 + 2b^2m + b^2)\log(c)^2 + a^2 + 2(abm^2 + 2abm + ab)\log(c)\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*log(c\*x^n))^3,x, algorithm="fricas")

[Out] 1/2\*(((b^2\*m^2 + 2\*b^2\*m + b^2)\*n^2\*log(x)^2 + a^2\*m^2 + 2\*a^2\*m + (b^2\*m^2 + 2\*b^2\*m + b^2)\*log(c)^2 + a^2 + 2\*(a\*b\*m^2 + 2\*a\*b\*m + a\*b)\*log(c) + 2\*(b^2\*m^2 + 2\*b^2\*m + b^2)\*n\*log(c) + (a\*b\*m^2 + 2\*a\*b\*m + a\*b)\*n)\*log(x))\*Ei(((b\*m + b)\*n\*log(x) + a\*m + (b\*m + b)\*log(c) + a)/(b\*n))\*e^((b\*m\*n\*log(d) - a\*m - (b\*m + b)\*log(c) - a)/(b\*n)) - ((b^2\*m + b^2)\*n^2\*x\*log(x) + (b^2\*m + b^2)\*n\*x\*log(c) + (b^2\*n^2 + (a\*b\*m + a\*b)\*n)\*x)\*e^(m\*log(d) + m\*log(x)))/(b^5\*n^5\*log(x)^2 + b^5\*n^3\*log(c)^2 + 2\*a\*b^4\*n^3\*log(c) + a^2\*b^3\*n^3 + 2\*(b^5\*n^4\*log(c) + a\*b^4\*n^4)\*log(x))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(a + b \log(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m/(a+b\*ln(c\*x\*\*n))\*\*3,x)

[Out] Integral((d\*x)\*\*m/(a + b\*log(c\*x\*\*n))\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(b \log(cx^n) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*log(c\*x^n))^3,x, algorithm="giac")

[Out] integrate((d\*x)^m/(b\*log(c\*x^n) + a)^3, x)

### 3.156 $\int (dx)^{-1+n} \log^3(cx^n) dx$

**Optimal.** Leaf size=74

$$\frac{(dx)^n \log^3(cx^n)}{dn} - \frac{3(dx)^n \log^2(cx^n)}{dn} + \frac{6(dx)^n \log(cx^n)}{dn} - \frac{6(dx)^n}{dn}$$

[Out]  $(-6*(d*x)^n)/(d*n) + (6*(d*x)^n*\text{Log}[c*x^n])/ (d*n) - (3*(d*x)^n*\text{Log}[c*x^n]^2)/(d*n) + ((d*x)^n*\text{Log}[c*x^n]^3)/(d*n)$

**Rubi [A]** time = 0.0531129, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2305, 2304}

$$\frac{(dx)^n \log^3(cx^n)}{dn} - \frac{3(dx)^n \log^2(cx^n)}{dn} + \frac{6(dx)^n \log(cx^n)}{dn} - \frac{6(dx)^n}{dn}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^(-1 + n)\*Log[c\*x^n]^3, x]

[Out]  $(-6*(d*x)^n)/(d*n) + (6*(d*x)^n*\text{Log}[c*x^n])/ (d*n) - (3*(d*x)^n*\text{Log}[c*x^n]^2)/(d*n) + ((d*x)^n*\text{Log}[c*x^n]^3)/(d*n)$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int (dx)^{-1+n} \log^3(cx^n) dx &= \frac{(dx)^n \log^3(cx^n)}{dn} - 3 \int (dx)^{-1+n} \log^2(cx^n) dx \\ &= -\frac{3(dx)^n \log^2(cx^n)}{dn} + \frac{(dx)^n \log^3(cx^n)}{dn} + 6 \int (dx)^{-1+n} \log(cx^n) dx \\ &= -\frac{6(dx)^n}{dn} + \frac{6(dx)^n \log(cx^n)}{dn} - \frac{3(dx)^n \log^2(cx^n)}{dn} + \frac{(dx)^n \log^3(cx^n)}{dn} \end{aligned}$$

**Mathematica [A]** time = 0.0076953, size = 40, normalized size = 0.54

$$\frac{(dx)^n (\log^3(cx^n) - 3 \log^2(cx^n) + 6 \log(cx^n) - 6)}{dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(-1 + n)*Log[c*x^n]^3,x]
```

```
[Out] ((d*x)^n*(-6 + 6*Log[c*x^n] - 3*Log[c*x^n]^2 + Log[c*x^n]^3))/(d*n)
```

**Maple [C]** time = 0.257, size = 2008, normalized size = 27.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(-1+n)*ln(c*x^n)^3,x)
```

```
[Out] 1/n*x*exp(1/2*(-1+n)*(-I*csgn(I*d*x)^3*Pi+I*csgn(I*d*x)^2*csgn(I*d)*Pi+I*cs
gn(I*d*x)^2*csgn(I*x)*Pi-I*csgn(I*d*x)*csgn(I*d)*csgn(I*x)*Pi+2*ln(x)+2*ln(
d))*ln(x^n)^3+3/2*(I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*csgn(I*x^n)*csgn(
I*c*x^n)*csgn(I*c)-I*Pi*csgn(I*c*x^n)^3+I*Pi*csgn(I*c*x^n)^2*csgn(I*c)+2*ln
(c)-2)/n*x*exp(1/2*(-1+n)*(-I*csgn(I*d*x)^3*Pi+I*csgn(I*d*x)^2*csgn(I*d)*Pi
+I*csgn(I*d*x)^2*csgn(I*x)*Pi-I*csgn(I*d*x)*csgn(I*d)*csgn(I*x)*Pi+2*ln(x)+
2*ln(d))*ln(x^n)^2+3/4*(-Pi^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4+2*Pi^2*csgn(I*
x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-Pi^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*
c)^2+2*Pi^2*csgn(I*x^n)*csgn(I*c*x^n)^5-4*Pi^2*csgn(I*x^n)*csgn(I*c*x^n)^4*
csgn(I*c)+2*Pi^2*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2-Pi^2*csgn(I*c*x^n)
^6+2*Pi^2*csgn(I*c*x^n)^5*csgn(I*c)-Pi^2*csgn(I*c*x^n)^4*csgn(I*c)^2-4*I*Pi
*csgn(I*c*x^n)^2*csgn(I*c)+4*I*ln(c)*Pi*csgn(I*c*x^n)^2*csgn(I*c)-4*I*Pi*cs
gn(I*x^n)*csgn(I*c*x^n)^2+4*I*Pi*csgn(I*c*x^n)^3+4*I*ln(c)*Pi*csgn(I*x^n)*c
sgn(I*c*x^n)^2-4*I*ln(c)*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-4*I*ln(c)*P
i*csgn(I*c*x^n)^3+4*I*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)+4*ln(c)^2-8*ln
(c)+8)/n*x*exp(1/2*(-1+n)*(-I*csgn(I*d*x)^3*Pi+I*csgn(I*d*x)^2*csgn(I*d)*Pi
+I*csgn(I*d*x)^2*csgn(I*x)*Pi-I*csgn(I*d*x)*csgn(I*d)*csgn(I*x)*Pi+2*ln(x)+
2*ln(d))*ln(x^n)+1/8*(-48+I*Pi^3*csgn(I*x^n)^3*csgn(I*c*x^n)^3*csgn(I*c)^3
+3*I*Pi^3*csgn(I*x^n)^2*csgn(I*c*x^n)^7-3*I*Pi^3*csgn(I*x^n)*csgn(I*c*x^n)^
8-3*I*Pi^3*csgn(I*c*x^n)^8*csgn(I*c)+3*I*Pi^3*csgn(I*c*x^n)^7*csgn(I*c)^2-I
*Pi^3*csgn(I*c*x^n)^6*csgn(I*c)^3-12*I*ln(c)^2*Pi*csgn(I*c*x^n)^3+6*Pi^2*cs
gn(I*c*x^n)^4*csgn(I*c)^2-24*ln(c)^2-12*Pi^2*csgn(I*x^n)^2*csgn(I*c*x^n)^3*
csgn(I*c)+6*Pi^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2+24*Pi^2*csgn(I*x
^n)*csgn(I*c*x^n)^4*csgn(I*c)-12*Pi^2*csgn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)
^2-6*ln(c)*Pi^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4+12*ln(c)*Pi^2*csgn(I*x^n)*csg
n(I*c*x^n)^5+12*ln(c)*Pi^2*csgn(I*c*x^n)^5*csgn(I*c)-6*ln(c)*Pi^2*csgn(I*c*
x^n)^4*csgn(I*c)^2+24*I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+24*I*Pi*csgn(I*c*x^n
)^2*csgn(I*c)+24*I*ln(c)*Pi*csgn(I*c*x^n)^3-I*Pi^3*csgn(I*x^n)^3*csgn(I*c*x
^n)^6+6*Pi^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4-12*Pi^2*csgn(I*x^n)*csgn(I*c*x^n
)^5+12*ln(c)*Pi^2*csgn(I*x^n)^2*csgn(I*c*x^n)^3*csgn(I*c)-6*ln(c)*Pi^2*csgn
(I*x^n)^2*csgn(I*c*x^n)^2*csgn(I*c)^2+48*ln(c)-12*Pi^2*csgn(I*c*x^n)^5*csgn
(I*c)-24*I*Pi*csgn(I*c*x^n)^3+I*Pi^3*csgn(I*c*x^n)^9-6*ln(c)*Pi^2*csgn(I*c*
x^n)^6+8*ln(c)^3+9*I*Pi^3*csgn(I*x^n)^2*csgn(I*c*x^n)^5*csgn(I*c)^2-3*I*Pi^
3*csgn(I*x^n)^2*csgn(I*c*x^n)^4*csgn(I*c)^3+9*I*Pi^3*csgn(I*x^n)*csgn(I*c*x
^n)^7*csgn(I*c)+24*I*ln(c)*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-12*I*ln(c)
^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csgn(I*c)-24*I*ln(c)*Pi*csgn(I*c*x^n)^2*cs
gn(I*c)+3*I*Pi^3*csgn(I*x^n)^3*csgn(I*c*x^n)^5*csgn(I*c)-3*I*Pi^3*csgn(I*x^
n)^3*csgn(I*c*x^n)^4*csgn(I*c)^2-9*I*Pi^3*csgn(I*x^n)^2*csgn(I*c*x^n)^6*cs
gn(I*c)-24*ln(c)*Pi^2*csgn(I*x^n)*csgn(I*c*x^n)^4*csgn(I*c)+12*ln(c)*Pi^2*cs
gn(I*x^n)*csgn(I*c*x^n)^3*csgn(I*c)^2-24*I*Pi*csgn(I*x^n)*csgn(I*c*x^n)*csg
n(I*c)-24*I*ln(c)*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+6*Pi^2*csgn(I*c*x^n)^6-9*I
*Pi^3*csgn(I*x^n)*csgn(I*c*x^n)^6*csgn(I*c)^2+3*I*Pi^3*csgn(I*x^n)*csgn(I*c
*x^n)^5*csgn(I*c)^3+12*I*ln(c)^2*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+12*I*ln(c)^
2*Pi*csgn(I*c*x^n)^2*csgn(I*c))/n*x*exp(1/2*(-1+n)*(-I*csgn(I*d*x)^3*Pi+I*c
sgn(I*d*x)^2*csgn(I*d)*Pi+I*csgn(I*d*x)^2*csgn(I*x)*Pi-I*csgn(I*d*x)*csgn(I
```

```
*d)*csgn(I*x)*Pi+2*ln(x)+2*ln(d))
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(-1+n)*log(c*x^n)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 0.995372, size = 197, normalized size = 2.66

$$\frac{(n^3 \log(x)^3 + \log(c)^3 + 3(n^2 \log(c) - n^2) \log(x)^2 - 3 \log(c)^2 + 3(n \log(c)^2 - 2n \log(c) + 2n) \log(x) + 6 \log(c) - 6)d^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(-1+n)*log(c*x^n)^3,x, algorithm="fricas")
```

```
[Out] (n^3*log(x)^3 + log(c)^3 + 3*(n^2*log(c) - n^2)*log(x)^2 - 3*log(c)^2 + 3*(n*log(c)^2 - 2*n*log(c) + 2*n)*log(x) + 6*log(c) - 6)*d^(n - 1)*x^n/n
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(-1+n)*ln(c*x**n)**3,x)
```

```
[Out] Timed out
```

**Giac [B]** time = 1.26781, size = 219, normalized size = 2.96

$$\frac{d^n n^2 x^n \log(x)^3}{d} + \frac{3 d^n n x^n \log(c) \log(x)^2}{d} + \frac{3 d^n x^n \log(c)^2 \log(x)}{d} - \frac{3 d^n n x^n \log(x)^2}{d} + \frac{d^n x^n \log(c)^3}{dn} - \frac{6 d^n x^n \log(c) \log(x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(-1+n)*log(c*x^n)^3,x, algorithm="giac")
```

```
[Out] d^n*n^2*x^n*log(x)^3/d + 3*d^n*n*x^n*log(c)*log(x)^2/d + 3*d^n*x^n*log(c)^2*log(x)/d - 3*d^n*n*x^n*log(x)^2/d + d^n*x^n*log(c)^3/(d*n) - 6*d^n*x^n*log(c)*log(x)/d - 3*d^n*x^n*log(c)^2/(d*n) + 6*d^n*x^n*log(x)/d + 6*d^n*x^n*log(c)/(d*n) - 6*d^n*x^n/(d*n)
```

### 3.157 $\int (dx)^{-1+n} \log^2(cx^n) dx$

**Optimal.** Leaf size=53

$$\frac{(dx)^n \log^2(cx^n)}{dn} - \frac{2(dx)^n \log(cx^n)}{dn} + \frac{2(dx)^n}{dn}$$

[Out]  $(2*(d*x)^n)/(d*n) - (2*(d*x)^n*\text{Log}[c*x^n])/ (d*n) + ((d*x)^n*\text{Log}[c*x^n]^2)/(d*n)$

**Rubi [A]** time = 0.031621, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2305, 2304}

$$\frac{(dx)^n \log^2(cx^n)}{dn} - \frac{2(dx)^n \log(cx^n)}{dn} + \frac{2(dx)^n}{dn}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^(-1 + n)\*Log[c\*x^n]^2, x]

[Out]  $(2*(d*x)^n)/(d*n) - (2*(d*x)^n*\text{Log}[c*x^n])/ (d*n) + ((d*x)^n*\text{Log}[c*x^n]^2)/(d*n)$

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*(d\*x)^p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int (dx)^{-1+n} \log^2(cx^n) dx &= \frac{(dx)^n \log^2(cx^n)}{dn} - 2 \int (dx)^{-1+n} \log(cx^n) dx \\ &= \frac{2(dx)^n}{dn} - \frac{2(dx)^n \log(cx^n)}{dn} + \frac{(dx)^n \log^2(cx^n)}{dn} \end{aligned}$$

**Mathematica [A]** time = 0.0056123, size = 30, normalized size = 0.57

$$\frac{(dx)^n (\log^2(cx^n) - 2 \log(cx^n) + 2)}{dn}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(-1 + n)\*Log[c\*x^n]^2, x]

[Out]  $((d*x)^n*(2 - 2*\text{Log}[c*x^n] + \text{Log}[c*x^n]^2))/(d*n)$

**Maple [C]** time = 0.121, size = 750, normalized size = 14.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((d*x)^{(-1+n)}*\ln(c*x^n)^2, x)$

[Out]  $\frac{1}{n*x}*\exp(1/2*(-1+n)*(-I*\text{csgn}(I*d*x)^3*\text{Pi}+I*\text{csgn}(I*d*x)^2*\text{csgn}(I*d)*\text{Pi}+I*\text{csgn}(I*d*x)^2*\text{csgn}(I*x)*\text{Pi}-I*\text{csgn}(I*d*x)*\text{csgn}(I*d)*\text{csgn}(I*x)*\text{Pi}+2*\ln(x)+2*\ln(d)))*\ln(x^n)^2+(I*\text{Pi}*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^2-I*\text{Pi}*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)*\text{csgn}(I*c)*-I*\text{Pi}*\text{csgn}(I*c*x^n)^3+I*\text{Pi}*\text{csgn}(I*c*x^n)^2*\text{csgn}(I*c)+2*\ln(c)-2)/n*x*\exp(1/2*(-1+n)*(-I*\text{csgn}(I*d*x)^3*\text{Pi}+I*\text{csgn}(I*d*x)^2*\text{csgn}(I*d)*\text{Pi}+I*\text{csgn}(I*d*x)^2*\text{csgn}(I*x)*\text{Pi}-I*\text{csgn}(I*d*x)*\text{csgn}(I*d)*\text{csgn}(I*x)*\text{Pi}+2*\ln(x)+2*\ln(d)))*\ln(x^n)+1/4*(-\text{Pi}^2*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^4+2*\text{Pi}^2*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^3*\text{csgn}(I*c)-\text{Pi}^2*\text{csgn}(I*x^n)^2*\text{csgn}(I*c*x^n)^2*\text{csgn}(I*c)^2+2*\text{Pi}^2*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^5-4*\text{Pi}^2*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^4*\text{csgn}(I*c)+2*\text{Pi}^2*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^3*\text{csgn}(I*c)^2-\text{Pi}^2*\text{csgn}(I*c*x^n)^6+2*\text{Pi}^2*\text{csgn}(I*c*x^n)^5*\text{csgn}(I*c)-\text{Pi}^2*\text{csgn}(I*c*x^n)^4*\text{csgn}(I*c)^2-4*I*\text{Pi}*\text{csgn}(I*c*x^n)^2*\text{csgn}(I*c)+4*I*\ln(c)*\text{Pi}*\text{csgn}(I*c*x^n)^2*\text{csgn}(I*c)-4*I*\text{Pi}*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^2+4*I*\text{Pi}*\text{csgn}(I*c*x^n)^3+4*I*\ln(c)*\text{Pi}*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)^2-4*I*\ln(c)*\text{Pi}*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)*\text{csgn}(I*c)-4*I*\ln(c)*\text{Pi}*\text{csgn}(I*c*x^n)^3+4*I*\text{Pi}*\text{csgn}(I*x^n)*\text{csgn}(I*c*x^n)*\text{csgn}(I*c)+4*\ln(c)^2-8*\ln(c)+8)/n*x*\exp(1/2*(-1+n)*(-I*\text{csgn}(I*d*x)^3*\text{Pi}+I*\text{csgn}(I*d*x)^2*\text{csgn}(I*d)*\text{Pi}+I*\text{csgn}(I*d*x)^2*\text{csgn}(I*x)*\text{Pi}-I*\text{csgn}(I*d*x)*\text{csgn}(I*d)*\text{csgn}(I*x)*\text{Pi}+2*\ln(x)+2*\ln(d)))$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((d*x)^{(-1+n)}*\log(c*x^n)^2, x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.02732, size = 113, normalized size = 2.13

$$\frac{(n^2 \log(x)^2 + \log(c)^2 + 2(n \log(c) - n) \log(x) - 2 \log(c) + 2)d^{n-1}x^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((d*x)^{(-1+n)}*\log(c*x^n)^2, x, \text{algorithm}="fricas")$

[Out]  $(n^2*\log(x)^2 + \log(c)^2 + 2*(n*\log(c) - n)*\log(x) - 2*\log(c) + 2)*d^{(n - 1)}*x^n/n$



**Sympy [A]** time = 113.492, size = 163, normalized size = 3.08

$$\begin{cases} \infty x \log(c)^2 & \text{for } d = 0 \wedge n = 0 \\ \frac{\log(c)^2 \log(x)}{d} & \text{for } n = 0 \\ 0^{n-1} \left( n^2 x \log(x)^2 - 2n^2 x \log(x) + 2n^2 x + 2nx \log(c) \log(x) - 2nx \log(c) + x \log(c)^2 \right) & \text{for } d = 0 \\ \frac{d^n n x^n \log(x)^2}{d} + \frac{2d^n x^n \log(c) \log(x)}{d} - \frac{2d^n x^n \log(x)}{d} + \frac{d^n x^n \log(c)^2}{dn} - \frac{2d^n x^n \log(c)}{dn} + \frac{2d^n x^n}{dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(-1+n)\*ln(c\*x\*\*n)\*\*2,x)

[Out] Piecewise((zoo\*x\*log(c)\*\*2, Eq(d, 0) & Eq(n, 0)), (log(c)\*\*2\*log(x)/d, Eq(n, 0)), (0\*\*(n - 1)\*(n\*\*2\*x\*log(x)\*\*2 - 2\*n\*\*2\*x\*log(x) + 2\*n\*\*2\*x + 2\*n\*x\*log(c)\*log(x) - 2\*n\*x\*log(c) + x\*log(c)\*\*2), Eq(d, 0)), (d\*\*n\*n\*x\*\*n\*log(x)\*\*2/d + 2\*d\*\*n\*x\*\*n\*log(c)\*log(x)/d - 2\*d\*\*n\*x\*\*n\*log(x)/d + d\*\*n\*x\*\*n\*log(c)\*\*2/(d\*n) - 2\*d\*\*n\*x\*\*n\*log(c)/(d\*n) + 2\*d\*\*n\*x\*\*n/(d\*n), True))

**Giac [A]** time = 1.2535, size = 123, normalized size = 2.32

$$\frac{d^n n x^n \log(x)^2}{d} + \frac{2 d^n x^n \log(c) \log(x)}{d} + \frac{d^n x^n \log(c)^2}{dn} - \frac{2 d^n x^n \log(x)}{d} - \frac{2 d^n x^n \log(c)}{dn} + \frac{2 d^n x^n}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(-1+n)\*log(c\*x^n)^2,x, algorithm="giac")

[Out] d^n\*n\*x^n\*log(x)^2/d + 2\*d^n\*x^n\*log(c)\*log(x)/d + d^n\*x^n\*log(c)^2/(d\*n) - 2\*d^n\*x^n\*log(x)/d - 2\*d^n\*x^n\*log(c)/(d\*n) + 2\*d^n\*x^n/(d\*n)

### 3.158 $\int (dx)^{-1+n} \log(cx^n) dx$

**Optimal.** Leaf size=32

$$\frac{(dx)^n \log(cx^n)}{dn} - \frac{(dx)^n}{dn}$$

[Out]  $-\left(\frac{(d*x)^n}{(d*n)}\right) + \left(\frac{(d*x)^n * \text{Log}[c*x^n]}{(d*n)}\right)$

**Rubi [A]** time = 0.0110705, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {2304}

$$\frac{(dx)^n \log(cx^n)}{dn} - \frac{(dx)^n}{dn}$$

Antiderivative was successfully verified.

[In] `Int[(d*x)^(-1 + n)*Log[c*x^n], x]`

[Out]  $-\left(\frac{(d*x)^n}{(d*n)}\right) + \left(\frac{(d*x)^n * \text{Log}[c*x^n]}{(d*n)}\right)$

#### Rule 2304

`Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_)^(m_.), x_Symbol] :>  
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]`

#### Rubi steps

$$\int (dx)^{-1+n} \log(cx^n) dx = -\frac{(dx)^n}{dn} + \frac{(dx)^n \log(cx^n)}{dn}$$

**Mathematica [A]** time = 0.0041659, size = 20, normalized size = 0.62

$$\frac{(dx)^n (\log(cx^n) - 1)}{dn}$$

Antiderivative was successfully verified.

[In] `Integrate[(d*x)^(-1 + n)*Log[c*x^n], x]`

[Out]  $\left(\frac{(d*x)^n * (-1 + \text{Log}[c*x^n])}{(d*n)}\right)$

**Maple [C]** time = 0.088, size = 263, normalized size = 8.2

$$\frac{x \ln(x^n)}{n} e^{\frac{(-1+n)(-i(\text{csgn}(ix))^3 \pi + i(\text{csgn}(ix))^2 \text{csgn}(id) \pi + i(\text{csgn}(ix))^2 \text{csgn}(ix) \pi - i \pi \text{csgn}(ix) \text{csgn}(id) \text{csgn}(ix) + 2 \ln(x) + 2 \ln(d))}{2}} + \frac{(i \pi \text{csgn}(ix^n) (\text{csgn}(ix^n)))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^(-1+n)*ln(c*x^n), x)`

```
[Out] 1/n*x*exp(1/2*(-1+n)*(-I*csgn(I*d*x)^3*Pi+I*csgn(I*d*x)^2*csgn(I*d)*Pi+I*cs
gn(I*d*x)^2*csgn(I*x)*Pi-I*csgn(I*d*x)*csgn(I*d)*csgn(I*x)*Pi+2*ln(x)+2*ln(
d))*ln(x^n)+1/2*(I*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*csgn(I*x^n)*csgn(I*
c*x^n)*csgn(I*c)-I*Pi*csgn(I*c*x^n)^3+I*Pi*csgn(I*c*x^n)^2*csgn(I*c)+2*ln(c
)-2)/n*x*exp(1/2*(-1+n)*(-I*csgn(I*d*x)^3*Pi+I*csgn(I*d*x)^2*csgn(I*d)*Pi+I
*csgn(I*d*x)^2*csgn(I*x)*Pi-I*csgn(I*d*x)*csgn(I*d)*csgn(I*x)*Pi+2*ln(x)+2*
ln(d))
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(-1+n)*log(c*x^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 0.993306, size = 55, normalized size = 1.72

$$\frac{(n \log(x) + \log(c) - 1)d^{n-1}x^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(-1+n)*log(c*x^n),x, algorithm="fricas")
```

```
[Out] (n*log(x) + log(c) - 1)*d^(n - 1)*x^n/n
```

**Sympy [A]** time = 39.6049, size = 68, normalized size = 2.12

$$\begin{cases} \infty \log(c) & \text{for } d = 0 \wedge n = 0 \\ \frac{\log(c) \log(x)}{d} & \text{for } n = 0 \\ 0^{n-1} (nx \log(x) - nx + x \log(c)) & \text{for } d = 0 \\ \frac{d^n x^n \log(x)}{d} + \frac{d^n x^n \log(c)}{dn} - \frac{d^n x^n}{dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(-1+n)*ln(c*x**n),x)
```

```
[Out] Piecewise((zoo*x*log(c), Eq(d, 0) & Eq(n, 0)), (log(c)*log(x)/d, Eq(n, 0)),
(0**(n - 1)*(n*x*log(x) - n*x + x*log(c)), Eq(d, 0)), (d**n*x**n*log(x)/d
+ d**n*x**n*log(c)/(d*n) - d**n*x**n/(d*n), True))
```

**Giac [A]** time = 1.38578, size = 57, normalized size = 1.78

$$\frac{d^n x^n \log(x)}{d} + \frac{d^n x^n \log(c)}{dn} - \frac{d^n x^n}{dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(-1+n)*log(c*x^n),x, algorithm="giac")
```

```
[Out] d^n*x^n*log(x)/d + d^n*x^n*log(c)/(d*n) - d^n*x^n/(d*n)
```

$$3.159 \quad \int \frac{(dx)^{-1+n}}{\log(cx^n)} dx$$

Optimal. Leaf size=27

$$\frac{x^{1-n}(dx)^{n-1}\text{li}(cx^n)}{cn}$$

[Out]  $(x^{(1-n)}(d*x)^{(-1+n)}\text{LogIntegral}[c*x^n])/(c*n)$

**Rubi [A]** time = 0.0373724, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {2308, 2307, 2298}

$$\frac{x^{1-n}(dx)^{n-1}\text{li}(cx^n)}{cn}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*x)^{(-1+n)}/\text{Log}[c*x^n], x]$

[Out]  $(x^{(1-n)}(d*x)^{(-1+n)}\text{LogIntegral}[c*x^n])/(c*n)$

#### Rule 2308

$\text{Int}[(d_*)(x_*)^{(m_*)}/\text{Log}[(c_*)(x_*)^{(n_*)}], x\_Symbol] \rightarrow \text{Dist}[(d*x)^m/x^m, \text{Int}[x^m/\text{Log}[c*x^n], x], x] /;$  FreeQ[{c, d, m, n}, x] && EqQ[m, n - 1]

#### Rule 2307

$\text{Int}[(x_*)^{(m_*)}/\text{Log}[(c_*)(x_*)^{(n_*)}], x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[1/\text{Log}[c*x], x], x, x^n], x] /;$  FreeQ[{c, m, n}, x] && EqQ[m, n - 1]

#### Rule 2298

$\text{Int}[\text{Log}[(c_*)(x_*)^{(-1)}], x\_Symbol] \rightarrow \text{Simp}[\text{LogIntegral}[c*x]/c, x] /;$  FreeQ[c, x]

#### Rubi steps

$$\begin{aligned} \int \frac{(dx)^{-1+n}}{\log(cx^n)} dx &= (x^{1-n}(dx)^{-1+n}) \int \frac{x^{-1+n}}{\log(cx^n)} dx \\ &= \frac{(x^{1-n}(dx)^{-1+n}) \text{Subst}\left(\int \frac{1}{\log(cx)} dx, x, x^n\right)}{n} \\ &= \frac{x^{1-n}(dx)^{-1+n}\text{li}(cx^n)}{cn} \end{aligned}$$

**Mathematica [A]** time = 0.0079955, size = 27, normalized size = 1.

$$\frac{x^{1-n}(dx)^{n-1}\text{li}(cx^n)}{cn}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(-1 + n)/Log[c\*x^n], x]

[Out] (x^(1 - n)\*(d\*x)^(-1 + n)\*LogIntegral[c\*x^n])/(c\*n)

**Maple [F]** time = 0.254, size = 0, normalized size = 0.

$$\int \frac{(dx)^{-1+n}}{\ln(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(-1+n)/ln(c\*x^n), x)

[Out] int((d\*x)^(-1+n)/ln(c\*x^n), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{n-1}}{\log(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(-1+n)/log(c\*x^n), x, algorithm="maxima")

[Out] integrate((d\*x)^(n - 1)/log(c\*x^n), x)

**Fricas [A]** time = 1.01431, size = 53, normalized size = 1.96

$$\frac{d^{n-1} \text{Ei}(n \log(x) + \log(c))}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(-1+n)/log(c\*x^n), x, algorithm="fricas")

[Out] d^(n - 1)\*Ei(n\*log(x) + log(c))/(c\*n)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{n-1}}{\log(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(-1+n)/ln(c\*x\*\*n), x)

[Out] Integral((d\*x)\*\*(n - 1)/log(c\*x\*\*n), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{n-1}}{\log(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(-1+n)/log(c*x^n),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(n - 1)/log(c*x^n), x)
```

$$3.160 \quad \int \frac{(dx)^{-1+n}}{\log^2(cx^n)} dx$$

**Optimal.** Leaf size=49

$$\frac{x^{1-n}(dx)^{n-1}\text{li}(cx^n)}{cn} - \frac{(dx)^n}{dn \log(cx^n)}$$

[Out]  $-\left(\frac{(d*x)^n}{(d*n*\text{Log}[c*x^n])}\right) + (x^{(1-n)}*(d*x)^{(-1+n)}*\text{LogIntegral}[c*x^n]) / (c*n)$

**Rubi [A]** time = 0.0527794, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {2306, 2308, 2307, 2298}

$$\frac{x^{1-n}(dx)^{n-1}\text{li}(cx^n)}{cn} - \frac{(dx)^n}{dn \log(cx^n)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*x)^{(-1+n)}/\text{Log}[c*x^n]^2, x]$

[Out]  $-\left(\frac{(d*x)^n}{(d*n*\text{Log}[c*x^n])}\right) + (x^{(1-n)}*(d*x)^{(-1+n)}*\text{LogIntegral}[c*x^n]) / (c*n)$

#### Rule 2306

$\text{Int}[(a + \text{Log}[c*x^n])*(b*x^m)^p, x\_Symbol] \rightarrow \text{Simp}[\frac{(d*x)^{m+1}*(a + b*\text{Log}[c*x^n])^{p+1}}{(b*d*n*(p+1))}, x] - \text{Dist}[\frac{m+1}{(b*n*(p+1))}, \text{Int}[(d*x)^m*(a + b*\text{Log}[c*x^n])^{p+1}, x], x] /;$  FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2308

$\text{Int}[(d*x)^m/\text{Log}[c*x^n], x\_Symbol] \rightarrow \text{Dist}[(d*x)^m/x^m, \text{Int}[x^m/\text{Log}[c*x^n], x], x] /;$  FreeQ[{c, d, m, n}, x] && EqQ[m, n - 1]

#### Rule 2307

$\text{Int}[x^m/\text{Log}[c*x^n], x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[1/\text{Log}[c*x], x], x, x^n], x] /;$  FreeQ[{c, m, n}, x] && EqQ[m, n - 1]

#### Rule 2298

$\text{Int}[\text{Log}[c*x]^{-1}, x\_Symbol] \rightarrow \text{Simp}[\text{LogIntegral}[c*x]/c, x] /;$  FreeQ[c, x]

#### Rubi steps



$$\begin{aligned}
\int \frac{(dx)^{-1+n}}{\log^2(cx^n)} dx &= -\frac{(dx)^n}{dn \log(cx^n)} + \int \frac{(dx)^{-1+n}}{\log(cx^n)} dx \\
&= -\frac{(dx)^n}{dn \log(cx^n)} + (x^{1-n}(dx)^{-1+n}) \int \frac{x^{-1+n}}{\log(cx^n)} dx \\
&= -\frac{(dx)^n}{dn \log(cx^n)} + \frac{(x^{1-n}(dx)^{-1+n}) \text{Subst}\left(\int \frac{1}{\log(cx)} dx, x, x^n\right)}{n} \\
&= -\frac{(dx)^n}{dn \log(cx^n)} + \frac{x^{1-n}(dx)^{-1+n} \text{li}(cx^n)}{cn}
\end{aligned}$$

**Mathematica [A]** time = 0.0172861, size = 49, normalized size = 1.

$$\frac{x^{1-n}(dx)^{n-1} \text{li}(cx^n)}{cn} - \frac{x(dx)^{n-1}}{n \log(cx^n)}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(-1 + n)/Log[c\*x^n]^2, x]

[Out] -((x\*(d\*x)^(-1 + n))/(n\*Log[c\*x^n])) + (x^(1 - n)\*(d\*x)^(-1 + n)\*LogIntegra  
1[c\*x^n])/(c\*n)

**Maple [F]** time = 1.202, size = 0, normalized size = 0.

$$\int \frac{(dx)^{-1+n}}{(\ln(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(-1+n)/ln(c\*x^n)^2, x)

[Out] int((d\*x)^(-1+n)/ln(c\*x^n)^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$d^n \int \frac{x^n}{dx \log(c) + dx \log(x^n)} dx - \frac{d^n x^n}{dn \log(c) + dn \log(x^n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(-1+n)/log(c\*x^n)^2, x, algorithm="maxima")

[Out] d^n\*integrate(x^n/(d\*x\*log(c) + d\*x\*log(x^n)), x) - d^n\*x^n/(d\*n\*log(c) + d  
\*n\*log(x^n))

**Fricas [A]** time = 1.00616, size = 132, normalized size = 2.69

$$\frac{d^{n-1}x^n - \frac{(n \log(x) + \log(c))d^{n-1} \text{Ei}(n \log(x) + \log(c))}{c}}{n^2 \log(x) + n \log(c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(-1+n)/log(c*x^n)^2,x, algorithm="fricas")
```

```
[Out] -(d^(n - 1)*x^n - (n*log(x) + log(c))*d^(n - 1)*Ei(n*log(x) + log(c))/c)/(n^2*log(x) + n*log(c))
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{n-1}}{\log(cx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(-1+n)/ln(c*x**n)**2,x)
```

```
[Out] Integral((d*x)**(n - 1)/log(c*x**n)**2, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{n-1}}{\log(cx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(-1+n)/log(c*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^(n - 1)/log(c*x^n)^2, x)
```

$$3.161 \quad \int \frac{(dx)^{-1+n}}{\log^3(cx^n)} dx$$

**Optimal.** Leaf size=77

$$\frac{x^{1-n}(dx)^{n-1}\text{li}(cx^n)}{2cn} - \frac{(dx)^n}{2dn \log^2(cx^n)} - \frac{(dx)^n}{2dn \log(cx^n)}$$

[Out]  $-(d*x)^n/(2*d*n*\text{Log}[c*x^n]^2) - (d*x)^n/(2*d*n*\text{Log}[c*x^n]) + (x^{(1-n)}*(d*x)^{(-1+n)}*\text{LogIntegral}[c*x^n])/(2*c*n)$

**Rubi [A]** time = 0.075966, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {2306, 2308, 2307, 2298}

$$\frac{x^{1-n}(dx)^{n-1}\text{li}(cx^n)}{2cn} - \frac{(dx)^n}{2dn \log^2(cx^n)} - \frac{(dx)^n}{2dn \log(cx^n)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*x)^{(-1+n)}/\text{Log}[c*x^n]^3, x]$

[Out]  $-(d*x)^n/(2*d*n*\text{Log}[c*x^n]^2) - (d*x)^n/(2*d*n*\text{Log}[c*x^n]) + (x^{(1-n)}*(d*x)^{(-1+n)}*\text{LogIntegral}[c*x^n])/(2*c*n)$

#### Rule 2306

$\text{Int}[(a + \text{Log}[(c_*)*(x_)^(n_)]*(b_*))^(p_)*((d_)*(x_))^(m_), x\_Symbol] \rightarrow \text{Simp}[(d*x)^(m+1)*(a + b*\text{Log}[c*x^n])^(p+1)/(b*d*n*(p+1)), x] - \text{Dist}[(m+1)/(b*n*(p+1)), \text{Int}[(d*x)^m*(a + b*\text{Log}[c*x^n])^(p+1), x], x] /;$  FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2308

$\text{Int}[(d_)*(x_))^(m_)/\text{Log}[(c_)*(x_)]^(n_), x\_Symbol] \rightarrow \text{Dist}[(d*x)^m/x^m, \text{Int}[x^m/\text{Log}[c*x^n], x], x] /;$  FreeQ[{c, d, m, n}, x] && EqQ[m, n - 1]

#### Rule 2307

$\text{Int}[(x_)]^(m_)/\text{Log}[(c_)*(x_)]^(n_), x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[1/\text{Log}[c*x], x], x, x^n], x] /;$  FreeQ[{c, m, n}, x] && EqQ[m, n - 1]

#### Rule 2298

$\text{Int}[\text{Log}[(c_)*(x_)]^(-1), x\_Symbol] \rightarrow \text{Simp}[\text{LogIntegral}[c*x]/c, x] /;$  FreeQ[c, x]

#### Rubi steps

$$\begin{aligned}
\int \frac{(dx)^{-1+n}}{\log^3(cx^n)} dx &= -\frac{(dx)^n}{2dn \log^2(cx^n)} + \frac{1}{2} \int \frac{(dx)^{-1+n}}{\log^2(cx^n)} dx \\
&= -\frac{(dx)^n}{2dn \log^2(cx^n)} - \frac{(dx)^n}{2dn \log(cx^n)} + \frac{1}{2} \int \frac{(dx)^{-1+n}}{\log(cx^n)} dx \\
&= -\frac{(dx)^n}{2dn \log^2(cx^n)} - \frac{(dx)^n}{2dn \log(cx^n)} + \frac{1}{2} (x^{1-n}(dx)^{-1+n}) \int \frac{x^{-1+n}}{\log(cx^n)} dx \\
&= -\frac{(dx)^n}{2dn \log^2(cx^n)} - \frac{(dx)^n}{2dn \log(cx^n)} + \frac{(x^{1-n}(dx)^{-1+n}) \text{Subst}\left(\int \frac{1}{\log(cx)} dx, x, x^n\right)}{2n} \\
&= -\frac{(dx)^n}{2dn \log^2(cx^n)} - \frac{(dx)^n}{2dn \log(cx^n)} + \frac{x^{1-n}(dx)^{-1+n} \text{li}(cx^n)}{2cn}
\end{aligned}$$

**Mathematica [A]** time = 0.0238329, size = 61, normalized size = 0.79

$$\frac{x^{-n}(dx)^n (\text{li}(cx^n) \log^2(cx^n) - cx^n (\log(cx^n) + 1))}{2cdn \log^2(cx^n)}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(-1 + n)/Log[c\*x^n]^3,x]

[Out] ((d\*x)^n\*(-(c\*x^n\*(1 + Log[c\*x^n])) + Log[c\*x^n]^2\*LogIntegral[c\*x^n]))/(2\*c\*d\*n\*x^n\*Log[c\*x^n]^2)

**Maple [F]** time = 1.195, size = 0, normalized size = 0.

$$\int \frac{(dx)^{-1+n}}{(\ln(cx^n))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(-1+n)/ln(c\*x^n)^3,x)

[Out] int((d\*x)^(-1+n)/ln(c\*x^n)^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$d^n \int \frac{x^n}{2(dx \log(c) + dx \log(x^n))} dx - \frac{d^n x^n \log(x^n) + (d^n \log(c) + d^n)x^n}{2(dn \log(c)^2 + 2dn \log(c) \log(x^n) + dn \log(x^n)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(-1+n)/log(c\*x^n)^3,x, algorithm="maxima")

[Out] d^n\*integrate(1/2\*x^n/(d\*x\*log(c) + d\*x\*log(x^n)), x) - 1/2\*(d^n\*x^n\*log(x^n) + (d^n\*log(c) + d^n)\*x^n)/(d^n\*log(c)^2 + 2\*d\*n\*log(c)\*log(x^n) + d\*n\*log(x^n)^2)

**Fricas [A]** time = 1.01033, size = 240, normalized size = 3.12

$$\frac{(n \log(x) + \log(c) + 1)d^{n-1}x^n - \frac{(n^2 \log(x)^2 + 2n \log(c) \log(x) + \log(c)^2)d^{n-1}\text{Ei}(n \log(x) + \log(c))}{c}}{2(n^3 \log(x)^2 + 2n^2 \log(c) \log(x) + n \log(c)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(-1+n)/log(c\*x^n)^3,x, algorithm="fricas")

[Out] -1/2\*((n\*log(x) + log(c) + 1)\*d^(n - 1)\*x^n - (n^2\*log(x)^2 + 2\*n\*log(c)\*log(x) + log(c)^2)\*d^(n - 1)\*Ei(n\*log(x) + log(c))/c)/(n^3\*log(x)^2 + 2\*n^2\*log(c)\*log(x) + n\*log(c)^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{n-1}}{\log(cx^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(-1+n)/ln(c\*x\*\*n)\*\*3,x)

[Out] Integral((d\*x)\*\*(n - 1)/log(c\*x\*\*n)\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{n-1}}{\log(cx^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(-1+n)/log(c\*x^n)^3,x, algorithm="giac")

[Out] integrate((d\*x)^(n - 1)/log(c\*x^n)^3, x)

### 3.162 $\int x^m \log^{\frac{3}{2}}(ax^n) dx$

**Optimal.** Leaf size=111

$$\frac{3\sqrt{\pi}n^{3/2}x^{m+1}(ax^n)^{-\frac{m+1}{n}}\operatorname{Erfi}\left(\frac{\sqrt{m+1}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{4(m+1)^{5/2}} + \frac{x^{m+1}\log^{\frac{3}{2}}(ax^n)}{m+1} - \frac{3nx^{m+1}\sqrt{\log(ax^n)}}{2(m+1)^2}$$

[Out] (3\*n^(3/2)\*Sqrt[Pi]\*x^(1+m)\*Erfi[(Sqrt[1+m]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(4\*(1+m)^(5/2)\*(a\*x^n)^((1+m)/n)) - (3\*n\*x^(1+m)\*Sqrt[Log[a\*x^n]])/(2\*(1+m)^2) + (x^(1+m)\*Log[a\*x^n]^(3/2))/(1+m)

**Rubi [A]** time = 0.125584, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2305, 2310, 2180, 2204}

$$\frac{3\sqrt{\pi}n^{3/2}x^{m+1}(ax^n)^{-\frac{m+1}{n}}\operatorname{Erfi}\left(\frac{\sqrt{m+1}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{4(m+1)^{5/2}} + \frac{x^{m+1}\log^{\frac{3}{2}}(ax^n)}{m+1} - \frac{3nx^{m+1}\sqrt{\log(ax^n)}}{2(m+1)^2}$$

Antiderivative was successfully verified.

[In] Int[x^m\*Log[a\*x^n]^(3/2),x]

[Out] (3\*n^(3/2)\*Sqrt[Pi]\*x^(1+m)\*Erfi[(Sqrt[1+m]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(4\*(1+m)^(5/2)\*(a\*x^n)^((1+m)/n)) - (3\*n\*x^(1+m)\*Sqrt[Log[a\*x^n]])/(2\*(1+m)^2) + (x^(1+m)\*Log[a\*x^n]^(3/2))/(1+m)

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m+1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m+1)), x] - Dist[(b\*n\*p)/(m+1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p-1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m+1)/(d\*n\*(c\*x^n)^((m+1)/n)), Subst[Int[E^(((m+1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned}
\int x^m \log^{\frac{3}{2}}(ax^n) dx &= \frac{x^{1+m} \log^{\frac{3}{2}}(ax^n)}{1+m} - \frac{(3n) \int x^m \sqrt{\log(ax^n)} dx}{2(1+m)} \\
&= -\frac{3nx^{1+m} \sqrt{\log(ax^n)}}{2(1+m)^2} + \frac{x^{1+m} \log^{\frac{3}{2}}(ax^n)}{1+m} + \frac{(3n^2) \int \frac{x^m}{\sqrt{\log(ax^n)}} dx}{4(1+m)^2} \\
&= -\frac{3nx^{1+m} \sqrt{\log(ax^n)}}{2(1+m)^2} + \frac{x^{1+m} \log^{\frac{3}{2}}(ax^n)}{1+m} + \frac{(3nx^{1+m} (ax^n)^{-\frac{1+m}{n}}) \text{Subst} \left( \int \frac{e^{\frac{(1+m)x}{n}}}{\sqrt{x}} dx, x, \log(ax^n) \right)}{4(1+m)^2} \\
&= -\frac{3nx^{1+m} \sqrt{\log(ax^n)}}{2(1+m)^2} + \frac{x^{1+m} \log^{\frac{3}{2}}(ax^n)}{1+m} + \frac{(3nx^{1+m} (ax^n)^{-\frac{1+m}{n}}) \text{Subst} \left( \int e^{\frac{(1+m)x^2}{n}} dx, x, \sqrt{\log(ax^n)} \right)}{2(1+m)^2} \\
&= \frac{3n^{3/2} \sqrt{\pi} x^{1+m} (ax^n)^{-\frac{1+m}{n}} \operatorname{erfi} \left( \frac{\sqrt{1+m} \sqrt{\log(ax^n)}}{\sqrt{n}} \right)}{4(1+m)^{5/2}} - \frac{3nx^{1+m} \sqrt{\log(ax^n)}}{2(1+m)^2} + \frac{x^{1+m} \log^{\frac{3}{2}}(ax^n)}{1+m}
\end{aligned}$$

**Mathematica [A]** time = 0.194285, size = 101, normalized size = 0.91

$$\frac{x^{m+1} \left( 3\sqrt{\pi} n^{3/2} (ax^n)^{-\frac{m+1}{n}} \operatorname{Erfi} \left( \frac{\sqrt{m+1} \sqrt{\log(ax^n)}}{\sqrt{n}} \right) + 2\sqrt{m+1} \sqrt{\log(ax^n)} (2(m+1) \log(ax^n) - 3n) \right)}{4(m+1)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*Log[a\*x^n]^(3/2),x]

[Out] (x^(1+m)\*((3\*n^(3/2)\*Sqrt[Pi]\*Erfi[(Sqrt[1+m]\*Sqrt[Log[a\*x^n]])]/Sqrt[n])/(a\*x^n)^((1+m)/n) + 2\*Sqrt[1+m]\*Sqrt[Log[a\*x^n]]\*(-3\*n + 2\*(1+m)\*Log[a\*x^n]))/(4\*(1+m)^(5/2))

**Maple [F]** time = 0.206, size = 0, normalized size = 0.

$$\int x^m (\ln(ax^n))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*ln(a\*x^n)^(3/2),x)

[Out] int(x^m\*ln(a\*x^n)^(3/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*log(a\*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate(x^m\*log(a\*x^n)^(3/2), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^m \log(ax^n)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*log(a\*x^n)^(3/2),x, algorithm="fricas")

[Out] integral(x^m\*log(a\*x^n)^(3/2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*ln(a\*x\*\*n)\*\*(3/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \log(ax^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*log(a\*x^n)^(3/2),x, algorithm="giac")

[Out] integrate(x^m\*log(a\*x^n)^(3/2), x)



### 3.163 $\int x^m \sqrt{\log(ax^n)} dx$

**Optimal.** Leaf size=86

$$\frac{x^{m+1} \sqrt{\log(ax^n)}}{m+1} - \frac{\sqrt{\pi} \sqrt{n} x^{m+1} (ax^n)^{-\frac{m+1}{n}} \operatorname{Erfi}\left(\frac{\sqrt{m+1} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2(m+1)^{3/2}}$$

```
[Out] -(Sqrt[n]*Sqrt[Pi]*x^(1+m)*Erfi[(Sqrt[1+m]*Sqrt[Log[a*x^n]])/Sqrt[n]])/
(2*(1+m)^(3/2)*(a*x^n)^((1+m)/n)) + (x^(1+m)*Sqrt[Log[a*x^n]])/(1+m
)
```

**Rubi [A]** time = 0.0665412, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2305, 2310, 2180, 2204}

$$\frac{x^{m+1} \sqrt{\log(ax^n)}}{m+1} - \frac{\sqrt{\pi} \sqrt{n} x^{m+1} (ax^n)^{-\frac{m+1}{n}} \operatorname{Erfi}\left(\frac{\sqrt{m+1} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2(m+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Sqrt[Log[a*x^n]],x]
```

```
[Out] -(Sqrt[n]*Sqrt[Pi]*x^(1+m)*Erfi[(Sqrt[1+m]*Sqrt[Log[a*x^n]])/Sqrt[n]])/
(2*(1+m)^(3/2)*(a*x^n)^((1+m)/n)) + (x^(1+m)*Sqrt[Log[a*x^n]])/(1+m
)
```

#### Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^p_.*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m+1)*(a+b*Log[c*x^n])^p)/(d*(m+1)), x] - Dist[(b*n
*p)/(m+1), Int[(d*x)^m*(a+b*Log[c*x^n])^(p-1), x], x] /; FreeQ[{a, b,
c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

#### Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^p_.*((d_.)*(x_))^(m_.), x_Symbol]
:> Dist[(d*x)^(m+1)/(d*n*(c*x^n)^((m+1)/n)), Subst[Int[E^((m+1)*x)
/n]*(a+b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

#### Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol]
:> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

#### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol]
:> Simp[(F^a*Sqrt[Pi]*Erfi[(c+d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

#### Rubi steps

$$\begin{aligned}
\int x^m \sqrt{\log(ax^n)} dx &= \frac{x^{1+m} \sqrt{\log(ax^n)}}{1+m} - \frac{n \int \frac{x^m}{\sqrt{\log(ax^n)}} dx}{2(1+m)} \\
&= \frac{x^{1+m} \sqrt{\log(ax^n)}}{1+m} - \frac{\left(x^{1+m} (ax^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int \frac{e^{\frac{(1+m)x}{n}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{2(1+m)} \\
&= \frac{x^{1+m} \sqrt{\log(ax^n)}}{1+m} - \frac{\left(x^{1+m} (ax^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int e^{\frac{(1+m)x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{1+m} \\
&= -\frac{\sqrt{n} \sqrt{\pi} x^{1+m} (ax^n)^{-\frac{1+m}{n}} \text{erfi}\left(\frac{\sqrt{1+m} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2(1+m)^{3/2}} + \frac{x^{1+m} \sqrt{\log(ax^n)}}{1+m}
\end{aligned}$$

**Mathematica [A]** time = 0.0513356, size = 86, normalized size = 1.

$$\frac{x^{m+1} \sqrt{\log(ax^n)}}{m+1} - \frac{\sqrt{\pi} \sqrt{n} x^{m+1} (ax^n)^{-\frac{m+1}{n}} \text{Erfi}\left(\frac{\sqrt{m+1} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2(m+1)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*Sqrt[Log[a\*x^n]],x]

[Out] -(Sqrt[n]\*Sqrt[Pi]\*x^(1+m)\*Erfi[(Sqrt[1+m]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(2\*(1+m)^(3/2)\*(a\*x^n)^((1+m)/n)) + (x^(1+m)\*Sqrt[Log[a\*x^n]])/(1+m)

**Maple [F]** time = 0.178, size = 0, normalized size = 0.

$$\int x^m \sqrt{\ln(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*ln(a\*x^n)^(1/2),x)

[Out] int(x^m\*ln(a\*x^n)^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*log(a\*x^n)^(1/2),x, algorithm="maxima")

[Out] integrate(x^m\*sqrt(log(a\*x^n)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^m \sqrt{\log(ax^n)}, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*log(a\*x^n)^(1/2),x, algorithm="fricas")

[Out] integral(x^m\*sqrt(log(a\*x^n)), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*ln(a\*x\*\*n)\*\*(1/2),x)

[Out] Integral(x\*\*m\*sqrt(log(a\*x\*\*n)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sqrt{\log(ax^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*log(a\*x^n)^(1/2),x, algorithm="giac")

[Out] integrate(x^m\*sqrt(log(a\*x^n)), x)

$$3.164 \quad \int \frac{x^m}{\sqrt{\log(ax^n)}} dx$$

**Optimal.** Leaf size=61

$$\frac{\sqrt{\pi} x^{m+1} (ax^n)^{-\frac{m+1}{n}} \operatorname{Erfi}\left(\frac{\sqrt{m+1}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{m+1}\sqrt{n}}$$

[Out] (Sqrt[Pi]\*x^(1 + m)\*Erfi[(Sqrt[1 + m]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(Sqrt[1 + m]\*Sqrt[n]\*(a\*x^n)^((1 + m)/n))

**Rubi [A]** time = 0.0472519, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {2310, 2180, 2204}

$$\frac{\sqrt{\pi} x^{m+1} (ax^n)^{-\frac{m+1}{n}} \operatorname{Erfi}\left(\frac{\sqrt{m+1}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{m+1}\sqrt{n}}$$

Antiderivative was successfully verified.

[In] Int[x^m/Sqrt[Log[a\*x^n]], x]

[Out] (Sqrt[Pi]\*x^(1 + m)\*Erfi[(Sqrt[1 + m]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(Sqrt[1 + m]\*Sqrt[n]\*(a\*x^n)^((1 + m)/n))

#### Rule 2310

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol]
:> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)*x)/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

#### Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol]
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

#### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol]
:> Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

#### Rubi steps

$$\begin{aligned} \int \frac{x^m}{\sqrt{\log(ax^n)}} dx &= \frac{\left(x^{1+m} (ax^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int \frac{e^{\frac{(1+m)x}{n}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n} \\ &= \frac{\left(2x^{1+m} (ax^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int e^{\frac{(1+m)x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{n} \\ &= \frac{\sqrt{\pi} x^{1+m} (ax^n)^{-\frac{1+m}{n}} \operatorname{erfi}\left(\frac{\sqrt{1+m}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{1+m}\sqrt{n}} \end{aligned}$$

**Mathematica [A]** time = 0.0105832, size = 61, normalized size = 1.

$$\frac{\sqrt{\pi} x^{m+1} (ax^n)^{-\frac{m+1}{n}} \operatorname{Erfi}\left(\frac{\sqrt{m+1}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{m+1}\sqrt{n}}$$

Antiderivative was successfully verified.

[In] Integrate[x^m/Sqrt[Log[a\*x^n]], x]

[Out] (Sqrt[Pi]\*x^(1+m)\*Erfi[(Sqrt[1+m]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(Sqrt[1+m]\*Sqrt[n]\*(a\*x^n)^((1+m)/n))

**Maple [F]** time = 0.201, size = 0, normalized size = 0.

$$\int x^m \frac{1}{\sqrt{\ln(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/ln(a\*x^n)^(1/2), x)

[Out] int(x^m/ln(a\*x^n)^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/log(a\*x^n)^(1/2), x, algorithm="maxima")

[Out] integrate(x^m/sqrt(log(a\*x^n)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x^m}{\sqrt{\log(ax^n)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>m</sup>/log(a\*x<sup>n</sup>)<sup>(1/2)</sup>,x, algorithm="fricas")

[Out] integral(x<sup>m</sup>/sqrt(log(a\*x<sup>n</sup>)), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m/ln(a\*x\*\*n)\*\*(1/2),x)

[Out] Integral(x\*\*m/sqrt(log(a\*x\*\*n)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\sqrt{\log(ax^n)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>m</sup>/log(a\*x<sup>n</sup>)<sup>(1/2)</sup>,x, algorithm="giac")

[Out] integrate(x<sup>m</sup>/sqrt(log(a\*x<sup>n</sup>)), x)

$$3.165 \quad \int \frac{x^m}{\log^2(ax^n)} dx$$

**Optimal.** Leaf size=83

$$\frac{2\sqrt{\pi}\sqrt{m+1}x^{m+1}(ax^n)^{-\frac{m+1}{n}} \operatorname{Erfi}\left(\frac{\sqrt{m+1}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^{m+1}}{n\sqrt{\log(ax^n)}}$$

[Out] (2\*Sqrt[1 + m]\*Sqrt[Pi]\*x^(1 + m)\*Erfi[(Sqrt[1 + m]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(n^(3/2)\*(a\*x^n)^((1 + m)/n)) - (2\*x^(1 + m))/(n\*Sqrt[Log[a\*x^n]])

**Rubi [A]** time = 0.0679234, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2306, 2310, 2180, 2204}

$$\frac{2\sqrt{\pi}\sqrt{m+1}x^{m+1}(ax^n)^{-\frac{m+1}{n}} \operatorname{Erfi}\left(\frac{\sqrt{m+1}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^{m+1}}{n\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Int[x^m/Log[a\*x^n]^(3/2), x]

[Out] (2\*Sqrt[1 + m]\*Sqrt[Pi]\*x^(1 + m)\*Erfi[(Sqrt[1 + m]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(n^(3/2)\*(a\*x^n)^((1 + m)/n)) - (2\*x^(1 + m))/(n\*Sqrt[Log[a\*x^n]])

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

#### Rubi steps

$$\begin{aligned}
\int \frac{x^m}{\log^{\frac{3}{2}}(ax^n)} dx &= -\frac{2x^{1+m}}{n\sqrt{\log(ax^n)}} + \frac{(2(1+m)) \int \frac{x^m}{\sqrt{\log(ax^n)}} dx}{n} \\
&= -\frac{2x^{1+m}}{n\sqrt{\log(ax^n)}} + \frac{\left(2(1+m)x^{1+m} (ax^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int \frac{e^{\frac{(1+m)x}{n}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{n^2} \\
&= -\frac{2x^{1+m}}{n\sqrt{\log(ax^n)}} + \frac{\left(4(1+m)x^{1+m} (ax^n)^{-\frac{1+m}{n}}\right) \text{Subst}\left(\int e^{\frac{(1+m)x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{n^2} \\
&= \frac{2\sqrt{1+m}\sqrt{\pi}x^{1+m} (ax^n)^{-\frac{1+m}{n}} \operatorname{erfi}\left(\frac{\sqrt{1+m}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^{1+m}}{n\sqrt{\log(ax^n)}}
\end{aligned}$$

**Mathematica [A]** time = 0.193792, size = 86, normalized size = 1.04

$$\frac{2\sqrt{\pi}\sqrt{m+1}e^{-\frac{(m+1)(\log(ax^n)-n\log(x))}{n}} \operatorname{Erfi}\left(\frac{\sqrt{m+1}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^{m+1}}{n\sqrt{\log(ax^n)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^m/Log[a\*x^n]^(3/2), x]

[Out] (2\*Sqrt[1 + m]\*Sqrt[Pi]\*Erfi[(Sqrt[1 + m]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(E^((1 + m)\*(-(n\*Log[x]) + Log[a\*x^n]))/n)\*n^(3/2) - (2\*x^(1 + m))/(n\*Sqrt[Log[a\*x^n]])

**Maple [F]** time = 0.185, size = 0, normalized size = 0.

$$\int x^m (\ln(ax^n))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/ln(a\*x^n)^(3/2), x)

[Out] int(x^m/ln(a\*x^n)^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/log(a\*x^n)^(3/2), x, algorithm="maxima")

[Out] integrate(x^m/log(a\*x^n)^(3/2), x)



---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{\log(ax^n)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/log(a\*x^n)^(3/2),x, algorithm="fricas")

[Out] integral(x^m/log(a\*x^n)^(3/2), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m/ln(a\*x\*\*n)\*\*(3/2),x)

[Out] Integral(x\*\*m/log(a\*x\*\*n)\*\*(3/2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\log(ax^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/log(a\*x^n)^(3/2),x, algorithm="giac")

[Out] integrate(x^m/log(a\*x^n)^(3/2), x)

$$3.166 \quad \int \frac{x^m}{\log^{\frac{5}{2}}(ax^n)} dx$$

**Optimal.** Leaf size=112

$$\frac{4\sqrt{\pi}(m+1)^{3/2}x^{m+1}(ax^n)^{-\frac{m+1}{n}} \operatorname{Erfi}\left(\frac{\sqrt{m+1}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{4(m+1)x^{m+1}}{3n^2\sqrt{\log(ax^n)}} - \frac{2x^{m+1}}{3n\log^{\frac{3}{2}}(ax^n)}$$

[Out] (4\*(1 + m)^(3/2)\*Sqrt[Pi]\*x^(1 + m)\*Erfi[(Sqrt[1 + m]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(3\*n^(5/2)\*(a\*x^n)^((1 + m)/n)) - (2\*x^(1 + m))/(3\*n\*Log[a\*x^n]^(3/2)) - (4\*(1 + m)\*x^(1 + m))/(3\*n^2\*Sqrt[Log[a\*x^n]])

**Rubi [A]** time = 0.0919285, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2306, 2310, 2180, 2204}

$$\frac{4\sqrt{\pi}(m+1)^{3/2}x^{m+1}(ax^n)^{-\frac{m+1}{n}} \operatorname{Erfi}\left(\frac{\sqrt{m+1}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{4(m+1)x^{m+1}}{3n^2\sqrt{\log(ax^n)}} - \frac{2x^{m+1}}{3n\log^{\frac{3}{2}}(ax^n)}$$

Antiderivative was successfully verified.

[In] Int[x^m/Log[a\*x^n]^(5/2), x]

[Out] (4\*(1 + m)^(3/2)\*Sqrt[Pi]\*x^(1 + m)\*Erfi[(Sqrt[1 + m]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/(3\*n^(5/2)\*(a\*x^n)^((1 + m)/n)) - (2\*x^(1 + m))/(3\*n\*Log[a\*x^n]^(3/2)) - (4\*(1 + m)\*x^(1 + m))/(3\*n^2\*Sqrt[Log[a\*x^n]])

#### Rule 2306

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*d\*n\*(p + 1)), x] - Dist[(m + 1)/(b\*n\*(p + 1)), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

#### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rubi steps

$$\begin{aligned}
\int \frac{x^m}{\log^{\frac{5}{2}}(ax^n)} dx &= -\frac{2x^{1+m}}{3n \log^{\frac{3}{2}}(ax^n)} + \frac{(2(1+m)) \int \frac{x^m}{\log^{\frac{3}{2}}(ax^n)} dx}{3n} \\
&= -\frac{2x^{1+m}}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{4(1+m)x^{1+m}}{3n^2 \sqrt{\log(ax^n)}} + \frac{(4(1+m)^2) \int \frac{x^m}{\sqrt{\log(ax^n)}} dx}{3n^2} \\
&= -\frac{2x^{1+m}}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{4(1+m)x^{1+m}}{3n^2 \sqrt{\log(ax^n)}} + \frac{(4(1+m)^2 x^{1+m} (ax^n)^{-\frac{1+m}{n}}) \text{Subst}\left(\int \frac{e^{\frac{(1+m)x}{\sqrt{x}}}}{\sqrt{x}} dx, x, \log(ax^n)\right)}{3n^3} \\
&= -\frac{2x^{1+m}}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{4(1+m)x^{1+m}}{3n^2 \sqrt{\log(ax^n)}} + \frac{(8(1+m)^2 x^{1+m} (ax^n)^{-\frac{1+m}{n}}) \text{Subst}\left(\int e^{\frac{(1+m)x^2}{n}} dx, x, \sqrt{\log(ax^n)}\right)}{3n^3} \\
&= \frac{4(1+m)^{3/2} \sqrt{\pi} x^{1+m} (ax^n)^{-\frac{1+m}{n}} \operatorname{erfi}\left(\frac{\sqrt{1+m} \sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{2x^{1+m}}{3n \log^{\frac{3}{2}}(ax^n)} - \frac{4(1+m)x^{1+m}}{3n^2 \sqrt{\log(ax^n)}}
\end{aligned}$$

**Mathematica [A]** time = 0.41263, size = 103, normalized size = 0.92

$$\frac{2 \left( \frac{2\sqrt{\pi}(m+1)^{3/2} e^{\frac{(m+1)(n \log(x) - \log(ax^n))}{n}} \operatorname{Erfi}\left(\frac{\sqrt{m+1} \sqrt{\log(ax^n)}}{\sqrt{n}}\right) - \frac{x^{m+1}(2(m+1) \log(ax^n) + n)}{\log^{\frac{3}{2}}(ax^n)}}{\sqrt{n}} \right)}{3n^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^m/Log[a\*x^n]^(5/2),x]

[Out] (2\*((2\*E^(((1+m)\*(n\*Log[x] - Log[a\*x^n]))/n)\*(1+m)^(3/2)\*Sqrt[Pi]\*Erfi[(Sqrt[1+m]\*Sqrt[Log[a\*x^n]])/Sqrt[n]])/Sqrt[n] - (x^(1+m)\*(n+2\*(1+m)\*Log[a\*x^n]))/Log[a\*x^n]^(3/2)))/(3\*n^2)

**Maple [F]** time = 0.188, size = 0, normalized size = 0.

$$\int x^m (\ln(ax^n))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m/ln(a\*x^n)^(5/2),x)

[Out] int(x^m/ln(a\*x^n)^(5/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/log(a\*x^n)^(5/2),x, algorithm="maxima")

[Out] integrate(x^m/log(a\*x^n)^(5/2), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m}{\log(ax^n)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/log(a\*x^n)^(5/2),x, algorithm="fricas")

[Out] integral(x^m/log(a\*x^n)^(5/2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m/ln(a\*x\*\*n)\*\*(5/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\log(ax^n)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m/log(a\*x^n)^(5/2),x, algorithm="giac")

[Out] integrate(x^m/log(a\*x^n)^(5/2), x)

### 3.167 $\int (dx)^m (a + b \log(cx^n))^p dx$

**Optimal.** Leaf size=106

$$\frac{(dx)^{m+1} e^{-\frac{a(m+1)}{bn}} (cx^n)^{-\frac{m+1}{n}} (a + b \log(cx^n))^p \left( -\frac{(m+1)(a+b \log(cx^n))}{bn} \right)^{-p} \Gamma\left(p + 1, -\frac{(m+1)(a+b \log(cx^n))}{bn}\right)}{d(m+1)}$$

[Out] ((d\*x)^(1 + m)\*Gamma[1 + p, -(((1 + m)\*(a + b\*Log[c\*x^n]))/(b\*n))])\*(a + b\*Log[c\*x^n])^p/(d\*E^((a\*(1 + m))/(b\*n))\*(1 + m)\*(c\*x^n)^((1 + m)/n)\*(-(((1 + m)\*(a + b\*Log[c\*x^n]))/(b\*n))))^p)

**Rubi [A]** time = 0.0694941, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2310, 2181}

$$\frac{(dx)^{m+1} e^{-\frac{a(m+1)}{bn}} (cx^n)^{-\frac{m+1}{n}} (a + b \log(cx^n))^p \left( -\frac{(m+1)(a+b \log(cx^n))}{bn} \right)^{-p} \Gamma\left(p + 1, -\frac{(m+1)(a+b \log(cx^n))}{bn}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^p,x]

[Out] ((d\*x)^(1 + m)\*Gamma[1 + p, -(((1 + m)\*(a + b\*Log[c\*x^n]))/(b\*n))])\*(a + b\*Log[c\*x^n])^p/(d\*E^((a\*(1 + m))/(b\*n))\*(1 + m)\*(c\*x^n)^((1 + m)/n)\*(-(((1 + m)\*(a + b\*Log[c\*x^n]))/(b\*n))))^p)

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^p]\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2181

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, -(f\*g\*Log[F])/d]\*(c + d\*x)]/(d\*(-(f\*g\*Log[F])/d)^(IntPart[m] + 1)\*(-(f\*g\*Log[F])\*(c + d\*x)/d)^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rubi steps

$$\begin{aligned} \int (dx)^m (a + b \log(cx^n))^p dx &= \frac{\left( (dx)^{1+m} (cx^n)^{-\frac{1+m}{n}} \right) \text{Subst} \left( \int e^{\frac{(1+m)x}{n}} (a + bx)^p dx, x, \log(cx^n) \right)}{dn} \\ &= \frac{e^{-\frac{a(1+m)}{bn}} (dx)^{1+m} (cx^n)^{-\frac{1+m}{n}} \Gamma \left( 1 + p, -\frac{(1+m)(a+b \log(cx^n))}{bn} \right) (a + b \log(cx^n))^p \left( -\frac{(1+m)(a+b \log(cx^n))}{bn} \right)^{-p}}{d(1+m)} \end{aligned}$$

**Mathematica [A]** time = 0.134605, size = 107, normalized size = 1.01

$$\frac{x^{-m} (dx)^m (a + b \log(cx^n))^p \exp \left( -\frac{(m+1)(a+b \log(cx^n)-bn \log(x))}{bn} \right) \left( -\frac{(m+1)(a+b \log(cx^n))}{bn} \right)^{-p} \Gamma \left( p + 1, -\frac{(m+1)(a+b \log(cx^n))}{bn} \right)}{m + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m\*(a + b\*Log[c\*x^n])^p,x]

[Out] ((d\*x)^m\*Gamma[1 + p, -(((1 + m)\*(a + b\*Log[c\*x^n]))/(b\*n))]\*(a + b\*Log[c\*x^n])^p)/(E^(((1 + m)\*(a - b\*n\*Log[x] + b\*Log[c\*x^n]))/(b\*n))\*(1 + m)\*x^m\*(-(((1 + m)\*(a + b\*Log[c\*x^n]))/(b\*n))))^p)

**Maple [F]** time = 1.076, size = 0, normalized size = 0.

$$\int (dx)^m (a + b \ln(cx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+b\*ln(c\*x^n))^p,x)

[Out] int((d\*x)^m\*(a+b\*ln(c\*x^n))^p,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*log(c\*x^n))^p,x, algorithm="maxima")

[Out] Exception raised: RuntimeError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx)^m (b \log(cx^n) + a)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*log(c\*x^n))^p,x, algorithm="fricas")

[Out] integral((d\*x)^m\*(b\*log(c\*x^n) + a)^p, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (a + b \log(cx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*(a+b\*ln(c\*x\*\*n))\*\*p,x)

[Out] Integral((d\*x)\*\*m\*(a + b\*log(c\*x\*\*n))\*\*p, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*log(c*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*(b*log(c*x^n) + a)^p, x)
```

### 3.168 $\int x^2 (a + b \log(cx^n))^p dx$

**Optimal.** Leaf size=89

$$3^{-p-1} x^3 e^{-\frac{3a}{bn}} (cx^n)^{-3/n} (a + b \log(cx^n))^p \left( -\frac{a + b \log(cx^n)}{bn} \right)^{-p} \text{Gamma} \left( p + 1, -\frac{3(a + b \log(cx^n))}{bn} \right)$$

[Out] (3^(-1 - p)\*x^3\*Gamma[1 + p, (-3\*(a + b\*Log[c\*x^n]))/(b\*n)]\*(a + b\*Log[c\*x^n])^p)/(E^((3\*a)/(b\*n))\*(c\*x^n)^(3/n)\*(-(a + b\*Log[c\*x^n]))/(b\*n))^p

**Rubi [A]** time = 0.0643562, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2310, 2181}

$$3^{-p-1} x^3 e^{-\frac{3a}{bn}} (cx^n)^{-3/n} (a + b \log(cx^n))^p \left( -\frac{a + b \log(cx^n)}{bn} \right)^{-p} \text{Gamma} \left( p + 1, -\frac{3(a + b \log(cx^n))}{bn} \right)$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*Log[c\*x^n])^p,x]

[Out] (3^(-1 - p)\*x^3\*Gamma[1 + p, (-3\*(a + b\*Log[c\*x^n]))/(b\*n)]\*(a + b\*Log[c\*x^n])^p)/(E^((3\*a)/(b\*n))\*(c\*x^n)^(3/n)\*(-(a + b\*Log[c\*x^n]))/(b\*n))^p

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^p\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^(m + 1)/n), Subst[Int[E^((m + 1)\*x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2181

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))\*((c\_.) + (d\_.)\*(x\_)^(m\_.)), x\_Symbol] := -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])\*(c + d\*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rubi steps

$$\begin{aligned} \int x^2 (a + b \log(cx^n))^p dx &= \frac{(x^3 (cx^n)^{-3/n}) \text{Subst} \left( \int e^{\frac{3x}{n}} (a + bx)^p dx, x, \log(cx^n) \right)}{n} \\ &= 3^{-1-p} e^{-\frac{3a}{bn}} x^3 (cx^n)^{-3/n} \Gamma \left( 1 + p, -\frac{3(a + b \log(cx^n))}{bn} \right) (a + b \log(cx^n))^p \left( -\frac{a + b \log(cx^n)}{bn} \right)^{-p} \end{aligned}$$

**Mathematica [A]** time = 0.0874848, size = 89, normalized size = 1.

$$3^{-p-1} x^3 e^{-\frac{3a}{bn}} (cx^n)^{-3/n} (a + b \log(cx^n))^p \left( -\frac{a + b \log(cx^n)}{bn} \right)^{-p} \text{Gamma} \left( p + 1, -\frac{3(a + b \log(cx^n))}{bn} \right)$$

Antiderivative was successfully verified.



[In] Integrate[x^2\*(a + b\*Log[c\*x^n])^p,x]

[Out]  $(3^{(-1 - p)} * x^3 * \Gamma[1 + p, (-3*(a + b*\text{Log}[c*x^n]))/(b*n)]) * (a + b*\text{Log}[c*x^n])^p / (E^{((3*a)/(b*n))} * (c*x^n)^{(3/n)} * (-((a + b*\text{Log}[c*x^n])/(b*n))))^p$

**Maple [F]** time = 0.479, size = 0, normalized size = 0.

$$\int x^2 (a + b \ln(cx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*ln(c\*x^n))^p,x)

[Out] int(x^2\*(a+b\*ln(c\*x^n))^p,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*log(c\*x^n))^p,x, algorithm="maxima")

[Out] Exception raised: RuntimeError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((b \log(cx^n) + a)^p x^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*log(c\*x^n))^p,x, algorithm="fricas")

[Out] integral((b\*log(c\*x^n) + a)^p\*x^2, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 (a + b \log(cx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*ln(c\*x\*\*n))\*\*p,x)

[Out] Integral(x\*\*2\*(a + b\*log(c\*x\*\*n))\*\*p, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \log(cx^n) + a)^p x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^p*x^2, x)
```

### 3.169 $\int x (a + b \log(cx^n))^p dx$

**Optimal.** Leaf size=89

$$2^{-p-1} x^2 e^{-\frac{2a}{bn}} (cx^n)^{-2/n} (a + b \log(cx^n))^p \left( -\frac{a + b \log(cx^n)}{bn} \right)^{-p} \Gamma\left(p + 1, -\frac{2(a + b \log(cx^n))}{bn}\right)$$

[Out]  $(2^{(-1-p)} x^2 \Gamma[1+p, (-2(a+b \log[cx^n]))/(bn)] (a+b \log[cx^n])^p) / (E^{((2a)/(bn))} (cx^n)^{(2/n)} (-((a+b \log[cx^n])/(bn)))^p)$

**Rubi [A]** time = 0.0515543, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2310, 2181}

$$2^{-p-1} x^2 e^{-\frac{2a}{bn}} (cx^n)^{-2/n} (a + b \log(cx^n))^p \left( -\frac{a + b \log(cx^n)}{bn} \right)^{-p} \Gamma\left(p + 1, -\frac{2(a + b \log(cx^n))}{bn}\right)$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*Log[c\*x^n])^p,x]

[Out]  $(2^{(-1-p)} x^2 \Gamma[1+p, (-2(a+b \log[cx^n]))/(bn)] (a+b \log[cx^n])^p) / (E^{((2a)/(bn))} (cx^n)^{(2/n)} (-((a+b \log[cx^n])/(bn)))^p)$

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m+1)/(d\*n\*(c\*x^n)^((m+1)/n)), Subst[Int[E^(((m+1)\*x)/n)\*(a+b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2181

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m+1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m]+1)\*(-((f\*g\*Log[F])\*(c + d\*x))/d))^FracPart[m]], x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rubi steps

$$\begin{aligned} \int x (a + b \log(cx^n))^p dx &= \frac{(x^2 (cx^n)^{-2/n}) \text{Subst}\left(\int e^{\frac{2x}{n}} (a + bx)^p dx, x, \log(cx^n)\right)}{n} \\ &= 2^{-1-p} e^{-\frac{2a}{bn}} x^2 (cx^n)^{-2/n} \Gamma\left(1 + p, -\frac{2(a + b \log(cx^n))}{bn}\right) (a + b \log(cx^n))^p \left(-\frac{a + b \log(cx^n)}{bn}\right)^{-p} \end{aligned}$$

**Mathematica [A]** time = 0.0812065, size = 89, normalized size = 1.

$$2^{-p-1} x^2 e^{-\frac{2a}{bn}} (cx^n)^{-2/n} (a + b \log(cx^n))^p \left( -\frac{a + b \log(cx^n)}{bn} \right)^{-p} \Gamma\left(p + 1, -\frac{2(a + b \log(cx^n))}{bn}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*Log[c\*x^n])^p,x]

[Out]  $(2^{(-1 - p)} * x^2 * \text{Gamma}[1 + p, (-2*(a + b*\text{Log}[c*x^n]))/(b*n)] * (a + b*\text{Log}[c*x^n])^p) / (E^{((2*a)/(b*n))} * (c*x^n)^{(2/n)} * (-((a + b*\text{Log}[c*x^n])/(b*n))))^p$

**Maple [F]** time = 0.411, size = 0, normalized size = 0.

$$\int x (a + b \ln(cx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*ln(c\*x^n))^p,x)

[Out] int(x\*(a+b\*ln(c\*x^n))^p,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*log(c\*x^n))^p,x, algorithm="maxima")

[Out] Exception raised: RuntimeError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((b \log(cx^n) + a)^p x, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*log(c\*x^n))^p,x, algorithm="fricas")

[Out] integral((b\*log(c\*x^n) + a)^p\*x, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x (a + b \log(cx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*ln(c\*x\*\*n))\*\*p,x)

[Out] Integral(x\*(a + b\*log(c\*x\*\*n))\*\*p, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \log(cx^n) + a)^p x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^p*x, x)
```

### 3.170 $\int (a + b \log(cx^n))^p dx$

**Optimal.** Leaf size=80

$$x e^{-\frac{a}{bn}} (cx^n)^{-1/n} (a + b \log(cx^n))^p \left( -\frac{a + b \log(cx^n)}{bn} \right)^{-p} \text{Gamma} \left( p + 1, -\frac{a + b \log(cx^n)}{bn} \right)$$

[Out] (x\*Gamma[1 + p, -((a + b\*Log[c\*x^n])/(b\*n))]\*(a + b\*Log[c\*x^n])^p)/(E^(a/(b\*n))\*(c\*x^n)^n^(-1)\*(-(a + b\*Log[c\*x^n])/(b\*n)))^p)

**Rubi [A]** time = 0.0375806, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2300, 2181}

$$x e^{-\frac{a}{bn}} (cx^n)^{-1/n} (a + b \log(cx^n))^p \left( -\frac{a + b \log(cx^n)}{bn} \right)^{-p} \text{Gamma} \left( p + 1, -\frac{a + b \log(cx^n)}{bn} \right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^p, x]

[Out] (x\*Gamma[1 + p, -((a + b\*Log[c\*x^n])/(b\*n))]\*(a + b\*Log[c\*x^n])^p)/(E^(a/(b\*n))\*(c\*x^n)^n^(-1)\*(-(a + b\*Log[c\*x^n])/(b\*n)))^p)

#### Rule 2300

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^p, x\_Symbol] := Dist[x/(n\*(c\*x^n)^(1/n)), Subst[Int[E^(x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

#### Rule 2181

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] := -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])\*(c + d\*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rubi steps

$$\begin{aligned} \int (a + b \log(cx^n))^p dx &= \frac{(x (cx^n)^{-1/n}) \text{Subst} \left( \int e^{\frac{x}{n}} (a + bx)^p dx, x, \log(cx^n) \right)}{n} \\ &= e^{-\frac{a}{bn}} x (cx^n)^{-1/n} \Gamma \left( 1 + p, -\frac{a + b \log(cx^n)}{bn} \right) (a + b \log(cx^n))^p \left( -\frac{a + b \log(cx^n)}{bn} \right)^{-p} \end{aligned}$$

**Mathematica [A]** time = 0.0675463, size = 80, normalized size = 1.

$$x e^{-\frac{a}{bn}} (cx^n)^{-1/n} (a + b \log(cx^n))^p \left( -\frac{a + b \log(cx^n)}{bn} \right)^{-p} \text{Gamma} \left( p + 1, -\frac{a + b \log(cx^n)}{bn} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^p,x]

[Out] (x\*Gamma[1 + p, -((a + b\*Log[c\*x^n])/(b\*n))]\*(a + b\*Log[c\*x^n])^p)/(E^(a/(b\*n))\*(c\*x^n)^n^(-1)\*(-((a + b\*Log[c\*x^n])/(b\*n))))^p)

**Maple [F]** time = 0.344, size = 0, normalized size = 0.

$$\int (a + b \ln(cx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^n))^p,x)

[Out] int((a+b\*ln(c\*x^n))^p,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^p,x, algorithm="maxima")

[Out] Exception raised: RuntimeError

**Fricas [A]** time = 1.30775, size = 132, normalized size = 1.65

$$e^{\left(-\frac{bnp \log\left(-\frac{1}{bn}\right) + b \log(c) + a}{bn}\right)} \Gamma\left(p + 1, -\frac{bn \log(x) + b \log(c) + a}{bn}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^p,x, algorithm="fricas")

[Out] e^(-((b\*n\*p\*log(-1/(b\*n)) + b\*log(c) + a)/(b\*n)))\*gamma(p + 1, -(b\*n\*log(x) + b\*log(c) + a)/(b\*n))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \log(cx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))\*\*p,x)

[Out] Integral((a + b\*log(c\*x\*\*n))\*\*p, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^p, x)
```



$$3.171 \quad \int \frac{(a+b \log(cx^n))^p}{x} dx$$

**Optimal.** Leaf size=26

$$\frac{(a+b \log(cx^n))^{p+1}}{bn(p+1)}$$

[Out] (a + b\*Log[c\*x^n])^(1 + p)/(b\*n\*(1 + p))

**Rubi [A]** time = 0.0317464, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2302, 30}

$$\frac{(a+b \log(cx^n))^{p+1}}{bn(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^p/x,x]

[Out] (a + b\*Log[c\*x^n])^(1 + p)/(b\*n\*(1 + p))

Rule 2302

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] := Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{(a+b \log(cx^n))^p}{x} dx &= \frac{\text{Subst}\left(\int x^p dx, x, a+b \log(cx^n)\right)}{bn} \\ &= \frac{(a+b \log(cx^n))^{1+p}}{bn(1+p)} \end{aligned}$$

**Mathematica [A]** time = 0.0077521, size = 26, normalized size = 1.

$$\frac{(a+b \log(cx^n))^{p+1}}{bn(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^p/x,x]

[Out] (a + b\*Log[c\*x^n])^(1 + p)/(b\*n\*(1 + p))

---

**Maple [A]** time = 0.04, size = 27, normalized size = 1.

$$\frac{(a + b \ln(cx^n))^{1+p}}{bn(1+p)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^n))^p/x,x)

[Out] (a+b\*ln(c\*x^n))^(1+p)/b/n/(1+p)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^p/x,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 1.03451, size = 99, normalized size = 3.81

$$\frac{(bn \log(x) + b \log(c) + a)(bn \log(x) + b \log(c) + a)^p}{bn^p + bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^p/x,x, algorithm="fricas")

[Out] (b\*n\*log(x) + b\*log(c) + a)\*(b\*n\*log(x) + b\*log(c) + a)^p/(b\*n\*p + b\*n)

---

**Sympy [A]** time = 2.6094, size = 56, normalized size = 2.15

$$- \begin{cases} -a^p \log(x) & \text{for } b = 0 \\ -(a + b \log(c))^p \log(x) & \text{for } n = 0 \\ \begin{cases} \frac{(a+b \log(cx^n))^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(a + b \log(cx^n)) & \text{otherwise} \end{cases} & \text{otherwise} \end{cases} \frac{1}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))\*\*p/x,x)

[Out] -Piecewise((-a\*\*p\*log(x), Eq(b, 0)), (- (a + b\*log(c))\*\*p\*log(x), Eq(n, 0)), (-Piecewise(((a + b\*log(c\*x\*\*n))\*\*p + 1)/(p + 1), Ne(p, -1)), (log(a + b\*log(c\*x\*\*n)), True))/(b\*n), True))

---

**Giac [A]** time = 1.31826, size = 36, normalized size = 1.38

$$\frac{(bn \log(x) + b \log(c) + a)^{p+1}}{bn(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^p/x,x, algorithm="giac")

[Out] (b\*n\*log(x) + b\*log(c) + a)^(p + 1)/(b\*n\*(p + 1))

$$3.172 \quad \int \frac{(a+b \log(cx^n))^p}{x^2} dx$$

**Optimal.** Leaf size=78

$$\frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} (a + b \log(cx^n))^p \left(\frac{a+b \log(cx^n)}{bn}\right)^{-p} \Gamma\left(p + 1, \frac{a+b \log(cx^n)}{bn}\right)}{x}$$

[Out] -((E^(a/(b\*n)))\*(c\*x^n)^n^(-1)\*Gamma[1 + p, (a + b\*Log[c\*x^n])/(b\*n)]\*(a + b\*Log[c\*x^n])^p)/(x\*((a + b\*Log[c\*x^n])/(b\*n))^p)

**Rubi [A]** time = 0.0544756, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2310, 2181}

$$\frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} (a + b \log(cx^n))^p \left(\frac{a+b \log(cx^n)}{bn}\right)^{-p} \Gamma\left(p + 1, \frac{a+b \log(cx^n)}{bn}\right)}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^p/x^2, x]

[Out] -((E^(a/(b\*n)))\*(c\*x^n)^n^(-1)\*Gamma[1 + p, (a + b\*Log[c\*x^n])/(b\*n)]\*(a + b\*Log[c\*x^n])^p)/(x\*((a + b\*Log[c\*x^n])/(b\*n))^p)

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^p\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2181

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] := -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])\*(c + d\*x))/d)^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rubi steps

$$\begin{aligned} \int \frac{(a + b \log(cx^n))^p}{x^2} dx &= \frac{(cx^n)^{\frac{1}{n}} \text{Subst}\left(\int e^{-\frac{x}{n}}(a + bx)^p dx, x, \log(cx^n)\right)}{nx} \\ &= \frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} \Gamma\left(1 + p, \frac{a+b \log(cx^n)}{bn}\right) (a + b \log(cx^n))^p \left(\frac{a+b \log(cx^n)}{bn}\right)^{-p}}{x} \end{aligned}$$

**Mathematica [A]** time = 0.0729687, size = 78, normalized size = 1.

$$\frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} (a + b \log(cx^n))^p \left(\frac{a+b \log(cx^n)}{bn}\right)^{-p} \Gamma\left(p + 1, \frac{a+b \log(cx^n)}{bn}\right)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^p/x^2,x]

[Out]  $-\left(\frac{E^{a/(b*n)}(c*x^n)^{n(-1)}\Gamma[1+p, (a + b*\text{Log}[c*x^n])/(b*n)]*(a + b*\text{Log}[c*x^n])^p}{x*((a + b*\text{Log}[c*x^n])/(b*n))^p}\right)$

**Maple [F]** time = 0.36, size = 0, normalized size = 0.

$$\int \frac{(a + b \ln(cx^n))^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^n))^p/x^2,x)

[Out] int((a+b\*ln(c\*x^n))^p/x^2,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^p/x^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \log(cx^n) + a)^p}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^p/x^2,x, algorithm="fricas")

[Out] integral((b\*log(c\*x^n) + a)^p/x^2, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \log(cx^n))^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))\*\*p/x\*\*2,x)

[Out] Integral((a + b\*log(c\*x\*\*n))\*\*p/x\*\*2, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \log(cx^n) + a)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^p/x^2, x)
```

$$3.173 \quad \int \frac{(a+b \log(cx^n))^p}{x^3} dx$$

**Optimal.** Leaf size=89

$$\frac{2^{-p-1} e^{\frac{2a}{bn}} (cx^n)^{2/n} (a+b \log(cx^n))^p \left(\frac{a+b \log(cx^n)}{bn}\right)^{-p} \Gamma\left(p+1, \frac{2(a+b \log(cx^n))}{bn}\right)}{x^2}$$

[Out]  $-\left(2^{-1-p} E^{\left(\frac{2a}{bn}\right)} (cx^n)^{2/n} \Gamma\left[1+p, \frac{2(a+b \log(cx^n))}{bn}\right]\right) / (b^n) * (a+b \log(cx^n))^p / (x^2 * \left(\frac{a+b \log(cx^n)}{bn}\right)^p)$

**Rubi [A]** time = 0.0583266, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2310, 2181}

$$\frac{2^{-p-1} e^{\frac{2a}{bn}} (cx^n)^{2/n} (a+b \log(cx^n))^p \left(\frac{a+b \log(cx^n)}{bn}\right)^{-p} \Gamma\left(p+1, \frac{2(a+b \log(cx^n))}{bn}\right)}{x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^p/x^3, x]

[Out]  $-\left(2^{-1-p} E^{\left(\frac{2a}{bn}\right)} (cx^n)^{2/n} \Gamma\left[1+p, \frac{2(a+b \log(cx^n))}{bn}\right]\right) / (b^n) * (a+b \log(cx^n))^p / (x^2 * \left(\frac{a+b \log(cx^n)}{bn}\right)^p)$

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m+1)/(d\*n\*(c\*x^n)^((m+1)/n)), Subst[Int[E^((m+1)\*x)/n]\*(a+b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2181

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))) \* ((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] :> -Simp[(F^(g\*(e - (c\*f)/d)) \* (c + d\*x)^FracPart[m] \* Gamma[m+1, -(f\*g\*Log[F])/d] \* (c + d\*x)) / (d \* (-(f\*g\*Log[F])/d)^(IntPart[m]+1) \* (-(f\*g\*Log[F]) \* (c + d\*x)/d)^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rubi steps

$$\int \frac{(a+b \log(cx^n))^p}{x^3} dx = \frac{(cx^n)^{2/n} \text{Subst}\left(\int e^{-\frac{2x}{n}} (a+bx)^p dx, x, \log(cx^n)\right)}{nx^2} = -\frac{2^{-1-p} e^{\frac{2a}{bn}} (cx^n)^{2/n} \Gamma\left(1+p, \frac{2(a+b \log(cx^n))}{bn}\right) (a+b \log(cx^n))^p \left(\frac{a+b \log(cx^n)}{bn}\right)^{-p}}{x^2}$$

**Mathematica [A]** time = 0.0781012, size = 89, normalized size = 1.

$$\frac{2^{-p-1} e^{\frac{2a}{bn}} (cx^n)^{2/n} (a+b \log(cx^n))^p \left(\frac{a+b \log(cx^n)}{bn}\right)^{-p} \Gamma\left(p+1, \frac{2(a+b \log(cx^n))}{bn}\right)}{x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^p/x^3,x]

[Out] -((2^(-1 - p)\*E^((2\*a)/(b\*n))\*(c\*x^n)^(2/n)\*Gamma[1 + p, (2\*(a + b\*Log[c\*x^n]))/(b\*n)]\*(a + b\*Log[c\*x^n])^p)/(x^2\*((a + b\*Log[c\*x^n])/(b\*n))^p))

**Maple [F]** time = 0.174, size = 0, normalized size = 0.

$$\int \frac{(a + b \ln(cx^n))^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^n))^p/x^3,x)

[Out] int((a+b\*ln(c\*x^n))^p/x^3,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^p/x^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \log(cx^n) + a)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^p/x^3,x, algorithm="fricas")

[Out] integral((b\*log(c\*x^n) + a)^p/x^3, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \log(cx^n))^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))\*\*p/x\*\*3,x)

[Out] Integral((a + b\*log(c\*x\*\*n))\*\*p/x\*\*3, x)



---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \log(cx^n) + a)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^n))^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^p/x^3, x)
```

$$3.174 \quad \int \frac{(a+b \log(cx^n))^p}{x^4} dx$$

**Optimal.** Leaf size=89

$$\frac{3^{-p-1} e^{\frac{3a}{bn}} (cx^n)^{3/n} (a+b \log(cx^n))^p \left(\frac{a+b \log(cx^n)}{bn}\right)^{-p} \Gamma\left(p+1, \frac{3(a+b \log(cx^n))}{bn}\right)}{x^3}$$

[Out]  $-\left(\left(3^{-1-p}\right) E^{\left(\frac{3a}{bn}\right)} \left(c x^n\right)^{\frac{3}{n}} \Gamma\left[1+p, \frac{3\left(a+b \operatorname{Log}\left[c x^n\right]\right)}{bn}\right]\right) / \left(b n\right) \cdot \left(a+b \operatorname{Log}\left[c x^n\right]\right)^p / \left(x^3 \left(a+b \operatorname{Log}\left[c x^n\right]\right) / \left(b n\right)\right)^p$

**Rubi [A]** time = 0.0585755, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2310, 2181}

$$\frac{3^{-p-1} e^{\frac{3a}{bn}} (cx^n)^{3/n} (a+b \log(cx^n))^p \left(\frac{a+b \log(cx^n)}{bn}\right)^{-p} \Gamma\left(p+1, \frac{3(a+b \log(cx^n))}{bn}\right)}{x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x^n])^p/x^4, x]

[Out]  $-\left(\left(3^{-1-p}\right) E^{\left(\frac{3a}{bn}\right)} \left(c x^n\right)^{\frac{3}{n}} \Gamma\left[1+p, \frac{3\left(a+b \operatorname{Log}\left[c x^n\right]\right)}{bn}\right]\right) / \left(b n\right) \cdot \left(a+b \operatorname{Log}\left[c x^n\right]\right)^p / \left(x^3 \left(a+b \operatorname{Log}\left[c x^n\right]\right) / \left(b n\right)\right)^p$

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[(d\*x)^(m+1)/(d\*n\*(c\*x^n)^((m+1)/n)), Subst[Int[E^((m+1)\*x/n)\*(a+b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2181

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m+1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m]+1)\*(-((f\*g\*Log[F])\*(c + d\*x)/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rubi steps

$$\begin{aligned} \int \frac{(a+b \log(cx^n))^p}{x^4} dx &= \frac{(cx^n)^{3/n} \operatorname{Subst}\left(\int e^{-\frac{3x}{n}} (a+bx)^p dx, x, \log(cx^n)\right)}{nx^3} \\ &= \frac{3^{-1-p} e^{\frac{3a}{bn}} (cx^n)^{3/n} \Gamma\left(1+p, \frac{3(a+b \log(cx^n))}{bn}\right) (a+b \log(cx^n))^p \left(\frac{a+b \log(cx^n)}{bn}\right)^{-p}}{x^3} \end{aligned}$$

**Mathematica [A]** time = 0.0795773, size = 89, normalized size = 1.

$$\frac{3^{-p-1} e^{\frac{3a}{bn}} (cx^n)^{3/n} (a+b \log(cx^n))^p \left(\frac{a+b \log(cx^n)}{bn}\right)^{-p} \Gamma\left(p+1, \frac{3(a+b \log(cx^n))}{bn}\right)}{x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x^n])^p/x^4,x]

[Out]  $-\left(\frac{3^{-1-p} E^{\left(\frac{3a}{bn}\right)} (cx^n)^{\frac{3}{n}} \Gamma[1+p]}{(bn)^3} \left(\frac{a + b \log[cx^n]}{bn}\right)^p\right) / (x^3 \left(\frac{a + b \log[cx^n]}{bn}\right)^p)$

**Maple [F]** time = 0.185, size = 0, normalized size = 0.

$$\int \frac{(a + b \ln(cx^n))^p}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^n))^p/x^4,x)

[Out] int((a+b\*ln(c\*x^n))^p/x^4,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^p/x^4,x, algorithm="maxima")

[Out] Exception raised: RuntimeError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \log(cx^n) + a)^p}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^p/x^4,x, algorithm="fricas")

[Out] integral((b\*log(c\*x^n) + a)^p/x^4, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \log(cx^n))^p}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*n))\*\*p/x\*\*4,x)

[Out] Integral((a + b\*log(c\*x\*\*n))\*\*p/x\*\*4, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \log(cx^n) + a)^p}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^n))^p/x^4,x, algorithm="giac")

[Out] integrate((b\*log(c\*x^n) + a)^p/x^4, x)

### 3.175 $\int (dx)^m (a + b \log(cx))^p dx$

**Optimal.** Leaf size=86

$$\frac{e^{-\frac{a(m+1)}{b}} (cx)^{-m-1} (dx)^{m+1} (a + b \log(cx))^p \left(-\frac{(m+1)(a+b \log(cx))}{b}\right)^{-p} \text{Gamma}\left(p + 1, -\frac{(m+1)(a+b \log(cx))}{b}\right)}{d(m+1)}$$

[Out]  $((c*x)^{-1 - m}*(d*x)^{(1 + m)}*\text{Gamma}[1 + p, -(((1 + m)*(a + b*\text{Log}[c*x]))/b)]*(a + b*\text{Log}[c*x])^p)/(d*E^{((a*(1 + m))/b)*(1 + m)*(-(((1 + m)*(a + b*\text{Log}[c*x]))/b))}^p)$

**Rubi [A]** time = 0.0717138, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2310, 2181}

$$\frac{e^{-\frac{a(m+1)}{b}} (cx)^{-m-1} (dx)^{m+1} (a + b \log(cx))^p \left(-\frac{(m+1)(a+b \log(cx))}{b}\right)^{-p} \text{Gamma}\left(p + 1, -\frac{(m+1)(a+b \log(cx))}{b}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*x)^m*(a + b*\text{Log}[c*x])^p, x]$

[Out]  $((c*x)^{-1 - m}*(d*x)^{(1 + m)}*\text{Gamma}[1 + p, -(((1 + m)*(a + b*\text{Log}[c*x]))/b)]*(a + b*\text{Log}[c*x])^p)/(d*E^{((a*(1 + m))/b)*(1 + m)*(-(((1 + m)*(a + b*\text{Log}[c*x]))/b))}^p)$

#### Rule 2310

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]^{(p_.)}*((d_.)*(x_.))^{(m_.)}, x\_Symbol]$   
 $]:> \text{Dist}[(d*x)^{(m+1)}/(d*n*(c*x^n)^{(m+1)/n}), \text{Subst}[\text{Int}[E^{((m+1)*x/n)*(a+b*x)^p}, x], x, \text{Log}[c*x^n]], x] /; \text{FreeQ}\{a, b, c, d, m, n, p\}, x]$

#### Rule 2181

$\text{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_.)))*((c_.) + (d_.)*(x_.))^{(m_.)}, x\_Symbol]$   
 $]:> -\text{Simp}[(F^{(g*(e - (c*f)/d))}*(c + d*x)^{\text{FracPart}[m]}*\text{Gamma}[m + 1, -((f*g*\text{Log}[F])/d)]*(c + d*x)]/(d*(-((f*g*\text{Log}[F])/d))^{\text{IntPart}[m] + 1}*(-((f*g*\text{Log}[F])*(c + d*x)/d))^{\text{FracPart}[m]}], x] /; \text{FreeQ}\{F, c, d, e, f, g, m\}, x] \&\& !\text{IntegerQ}[m]$

#### Rubi steps

$$\int (dx)^m (a + b \log(cx))^p dx = \frac{((cx)^{-1-m} (dx)^{1+m}) \text{Subst}\left(\int e^{(1+m)x} (a + bx)^p dx, x, \log(cx)\right)}{d}$$

$$= \frac{e^{-\frac{a(1+m)}{b}} (cx)^{-1-m} (dx)^{1+m} \Gamma\left(1 + p, -\frac{(1+m)(a+b \log(cx))}{b}\right) (a + b \log(cx))^p \left(-\frac{(1+m)(a+b \log(cx))}{b}\right)^{-p}}{d(1+m)}$$

**Mathematica [A]** time = 0.108523, size = 82, normalized size = 0.95

$$\frac{e^{-\frac{a(m+1)}{b}} (cx)^{-m} (dx)^m (a + b \log(cx))^p \left(-\frac{(m+1)(a+b \log(cx))}{b}\right)^{-p} \text{Gamma}\left(p + 1, -\frac{(m+1)(a+b \log(cx))}{b}\right)}{c(m+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m\*(a + b\*Log[c\*x])^p,x]

[Out] ((d\*x)^m\*Gamma[1 + p, -(((1 + m)\*(a + b\*Log[c\*x]))/b)]\*(a + b\*Log[c\*x])^p)/  
(c\*E^((a\*(1 + m))/b)\*(1 + m)\*(c\*x)^m\*(-(((1 + m)\*(a + b\*Log[c\*x]))/b))^p)

**Maple [F]** time = 0.198, size = 0, normalized size = 0.

$$\int (dx)^m (a + b \ln(cx))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+b\*ln(c\*x))^p,x)

[Out] int((d\*x)^m\*(a+b\*ln(c\*x))^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (b \log(cx) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*log(c\*x))^p,x, algorithm="maxima")

[Out] integrate((d\*x)^m\*(b\*log(c\*x) + a)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx)^m (b \log(cx) + a)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*log(c\*x))^p,x, algorithm="fricas")

[Out] integral((d\*x)^m\*(b\*log(c\*x) + a)^p, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (a + b \log(cx))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*(a+b\*ln(c\*x))\*\*p,x)

[Out] Integral((d\*x)\*\*m\*(a + b\*log(c\*x))\*\*p, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (b \log(cx) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*log(c*x))^p,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*(b*log(c*x) + a)^p, x)
```

### 3.176 $\int x^2(a + b \log(cx))^p dx$

**Optimal.** Leaf size=63

$$\frac{3^{-p-1} e^{-\frac{3a}{b}} (a + b \log(cx))^p \left(-\frac{a+b \log(cx)}{b}\right)^{-p} \Gamma\left(p+1, -\frac{3(a+b \log(cx))}{b}\right)}{c^3}$$

[Out]  $(3^{(-1-p)} \Gamma[1+p, (-3*(a+b \text{Log}[c*x]))/b] * (a+b \text{Log}[c*x])^p) / (c^3 * E^{((3*a)/b)} * (-((a+b \text{Log}[c*x])/b))^p)$

**Rubi [A]** time = 0.0588589, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2309, 2181}

$$\frac{3^{-p-1} e^{-\frac{3a}{b}} (a + b \log(cx))^p \left(-\frac{a+b \log(cx)}{b}\right)^{-p} \Gamma\left(p+1, -\frac{3(a+b \log(cx))}{b}\right)}{c^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*(a + b*\text{Log}[c*x])^p, x]$

[Out]  $(3^{(-1-p)} \Gamma[1+p, (-3*(a+b \text{Log}[c*x]))/b] * (a+b \text{Log}[c*x])^p) / (c^3 * E^{((3*a)/b)} * (-((a+b \text{Log}[c*x])/b))^p)$

#### Rule 2309

$\text{Int}[(a_. + \text{Log}[(c_.)*(x_.)]*(b_.))^{(p_.)}*(x_.)^{(m_.)}, x\_Symbol] \rightarrow \text{Dist}[1/c^{(m+1)}, \text{Subst}[\text{Int}[E^{(m+1)*x}*(a + b*x)^p, x], x, \text{Log}[c*x]], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\amp; \ \text{IntegerQ}[m]$

#### Rule 2181

$\text{Int}[(F_)^{((g_.)*(e_.) + (f_.)*(x_.))}*((c_.) + (d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow -\text{Simp}[(F^{(g*(e - (c*f)/d))}*(c + d*x)^{\text{FracPart}[m]}*\Gamma[m+1, (-((f*g*\text{Log}[F])/d))*(c + d*x)]) / (d*(-((f*g*\text{Log}[F])/d))^{\text{IntPart}[m]+1}*(-((f*g*\text{Log}[F])*(c + d*x))/d))^{\text{FracPart}[m]}], x] /; \text{FreeQ}[\{F, c, d, e, f, g, m\}, x] \ \&\amp; \ !\text{IntegerQ}[m]$

#### Rubi steps

$$\begin{aligned} \int x^2(a + b \log(cx))^p dx &= \frac{\text{Subst}\left(\int e^{3x}(a + bx)^p dx, x, \log(cx)\right)}{c^3} \\ &= \frac{3^{-1-p} e^{-\frac{3a}{b}} \Gamma\left(1+p, -\frac{3(a+b \log(cx))}{b}\right) (a + b \log(cx))^p \left(-\frac{a+b \log(cx)}{b}\right)^{-p}}{c^3} \end{aligned}$$

**Mathematica [A]** time = 0.0363865, size = 63, normalized size = 1.

$$\frac{3^{-p-1} e^{-\frac{3a}{b}} (a + b \log(cx))^p \left(-\frac{a+b \log(cx)}{b}\right)^{-p} \Gamma\left(p+1, -\frac{3(a+b \log(cx))}{b}\right)}{c^3}$$

Antiderivative was successfully verified.



[In] Integrate[x^2\*(a + b\*Log[c\*x])^p,x]

[Out]  $(3^{(-1 - p)} \Gamma[1 + p, (-3*(a + b \operatorname{Log}[c*x]))/b] * (a + b \operatorname{Log}[c*x])^p) / (c^3 * E^{((3*a)/b) * (-((a + b \operatorname{Log}[c*x])/b))}^p)$

**Maple [F]** time = 0.058, size = 0, normalized size = 0.

$$\int x^2 (a + b \ln(cx))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*ln(c\*x))^p,x)

[Out] int(x^2\*(a+b\*ln(c\*x))^p,x)

**Maxima [A]** time = 1.1952, size = 59, normalized size = 0.94

$$\frac{(b \log(cx) + a)^{p+1} e^{\left(-\frac{3a}{b}\right)} E_{-p}\left(-\frac{3(b \log(cx) + a)}{b}\right)}{bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*log(c\*x))^p,x, algorithm="maxima")

[Out]  $-(b \log(c*x) + a)^{(p + 1)} * e^{(-3*a/b)} * \exp\_integral\_e(-p, -3*(b \log(c*x) + a)/b) / (b*c^3)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}((b \log(cx) + a)^p x^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*log(c\*x))^p,x, algorithm="fricas")

[Out] integral((b\*log(c\*x) + a)^p\*x^2, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 (a + b \log(cx))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*ln(c\*x))\*\*p,x)

[Out] Integral(x\*\*2\*(a + b\*log(c\*x))\*\*p, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \log(cx) + a)^p x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x) + a)^p*x^2, x)
```

### 3.177 $\int x(a + b \log(cx))^p dx$

**Optimal.** Leaf size=63

$$\frac{2^{-p-1} e^{-\frac{2a}{b}} (a + b \log(cx))^p \left(-\frac{a+b \log(cx)}{b}\right)^{-p} \Gamma\left(p+1, -\frac{2(a+b \log(cx))}{b}\right)}{c^2}$$

[Out]  $(2^{(-1-p)} \Gamma[1+p, (-2*(a+b \log[c*x]))/b] * (a+b \log[c*x])^p) / (c^2 * E^{((2*a)/b) * (-((a+b \log[c*x])/b))^p})$

**Rubi [A]** time = 0.0474052, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2309, 2181}

$$\frac{2^{-p-1} e^{-\frac{2a}{b}} (a + b \log(cx))^p \left(-\frac{a+b \log(cx)}{b}\right)^{-p} \Gamma\left(p+1, -\frac{2(a+b \log(cx))}{b}\right)}{c^2}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*Log[c\*x])^p, x]

[Out]  $(2^{(-1-p)} \Gamma[1+p, (-2*(a+b \log[c*x]))/b] * (a+b \log[c*x])^p) / (c^2 * E^{((2*a)/b) * (-((a+b \log[c*x])/b))^p})$

#### Rule 2309

Int[((a\_.) + Log[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*(x\_.)^(m\_.), x\_Symbol] :> Dist[1/c^(m+1), Subst[Int[E^((m+1)\*x)\*(a+b\*x)^p, x], x, Log[c\*x]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]

#### Rule 2181

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_.)))\*((c\_.) + (d\_.)\*(x\_.))^(m\_.), x\_Symbol] :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m+1, (-((f\*g\*Log[F])/d)\*(c + d\*x))]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m]+1)\*(-((f\*g\*Log[F])\*(c + d\*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rubi steps

$$\begin{aligned} \int x(a + b \log(cx))^p dx &= \frac{\text{Subst}\left(\int e^{2x}(a + bx)^p dx, x, \log(cx)\right)}{c^2} \\ &= \frac{2^{-1-p} e^{-\frac{2a}{b}} \Gamma\left(1+p, -\frac{2(a+b \log(cx))}{b}\right) (a + b \log(cx))^p \left(-\frac{a+b \log(cx)}{b}\right)^{-p}}{c^2} \end{aligned}$$

**Mathematica [A]** time = 0.0312426, size = 63, normalized size = 1.

$$\frac{2^{-p-1} e^{-\frac{2a}{b}} (a + b \log(cx))^p \left(-\frac{a+b \log(cx)}{b}\right)^{-p} \Gamma\left(p+1, -\frac{2(a+b \log(cx))}{b}\right)}{c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*Log[c\*x])^p,x]

[Out]  $(2^{(-1 - p)} \text{Gamma}[1 + p, (-2*(a + b \text{Log}[c*x]))/b] * (a + b \text{Log}[c*x])^p) / (c^{2*E^{((2*a)/b)} * (-((a + b \text{Log}[c*x])/b))}^p)$

**Maple [F]** time = 0.051, size = 0, normalized size = 0.

$$\int x(a + b \ln(cx))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*ln(c\*x))^p,x)

[Out] int(x\*(a+b\*ln(c\*x))^p,x)

**Maxima [A]** time = 1.26148, size = 59, normalized size = 0.94

$$-\frac{(b \log(cx) + a)^{p+1} e^{\left(-\frac{2a}{b}\right)} E_{-p}\left(-\frac{2(b \log(cx) + a)}{b}\right)}{bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*log(c\*x))^p,x, algorithm="maxima")

[Out]  $-(b \log(c*x) + a)^{(p + 1)} * e^{(-2*a/b)} * \text{exp\_integral\_e}(-p, -2*(b \log(c*x) + a)/b) / (b*c^2)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((b \log(cx) + a)^p x, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*log(c\*x))^p,x, algorithm="fricas")

[Out] integral((b\*log(c\*x) + a)^p\*x, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x(a + b \log(cx))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*ln(c\*x))\*\*p,x)

[Out] Integral(x\*(a + b\*log(c\*x))\*\*p, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \log(cx) + a)^p x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*log(c*x))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x) + a)^p*x, x)
```

### 3.178 $\int (a + b \log(cx))^p dx$

**Optimal.** Leaf size=56

$$\frac{e^{-\frac{a}{b}}(a + b \log(cx))^p \left(-\frac{a+b \log(cx)}{b}\right)^{-p} \Gamma\left(p + 1, -\frac{a+b \log(cx)}{b}\right)}{c}$$

[Out] (Gamma[1 + p, -((a + b\*Log[c\*x])/b)]\*(a + b\*Log[c\*x])^p)/(c\*E^(a/b)\*(-((a + b\*Log[c\*x])/b))^p)

**Rubi [A]** time = 0.0329044, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2299, 2181}

$$\frac{e^{-\frac{a}{b}}(a + b \log(cx))^p \left(-\frac{a+b \log(cx)}{b}\right)^{-p} \Gamma\left(p + 1, -\frac{a+b \log(cx)}{b}\right)}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x])^p,x]

[Out] (Gamma[1 + p, -((a + b\*Log[c\*x])/b)]\*(a + b\*Log[c\*x])^p)/(c\*E^(a/b)\*(-((a + b\*Log[c\*x])/b))^p)

#### Rule 2299

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^p], x\_Symbol] := Dist[1/(n\*c^(1/n)), Subst[Int[E^(x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[1/n]

#### Rule 2181

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] := -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])\*(c + d\*x)/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rubi steps

$$\begin{aligned} \int (a + b \log(cx))^p dx &= \frac{\text{Subst}\left(\int e^x(a + bx)^p dx, x, \log(cx)\right)}{c} \\ &= \frac{e^{-\frac{a}{b}} \Gamma\left(1 + p, -\frac{a+b \log(cx)}{b}\right) (a + b \log(cx))^p \left(-\frac{a+b \log(cx)}{b}\right)^{-p}}{c} \end{aligned}$$

**Mathematica [A]** time = 0.0260325, size = 56, normalized size = 1.

$$\frac{e^{-\frac{a}{b}}(a + b \log(cx))^p \left(-\frac{a+b \log(cx)}{b}\right)^{-p} \Gamma\left(p + 1, -\frac{a+b \log(cx)}{b}\right)}{c}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x])^p,x]

[Out] (Gamma[1 + p, -((a + b\*Log[c\*x])/b)]\*(a + b\*Log[c\*x])^p)/(c\*E^(a/b)\*(-((a + b\*Log[c\*x])/b))^p)

**Maple [F]** time = 0.048, size = 0, normalized size = 0.

$$\int (a + b \ln(cx))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x))^p,x)

[Out] int((a+b\*ln(c\*x))^p,x)

**Maxima [A]** time = 1.24848, size = 59, normalized size = 1.05

$$\frac{(b \log(cx) + a)^{p+1} e^{-\frac{a}{b}} E_{-p}\left(-\frac{b \log(cx) + a}{b}\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x))^p,x, algorithm="maxima")

[Out] -(b\*log(c\*x) + a)^(p + 1)\*e^(-a/b)\*exp\_integral\_e(-p, -(b\*log(c\*x) + a)/b)/(b\*c)

**Fricas [A]** time = 1.05313, size = 86, normalized size = 1.54

$$\frac{e^{\left(\frac{bp \log\left(-\frac{1}{b}\right) + a}{b}\right)} \Gamma\left(p + 1, -\frac{b \log(cx) + a}{b}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x))^p,x, algorithm="fricas")

[Out] e^(-(b\*p\*log(-1/b) + a)/b)\*gamma(p + 1, -(b\*log(c\*x) + a)/b)/c

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \log(cx))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x))\*\*p,x)

[Out] Integral((a + b\*log(c\*x))\*\*p, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \log(cx) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x))^p,x, algorithm="giac")

[Out] integrate((b\*log(c\*x) + a)^p, x)



$$3.179 \quad \int \frac{(a+b \log(cx))^p}{x} dx$$

**Optimal.** Leaf size=21

$$\frac{(a+b \log(cx))^{p+1}}{b(p+1)}$$

[Out] (a + b\*Log[c\*x])^(1 + p)/(b\*(1 + p))

**Rubi [A]** time = 0.0276097, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2302, 30}

$$\frac{(a+b \log(cx))^{p+1}}{b(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x])^p/x,x]

[Out] (a + b\*Log[c\*x])^(1 + p)/(b\*(1 + p))

**Rule 2302**

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] := Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

**Rule 30**

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

**Rubi steps**

$$\begin{aligned} \int \frac{(a+b \log(cx))^p}{x} dx &= \frac{\text{Subst}\left(\int x^p dx, x, a+b \log(cx)\right)}{b} \\ &= \frac{(a+b \log(cx))^{1+p}}{b(1+p)} \end{aligned}$$

**Mathematica [A]** time = 0.0062163, size = 21, normalized size = 1.

$$\frac{(a+b \log(cx))^{p+1}}{b(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x])^p/x,x]

[Out] (a + b\*Log[c\*x])^(1 + p)/(b\*(1 + p))

**Maple [A]** time = 0.037, size = 22, normalized size = 1.1

$$\frac{(a + b \ln(cx))^{1+p}}{b(1+p)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x))^p/x,x)

[Out] (a+b\*ln(c\*x))^(1+p)/b/(1+p)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x))^p/x,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.03041, size = 63, normalized size = 3.

$$\frac{(b \log(cx) + a)(b \log(cx) + a)^p}{bp + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x))^p/x,x, algorithm="fricas")

[Out] (b\*log(c\*x) + a)\*(b\*log(c\*x) + a)^p/(b\*p + b)

**Sympy [A]** time = 1.33317, size = 39, normalized size = 1.86

$$-\begin{cases} -a^p \log(x) & \text{for } b = 0 \\ \frac{(a+b \log(cx))^{p+1}}{p+1} & \text{for } p \neq -1 \\ \frac{\log(a + b \log(cx))}{b} & \text{otherwise} \end{cases} \quad \text{otherwise}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x))\*\*p/x,x)

[Out] -Piecewise((-a\*\*p\*log(x), Eq(b, 0)), (-Piecewise(((a + b\*log(c\*x))\*\*(p + 1))/(p + 1), Ne(p, -1)), (log(a + b\*log(c\*x)), True))/b, True))

**Giac [A]** time = 1.28614, size = 28, normalized size = 1.33

$$\frac{(b \log(cx) + a)^{p+1}}{b(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x))^p/x,x, algorithm="giac")

[Out] (b\*log(c\*x) + a)^(p + 1)/(b\*(p + 1))

$$3.180 \quad \int \frac{(a+b \log(cx))^p}{x^2} dx$$

**Optimal.** Leaf size=52

$$-ce^{a/b}(a+b \log(cx))^p \left(\frac{a+b \log(cx)}{b}\right)^{-p} \text{Gamma}\left(p+1, \frac{a+b \log(cx)}{b}\right)$$

[Out]  $-\left(\left(c \cdot E^{(a/b)} \cdot \text{Gamma}[1+p, (a+b \cdot \text{Log}[c \cdot x])/b]\right) \cdot (a+b \cdot \text{Log}[c \cdot x])^p\right) / \left((a+b \cdot \text{Log}[c \cdot x])/b\right)^p$

**Rubi [A]** time = 0.0493041, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2309, 2181}

$$-ce^{a/b}(a+b \log(cx))^p \left(\frac{a+b \log(cx)}{b}\right)^{-p} \text{Gamma}\left(p+1, \frac{a+b \log(cx)}{b}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x])^p/x^2,x]

[Out]  $-\left(\left(c \cdot E^{(a/b)} \cdot \text{Gamma}[1+p, (a+b \cdot \text{Log}[c \cdot x])/b]\right) \cdot (a+b \cdot \text{Log}[c \cdot x])^p\right) / \left((a+b \cdot \text{Log}[c \cdot x])/b\right)^p$

#### Rule 2309

Int[((a\_.) + Log[(c\_.)\*(x\_.)]\*(b\_.))^p\*(x\_)^(m\_.), x\_Symbol] := Dist[1/c^(m+1), Subst[Int[E^((m+1)\*x)\*(a+b\*x)^p, x], x, Log[c\*x]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]

#### Rule 2181

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] := -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m+1, (-((f\*g\*Log[F])/d))\*(c + d\*x)])/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m]+1)\*(-((f\*g\*Log[F])\*(c + d\*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rubi steps

$$\begin{aligned} \int \frac{(a+b \log(cx))^p}{x^2} dx &= c \text{Subst} \left( \int e^{-x}(a+bx)^p dx, x, \log(cx) \right) \\ &= -ce^{a/b} \Gamma\left(1+p, \frac{a+b \log(cx)}{b}\right) (a+b \log(cx))^p \left(\frac{a+b \log(cx)}{b}\right)^{-p} \end{aligned}$$

**Mathematica [A]** time = 0.0329271, size = 48, normalized size = 0.92

$$-ce^{a/b} \left(\frac{a}{b} + \log(cx)\right)^{-p} (a+b \log(cx))^p \text{Gamma}\left(p+1, \frac{a}{b} + \log(cx)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x])^p/x^2,x]

[Out]  $-\left(\frac{c \cdot E^{a/b} \cdot \Gamma(1 + p, a/b + \text{Log}[c \cdot x]) \cdot (a + b \cdot \text{Log}[c \cdot x])^p}{(a/b + \text{Log}[c \cdot x])^p}\right)$

**Maple [F]** time = 0.054, size = 0, normalized size = 0.

$$\int \frac{(a + b \ln(cx))^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*ln(c*x))^p/x^2,x)`

[Out] `int((a+b*ln(c*x))^p/x^2,x)`

**Maxima [A]** time = 1.25913, size = 54, normalized size = 1.04

$$-\frac{(b \log(cx) + a)^{p+1} c e^{\frac{a}{b}} E_{-p}\left(\frac{b \log(cx) + a}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x))^p/x^2,x, algorithm="maxima")`

[Out]  $-(b \cdot \log(c \cdot x) + a)^{p + 1} \cdot c \cdot e^{a/b} \cdot \text{exp\_integral\_e}(-p, (b \cdot \log(c \cdot x) + a)/b) / b$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \log(cx) + a)^p}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x))^p/x^2,x, algorithm="fricas")`

[Out] `integral((b*log(c*x) + a)^p/x^2, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \log(cx))^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*ln(c*x))**p/x**2,x)`

[Out] `Integral((a + b*log(c*x))**p/x**2, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \log(cx) + a)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x) + a)^p/x^2, x)
```

$$3.181 \quad \int \frac{(a+b \log(cx))^p}{x^3} dx$$

**Optimal.** Leaf size=63

$$c^2 (-2^{-p-1}) e^{\frac{2a}{b}} (a+b \log(cx))^p \left( \frac{a+b \log(cx)}{b} \right)^{-p} \text{Gamma} \left( p+1, \frac{2(a+b \log(cx))}{b} \right)$$

[Out]  $-(2^{-(1+p)} c^2 E^{(2a/b)} \text{Gamma}[1+p, (2(a+b \text{Log}[c*x]))/b]) (a+b \text{Log}[c*x])^p / ((a+b \text{Log}[c*x])/b)^p$

**Rubi [A]** time = 0.0548788, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2309, 2181}

$$c^2 (-2^{-p-1}) e^{\frac{2a}{b}} (a+b \log(cx))^p \left( \frac{a+b \log(cx)}{b} \right)^{-p} \text{Gamma} \left( p+1, \frac{2(a+b \log(cx))}{b} \right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x])^p/x^3,x]

[Out]  $-(2^{-(1+p)} c^2 E^{(2a/b)} \text{Gamma}[1+p, (2(a+b \text{Log}[c*x]))/b]) (a+b \text{Log}[c*x])^p / ((a+b \text{Log}[c*x])/b)^p$

#### Rule 2309

Int[((a\_.) + Log[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/c^(m + 1), Subst[Int[E^((m + 1)\*x)\*(a + b\*x)^p, x], x, Log[c\*x]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]

#### Rule 2181

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-(f\*g\*Log[F])/d)\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-(f\*g\*Log[F])\*(c + d\*x)/d)^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rubi steps

$$\begin{aligned} \int \frac{(a+b \log(cx))^p}{x^3} dx &= c^2 \text{Subst} \left( \int e^{-2x} (a+bx)^p dx, x, \log(cx) \right) \\ &= -2^{-1-p} c^2 e^{\frac{2a}{b}} \Gamma \left( 1+p, \frac{2(a+b \log(cx))}{b} \right) (a+b \log(cx))^p \left( \frac{a+b \log(cx)}{b} \right)^{-p} \end{aligned}$$

**Mathematica [A]** time = 0.0349996, size = 63, normalized size = 1.

$$c^2 (-2^{-p-1}) e^{\frac{2a}{b}} (a+b \log(cx))^p \left( \frac{a+b \log(cx)}{b} \right)^{-p} \text{Gamma} \left( p+1, \frac{2(a+b \log(cx))}{b} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x])^p/x^3,x]

[Out]  $-\left(\frac{2^{-1-p} c^2 E^{\left(\frac{2a}{b}\right)} \Gamma[1+p, (2(a+b\text{Log}[c*x]))/b]}{b}\right) (a+b\text{Log}[c*x])^p / \left(\frac{a+b\text{Log}[c*x]}{b}\right)^p$

**Maple [F]** time = 0.047, size = 0, normalized size = 0.

$$\int \frac{(a + b \ln(cx))^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*ln(c*x))^p/x^3,x)`

[Out] `int((a+b*ln(c*x))^p/x^3,x)`

**Maxima [A]** time = 1.33496, size = 59, normalized size = 0.94

$$\frac{(b \log(cx) + a)^{p+1} c^2 e^{\left(\frac{2a}{b}\right)} E_{-p}\left(\frac{2(b \log(cx) + a)}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x))^p/x^3,x, algorithm="maxima")`

[Out]  $-(b \log(cx) + a)^{p+1} c^2 e^{\left(\frac{2a}{b}\right)} \text{exp\_integral\_e}(-p, 2(b \log(cx) + a)/b) / b$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \log(cx) + a)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x))^p/x^3,x, algorithm="fricas")`

[Out] `integral((b*log(c*x) + a)^p/x^3, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \log(cx))^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*ln(c*x))**p/x**3,x)`

[Out] `Integral((a + b*log(c*x))**p/x**3, x)`



**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \log(cx) + a)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x))^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x) + a)^p/x^3, x)
```

$$3.182 \quad \int \frac{(a+b \log(cx))^p}{x^4} dx$$

**Optimal.** Leaf size=63

$$c^3 (-3^{-p-1}) e^{\frac{3a}{b}} (a+b \log(cx))^p \left( \frac{a+b \log(cx)}{b} \right)^{-p} \text{Gamma} \left( p+1, \frac{3(a+b \log(cx))}{b} \right)$$

[Out]  $-\left(\left(3^{-1-p}\right)*c^3 * E^{\left(\left(3*a\right)/b\right)} * \text{Gamma}\left[1+p, \left(3*\left(a+b*\text{Log}\left[c*x\right]\right)\right)/b\right] * \left(a+b*\text{Log}\left[c*x\right]\right)^p\right) / \left(\left(a+b*\text{Log}\left[c*x\right]\right)/b\right)^p$

**Rubi [A]** time = 0.0543657, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2309, 2181}

$$c^3 (-3^{-p-1}) e^{\frac{3a}{b}} (a+b \log(cx))^p \left( \frac{a+b \log(cx)}{b} \right)^{-p} \text{Gamma} \left( p+1, \frac{3(a+b \log(cx))}{b} \right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*x])^p/x^4, x]

[Out]  $-\left(\left(3^{-1-p}\right)*c^3 * E^{\left(\left(3*a\right)/b\right)} * \text{Gamma}\left[1+p, \left(3*\left(a+b*\text{Log}\left[c*x\right]\right)\right)/b\right] * \left(a+b*\text{Log}\left[c*x\right]\right)^p\right) / \left(\left(a+b*\text{Log}\left[c*x\right]\right)/b\right)^p$

#### Rule 2309

Int[((a\_.) + Log[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*(x\_.)^(m\_.), x\_Symbol] :> Dist[1/c^(m + 1), Subst[Int[E^((m + 1)\*x)\*(a + b\*x)^p, x], x, Log[c\*x]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[m]

#### Rule 2181

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_.)))\*((c\_.) + (d\_.)\*(x\_.))^(m\_.), x\_Symbol] :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])\*(c + d\*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rubi steps

$$\begin{aligned} \int \frac{(a+b \log(cx))^p}{x^4} dx &= c^3 \text{Subst} \left( \int e^{-3x} (a+bx)^p dx, x, \log(cx) \right) \\ &= -3^{-1-p} c^3 e^{\frac{3a}{b}} \Gamma \left( 1+p, \frac{3(a+b \log(cx))}{b} \right) (a+b \log(cx))^p \left( \frac{a+b \log(cx)}{b} \right)^{-p} \end{aligned}$$

**Mathematica [A]** time = 0.0346417, size = 63, normalized size = 1.

$$c^3 (-3^{-p-1}) e^{\frac{3a}{b}} (a+b \log(cx))^p \left( \frac{a+b \log(cx)}{b} \right)^{-p} \text{Gamma} \left( p+1, \frac{3(a+b \log(cx))}{b} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*x])^p/x^4, x]

[Out]  $-\left(3^{(-1-p)}c^3E^{\left(\frac{3a}{b}\right)}\Gamma\left[1+p,\frac{3(a+b\text{Log}[c*x])}{b}\right]*(a+b\text{Log}[c*x])^p\right)/\left(\frac{a+b\text{Log}[c*x]}{b}\right)^p$

**Maple [F]** time = 0.048, size = 0, normalized size = 0.

$$\int \frac{(a + b \ln(cx))^p}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*ln(c*x))^p/x^4,x)`

[Out] `int((a+b*ln(c*x))^p/x^4,x)`

**Maxima [A]** time = 1.26159, size = 59, normalized size = 0.94

$$\frac{(b \log(cx) + a)^{p+1} c^3 e^{\left(\frac{3a}{b}\right)} E_{-p}\left(\frac{3(b \log(cx) + a)}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x))^p/x^4,x, algorithm="maxima")`

[Out]  $-(b \log(cx) + a)^{p+1} c^3 e^{\left(\frac{3a}{b}\right)} \exp\_integral\_e(-p, \frac{3(b \log(cx) + a)}{b})/b$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \log(cx) + a)^p}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*log(c*x))^p/x^4,x, algorithm="fricas")`

[Out] `integral((b*log(c*x) + a)^p/x^4, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \log(cx))^p}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*ln(c*x))**p/x**4,x)`

[Out] `Integral((a + b*log(c*x))**p/x**4, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \log(cx) + a)^p}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x))^p/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x) + a)^p/x^4, x)
```

### 3.183 $\int (dx)^m (a + b \log(c\sqrt{x}))^p dx$

**Optimal.** Leaf size=107

$$\frac{2^{-p} e^{-\frac{2a(m+1)}{b}} (c\sqrt{x})^{-2(m+1)} (dx)^{m+1} (a + b \log(c\sqrt{x}))^p \left( -\frac{(m+1)(a+b \log(c\sqrt{x}))}{b} \right)^{-p} \Gamma\left(p+1, -\frac{2(m+1)(a+b \log(c\sqrt{x}))}{b}\right)}{d(m+1)}$$

[Out] ((d\*x)^(1+m)\*Gamma[1+p, (-2\*(1+m)\*(a+b\*Log[c\*Sqrt[x]]))]/b)\*(a+b\*Log[c\*Sqrt[x]])^p/(2^p\*d\*E^((2\*a\*(1+m))/b)\*(1+m)\*(c\*Sqrt[x])^(2\*(1+m)))\*(-(((1+m)\*(a+b\*Log[c\*Sqrt[x]]))/b))^p

**Rubi [A]** time = 0.0839944, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2310, 2181}

$$\frac{2^{-p} e^{-\frac{2a(m+1)}{b}} (c\sqrt{x})^{-2(m+1)} (dx)^{m+1} (a + b \log(c\sqrt{x}))^p \left( -\frac{(m+1)(a+b \log(c\sqrt{x}))}{b} \right)^{-p} \Gamma\left(p+1, -\frac{2(m+1)(a+b \log(c\sqrt{x}))}{b}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m\*(a + b\*Log[c\*Sqrt[x]])^p, x]

[Out] ((d\*x)^(1+m)\*Gamma[1+p, (-2\*(1+m)\*(a+b\*Log[c\*Sqrt[x]]))]/b)\*(a+b\*Log[c\*Sqrt[x]])^p/(2^p\*d\*E^((2\*a\*(1+m))/b)\*(1+m)\*(c\*Sqrt[x])^(2\*(1+m)))\*(-(((1+m)\*(a+b\*Log[c\*Sqrt[x]]))/b))^p

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^p\*(d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m+1)/(d\*n\*(c\*x^n)^(m+1/n)), Subst[Int[E^((m+1)\*x)/n]\*(a+b\*x)^p, x], x, Log[c\*x^n]], x /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2181

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m+1, (-((f\*g\*Log[F])/d)\*(c + d\*x))]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m]+1)\*(-((f\*g\*Log[F])/d)\*(c + d\*x)/d))^FracPart[m]], x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rubi steps

$$\begin{aligned} \int (dx)^m (a + b \log(c\sqrt{x}))^p dx &= \frac{\left(2 (c\sqrt{x})^{-2(1+m)} (dx)^{1+m}\right) \text{Subst}\left(\int e^{2(1+m)x} (a + bx)^p dx, x, \log(c\sqrt{x})\right)}{d} \\ &= \frac{2^{-p} e^{-\frac{2a(1+m)}{b}} (c\sqrt{x})^{-2(1+m)} (dx)^{1+m} \Gamma\left(1+p, -\frac{2(1+m)(a+b \log(c\sqrt{x}))}{b}\right) (a + b \log(c\sqrt{x}))^p}{d(1+m)} \end{aligned}$$

**Mathematica [A]** time = 0.187964, size = 103, normalized size = 0.96

$$\frac{2^{-p} e^{-\frac{2a(m+1)}{b}} (c\sqrt{x})^{-2m} (dx)^m (a + b \log(c\sqrt{x}))^p \left( -\frac{(m+1)(a+b \log(c\sqrt{x}))}{b} \right)^{-p} \Gamma\left(p+1, -\frac{2(m+1)(a+b \log(c\sqrt{x}))}{b}\right)}{c^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*Log[c*Sqrt[x]])^p,x]
```

```
[Out] ((d*x)^m*Gamma[1 + p, (-2*(1 + m)*(a + b*Log[c*Sqrt[x]]))/b]*(a + b*Log[c*Sqrt[x]])^p)/(2^p*c^2*E^((2*a*(1 + m))/b)*(1 + m)*(c*Sqrt[x])^(2*m)*(-(((1 + m)*(a + b*Log[c*Sqrt[x]]))/b)))^p)
```

**Maple [F]** time = 0.086, size = 0, normalized size = 0.

$$\int (dx)^m (a + b \ln(c\sqrt{x}))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*ln(c*x^(1/2)))^p,x)
```

```
[Out] int((d*x)^m*(a+b*ln(c*x^(1/2)))^p,x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (b \log(c\sqrt{x}) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*log(c*x^(1/2)))^p,x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m*(b*log(c*sqrt(x)) + a)^p, x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx)^m (b \log(c\sqrt{x}) + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*log(c*x^(1/2)))^p,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m*(b*log(c*sqrt(x)) + a)^p, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*ln(c*x**(1/2)))**p,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (b \log(c\sqrt{x}) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*log(c*x^(1/2)))^p,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*(b*log(c*sqrt(x)) + a)^p, x)
```

### 3.184 $\int x^2 (a + b \log(c\sqrt{x}))^p dx$

**Optimal.** Leaf size=80

$$\frac{2^{-p} 3^{-p-1} e^{-\frac{6a}{b}} (a + b \log(c\sqrt{x}))^p \left(-\frac{a+b \log(c\sqrt{x})}{b}\right)^{-p} \Gamma\left(p+1, -\frac{6(a+b \log(c\sqrt{x}))}{b}\right)}{c^6}$$

[Out] (3^(-1 - p)\*Gamma[1 + p, (-6\*(a + b\*Log[c\*Sqrt[x]]))]/b)\*(a + b\*Log[c\*Sqrt[x]])^p/(2^p\*c^6\*E^(((6\*a)/b)\*(-(a + b\*Log[c\*Sqrt[x]])/b))^p)

**Rubi [A]** time = 0.0620918, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2310, 2181}

$$\frac{2^{-p} 3^{-p-1} e^{-\frac{6a}{b}} (a + b \log(c\sqrt{x}))^p \left(-\frac{a+b \log(c\sqrt{x})}{b}\right)^{-p} \Gamma\left(p+1, -\frac{6(a+b \log(c\sqrt{x}))}{b}\right)}{c^6}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*Log[c\*Sqrt[x]])^p,x]

[Out] (3^(-1 - p)\*Gamma[1 + p, (-6\*(a + b\*Log[c\*Sqrt[x]]))]/b)\*(a + b\*Log[c\*Sqrt[x]])^p/(2^p\*c^6\*E^(((6\*a)/b)\*(-(a + b\*Log[c\*Sqrt[x]])/b))^p)

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^p\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2181

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))\*((c\_.) + (d\_.)\*(x\_)^(m\_)), x\_Symbol] :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])\*(c + d\*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rubi steps

$$\begin{aligned} \int x^2 (a + b \log(c\sqrt{x}))^p dx &= \frac{2 \text{Subst}\left(\int e^{6x} (a + bx)^p dx, x, \log(c\sqrt{x})\right)}{c^6} \\ &= \frac{2^{-p} 3^{-1-p} e^{-\frac{6a}{b}} \Gamma\left(1 + p, -\frac{6(a+b \log(c\sqrt{x}))}{b}\right) (a + b \log(c\sqrt{x}))^p \left(-\frac{a+b \log(c\sqrt{x})}{b}\right)^{-p}}{c^6} \end{aligned}$$

**Mathematica [A]** time = 0.0516996, size = 80, normalized size = 1.

$$\frac{2^{-p} 3^{-p-1} e^{-\frac{6a}{b}} (a + b \log(c\sqrt{x}))^p \left(-\frac{a+b \log(c\sqrt{x})}{b}\right)^{-p} \Gamma\left(p+1, -\frac{6(a+b \log(c\sqrt{x}))}{b}\right)}{c^6}$$



Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Log[c*Sqrt[x]])^p,x]
```

```
[Out] (3^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*Sqrt[x]]))/b]*(a + b*Log[c*Sqrt[x]])^p)/(2^p*c^6*E^((6*a)/b)*(-(a + b*Log[c*Sqrt[x]])/b))^p
```

**Maple [F]** time = 0.043, size = 0, normalized size = 0.

$$\int x^2 (a + b \ln(c\sqrt{x}))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*ln(c*x^(1/2))))^p,x)
```

```
[Out] int(x^2*(a+b*ln(c*x^(1/2))))^p,x)
```

**Maxima [A]** time = 1.28815, size = 65, normalized size = 0.81

$$\frac{2 (b \log(c\sqrt{x}) + a)^{p+1} e^{\left(-\frac{6a}{b}\right)} E_{-p}\left(-\frac{6(b \log(c\sqrt{x}) + a)}{b}\right)}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^(1/2))))^p,x, algorithm="maxima")
```

```
[Out] -2*(b*log(c*sqrt(x)) + a)^(p + 1)*e^(-6*a/b)*exp_integral_e(-p, -6*(b*log(c*sqrt(x)) + a)/b)/(b*c^6)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \log(c\sqrt{x}) + a\right)^p x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*log(c*x^(1/2))))^p,x, algorithm="fricas")
```

```
[Out] integral((b*log(c*sqrt(x)) + a)^p*x^2, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*ln(c*x**(1/2))))**p,x)
```

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \log(c\sqrt{x}) + a)^p x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*log(c\*x^(1/2)))^p,x, algorithm="giac")

[Out] integrate((b\*log(c\*sqrt(x)) + a)^p\*x^2, x)

### 3.185 $\int x \left( a + b \log(c\sqrt{x}) \right)^p dx$

**Optimal.** Leaf size=75

$$\frac{2^{-2p-1} e^{-\frac{4a}{b}} \left( a + b \log(c\sqrt{x}) \right)^p \left( -\frac{a+b \log(c\sqrt{x})}{b} \right)^{-p} \Gamma\left( p+1, -\frac{4(a+b \log(c\sqrt{x}))}{b} \right)}{c^4}$$

[Out]  $(2^{(-1 - 2*p)} * \Gamma[1 + p, (-4*(a + b*\text{Log}[c*\text{Sqrt}[x]])) / b]) * (a + b*\text{Log}[c*\text{Sqrt}[x]])^p / (c^4 * E^{((4*a)/b)} * (-((a + b*\text{Log}[c*\text{Sqrt}[x]])/b))^p)$

**Rubi [A]** time = 0.0548699, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2310, 2181}

$$\frac{2^{-2p-1} e^{-\frac{4a}{b}} \left( a + b \log(c\sqrt{x}) \right)^p \left( -\frac{a+b \log(c\sqrt{x})}{b} \right)^{-p} \Gamma\left( p+1, -\frac{4(a+b \log(c\sqrt{x}))}{b} \right)}{c^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*(a + b*\text{Log}[c*\text{Sqrt}[x]])^p, x]$

[Out]  $(2^{(-1 - 2*p)} * \Gamma[1 + p, (-4*(a + b*\text{Log}[c*\text{Sqrt}[x]])) / b]) * (a + b*\text{Log}[c*\text{Sqrt}[x]])^p / (c^4 * E^{((4*a)/b)} * (-((a + b*\text{Log}[c*\text{Sqrt}[x]])/b))^p)$

#### Rule 2310

$\text{Int}[(a_. + \text{Log}[(c_.)*(x_.)^{(n_.)}] * (b_.))^p * ((d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Dist}[(d*x)^{(m+1)} / (d*n*(c*x^n)^{(m+1)/n}), \text{Subst}[\text{Int}[E^{((m+1)*x/n)} * (a + b*x)^p, x], x, \text{Log}[c*x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p\}, x]$

#### Rule 2181

$\text{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_.))) * ((c_.) + (d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow -\text{Simp}[(F^{(g*(e - (c*f)/d))} * (c + d*x)^{\text{FracPart}[m]} * \Gamma[m+1, (-((f*g*\text{Log}[F])/d)) * (c + d*x)]) / (d * (-((f*g*\text{Log}[F])/d))^{\text{IntPart}[m] + 1} * (-((f*g*\text{Log}[F]) * (c + d*x))/d))^{\text{FracPart}[m]}], x] /; \text{FreeQ}\{F, c, d, e, f, g, m\}, x \&\& \text{IntegerQ}[m]$

#### Rubi steps

$$\begin{aligned} \int x \left( a + b \log(c\sqrt{x}) \right)^p dx &= \frac{2 \text{Subst} \left( \int e^{4x} (a + bx)^p dx, x, \log(c\sqrt{x}) \right)}{c^4} \\ &= \frac{2^{-1-2p} e^{-\frac{4a}{b}} \Gamma \left( 1 + p, -\frac{4(a+b \log(c\sqrt{x}))}{b} \right) \left( a + b \log(c\sqrt{x}) \right)^p \left( -\frac{a+b \log(c\sqrt{x})}{b} \right)^{-p}}{c^4} \end{aligned}$$

**Mathematica [A]** time = 0.0445954, size = 75, normalized size = 1.

$$\frac{2^{-2p-1} e^{-\frac{4a}{b}} \left( a + b \log(c\sqrt{x}) \right)^p \left( -\frac{a+b \log(c\sqrt{x})}{b} \right)^{-p} \Gamma\left( p+1, -\frac{4(a+b \log(c\sqrt{x}))}{b} \right)}{c^4}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*Log[c\*Sqrt[x]])^p,x]

[Out] (2^(-1 - 2\*p)\*Gamma[1 + p, (-4\*(a + b\*Log[c\*Sqrt[x]]))/b]\*(a + b\*Log[c\*Sqrt[x]])^p)/(c^4\*E^((4\*a)/b)\*(-(a + b\*Log[c\*Sqrt[x]])/b))^p

**Maple [F]** time = 0.043, size = 0, normalized size = 0.

$$\int x (a + b \ln(c\sqrt{x}))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*ln(c\*x^(1/2))))^p,x)

[Out] int(x\*(a+b\*ln(c\*x^(1/2))))^p,x)

**Maxima [A]** time = 1.33478, size = 65, normalized size = 0.87

$$\frac{2 (b \log(c\sqrt{x}) + a)^{p+1} e^{\left(-\frac{4a}{b}\right)} E_{-p} \left(-\frac{4 (b \log(c\sqrt{x}) + a)}{b}\right)}{bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*log(c\*x^(1/2))))^p,x, algorithm="maxima")

[Out] -2\*(b\*log(c\*sqrt(x)) + a)^(p + 1)\*e^(-4\*a/b)\*exp\_integral\_e(-p, -4\*(b\*log(c\*sqrt(x)) + a)/b)/(b\*c^4)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \log(c\sqrt{x}) + a\right)^p x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*log(c\*x^(1/2))))^p,x, algorithm="fricas")

[Out] integral((b\*log(c\*sqrt(x)) + a)^p\*x, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x (a + b \log(c\sqrt{x}))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*ln(c\*x\*\*(1/2))))\*\*p,x)

[Out] Integral(x\*(a + b\*log(c\*sqrt(x)))\*\*p, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \log(c\sqrt{x}) + a)^p x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*log(c\*x^(1/2)))^p,x, algorithm="giac")

[Out] integrate((b\*log(c\*sqrt(x)) + a)^p\*x, x)

### 3.186 $\int (a + b \log(c\sqrt{x}))^p dx$

**Optimal.** Leaf size=73

$$\frac{2^{-p} e^{-\frac{2a}{b}} (a + b \log(c\sqrt{x}))^p \left(-\frac{a+b \log(c\sqrt{x})}{b}\right)^{-p} \Gamma\left(p+1, -\frac{2(a+b \log(c\sqrt{x}))}{b}\right)}{c^2}$$

[Out] (Gamma[1 + p, (-2\*(a + b\*Log[c\*Sqrt[x]]))/b]\*(a + b\*Log[c\*Sqrt[x]])^p)/(2^p \* c^2 \* E^((2\*a)/b)\*(-((a + b\*Log[c\*Sqrt[x]])/b))^p)

**Rubi [A]** time = 0.0351927, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2299, 2181}

$$\frac{2^{-p} e^{-\frac{2a}{b}} (a + b \log(c\sqrt{x}))^p \left(-\frac{a+b \log(c\sqrt{x})}{b}\right)^{-p} \Gamma\left(p+1, -\frac{2(a+b \log(c\sqrt{x}))}{b}\right)}{c^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*Sqrt[x]])^p, x]

[Out] (Gamma[1 + p, (-2\*(a + b\*Log[c\*Sqrt[x]]))/b]\*(a + b\*Log[c\*Sqrt[x]])^p)/(2^p \* c^2 \* E^((2\*a)/b)\*(-((a + b\*Log[c\*Sqrt[x]])/b))^p)

#### Rule 2299

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^p, x\_Symbol] :> Dist[1/(n\*c^(1/n)), Subst[Int[E^(x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[1/n]

#### Rule 2181

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))\*\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])\*(c + d\*x)/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rubi steps

$$\begin{aligned} \int (a + b \log(c\sqrt{x}))^p dx &= \frac{2 \text{Subst}\left(\int e^{2x} (a + bx)^p dx, x, \log(c\sqrt{x})\right)}{c^2} \\ &= \frac{2^{-p} e^{-\frac{2a}{b}} \Gamma\left(1 + p, -\frac{2(a+b \log(c\sqrt{x}))}{b}\right) (a + b \log(c\sqrt{x}))^p \left(-\frac{a+b \log(c\sqrt{x})}{b}\right)^{-p}}{c^2} \end{aligned}$$

**Mathematica [A]** time = 0.0260003, size = 73, normalized size = 1.

$$\frac{2^{-p} e^{-\frac{2a}{b}} (a + b \log(c\sqrt{x}))^p \left(-\frac{a+b \log(c\sqrt{x})}{b}\right)^{-p} \Gamma\left(p+1, -\frac{2(a+b \log(c\sqrt{x}))}{b}\right)}{c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*Sqrt[x]])^p, x]

[Out] (Gamma[1 + p, (-2\*(a + b\*Log[c\*Sqrt[x]]))/b]\*(a + b\*Log[c\*Sqrt[x]])^p)/(2^p \* c^2 \* E^((2\*a)/b) \* (-((a + b\*Log[c\*Sqrt[x]])/b))^p)

**Maple [F]** time = 0.044, size = 0, normalized size = 0.

$$\int (a + b \ln(c\sqrt{x}))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^(1/2)))^p, x)

[Out] int((a+b\*ln(c\*x^(1/2)))^p, x)

**Maxima [A]** time = 1.24685, size = 65, normalized size = 0.89

$$\frac{2(b \log(c\sqrt{x}) + a)^{p+1} e^{\left(-\frac{2a}{b}\right)} E_{-p}\left(-\frac{2(b \log(c\sqrt{x}) + a)}{b}\right)}{bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^(1/2)))^p, x, algorithm="maxima")

[Out] -2\*(b\*log(c\*sqrt(x)) + a)^(p + 1)\*e^(-2\*a/b)\*exp\_integral\_e(-p, -2\*(b\*log(c\*sqrt(x)) + a)/b)/(b\*c^2)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \log(c\sqrt{x}) + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^(1/2)))^p, x, algorithm="fricas")

[Out] integral((b\*log(c\*sqrt(x)) + a)^p, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \log(c\sqrt{x}))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*(1/2)))\*\*p, x)

```
[Out] Integral((a + b*log(c*sqrt(x)))**p, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \log(c\sqrt{x}) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^(1/2)))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*sqrt(x)) + a)^p, x)
```



$$3.187 \quad \int \frac{(a+b \log(c\sqrt{x}))^p}{x} dx$$

**Optimal.** Leaf size=26

$$\frac{2(a+b \log(c\sqrt{x}))^{p+1}}{b(p+1)}$$

[Out] (2\*(a + b\*Log[c\*Sqrt[x]])^(1 + p))/(b\*(1 + p))

**Rubi [A]** time = 0.0284044, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2302, 30}

$$\frac{2(a+b \log(c\sqrt{x}))^{p+1}}{b(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*Sqrt[x]])^p/x, x]

[Out] (2\*(a + b\*Log[c\*Sqrt[x]])^(1 + p))/(b\*(1 + p))

#### Rule 2302

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] := Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{(a+b \log(c\sqrt{x}))^p}{x} dx &= \frac{2 \text{Subst}\left(\int x^p dx, x, a + b \log(c\sqrt{x})\right)}{b} \\ &= \frac{2(a+b \log(c\sqrt{x}))^{1+p}}{b(1+p)} \end{aligned}$$

**Mathematica [A]** time = 0.0066608, size = 26, normalized size = 1.

$$\frac{2(a+b \log(c\sqrt{x}))^{p+1}}{b(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*Sqrt[x]])^p/x, x]

[Out] (2\*(a + b\*Log[c\*Sqrt[x]])^(1 + p))/(b\*(1 + p))

---

**Maple [A]** time = 0.04, size = 25, normalized size = 1.

$$2 \frac{(a + b \ln(c\sqrt{x}))^{1+p}}{b(1+p)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^(1/2)))^p/x,x)

[Out] 2\*(a+b\*ln(c\*x^(1/2)))^(1+p)/b/(1+p)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^(1/2)))^p/x,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 1.04289, size = 82, normalized size = 3.15

$$\frac{2(b \log(c\sqrt{x}) + a)(b \log(c\sqrt{x}) + a)^p}{bp + b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^(1/2)))^p/x,x, algorithm="fricas")

[Out] 2\*(b\*log(c\*sqrt(x)) + a)\*(b\*log(c\*sqrt(x)) + a)^p/(b\*p + b)

---

**Sympy [A]** time = 16.2973, size = 48, normalized size = 1.85

$$-\begin{cases} -a^p \log(x) & \text{for } b = 0 \\ 2 \begin{cases} \frac{(a+b \log(c\sqrt{x}))^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(a + b \log(c\sqrt{x})) & \text{otherwise} \end{cases} & \text{otherwise} \\ -\frac{\log(a + b \log(c\sqrt{x}))}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*(1/2)))\*\*p/x,x)

[Out] -Piecewise((-a\*\*p\*log(x), Eq(b, 0)), (-2\*Piecewise(((a + b\*log(c\*sqrt(x))))\*(p + 1)/(p + 1), Ne(p, -1)), (log(a + b\*log(c\*sqrt(x))), True))/b, True))

---

**Giac [A]** time = 1.26272, size = 34, normalized size = 1.31

$$\frac{2 \left( b \log(c) + \frac{1}{2} b \log(x) + a \right)^{p+1}}{b(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^(1/2)))^p/x,x, algorithm="giac")

[Out] 2\*(b\*log(c) + 1/2\*b\*log(x) + a)^(p + 1)/(b\*(p + 1))

$$3.188 \quad \int \frac{(a+b \log(c\sqrt{x}))^p}{x^2} dx$$

**Optimal.** Leaf size=73

$$c^2 (-2^{-p}) e^{\frac{2a}{b}} (a+b \log(c\sqrt{x}))^p \left( \frac{a+b \log(c\sqrt{x})}{b} \right)^{-p} \text{Gamma} \left( p+1, \frac{2(a+b \log(c\sqrt{x}))}{b} \right)$$

[Out]  $-\left(\left(c^2 E^{\left(\frac{2a}{b}\right)} \text{Gamma}\left[1+p, \frac{2(a+b \text{Log}[c \text{Sqrt}[x]])}{b}\right]\right)/b\right) (a+b \text{Log}[c \text{Sqrt}[x]])^p / \left(2^p \left(\frac{a+b \text{Log}[c \text{Sqrt}[x]]}{b}\right)^p\right)$

**Rubi [A]** time = 0.0521968, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2310, 2181}

$$c^2 (-2^{-p}) e^{\frac{2a}{b}} (a+b \log(c\sqrt{x}))^p \left( \frac{a+b \log(c\sqrt{x})}{b} \right)^{-p} \text{Gamma} \left( p+1, \frac{2(a+b \log(c\sqrt{x}))}{b} \right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*Sqrt[x]])^p/x^2,x]

[Out]  $-\left(\left(c^2 E^{\left(\frac{2a}{b}\right)} \text{Gamma}\left[1+p, \frac{2(a+b \text{Log}[c \text{Sqrt}[x]])}{b}\right]\right)/b\right) (a+b \text{Log}[c \text{Sqrt}[x]])^p / \left(2^p \left(\frac{a+b \text{Log}[c \text{Sqrt}[x]]}{b}\right)^p\right)$

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m+1)/(d\*n\*(c\*x^n)^(m+1/n)), Subst[Int[E^((m+1)\*x/n)\*(a+b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2181

Int[(F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_)^(m\_), x\_Symbol] :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m+1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m]+1)\*(-((f\*g\*Log[F])\*(c + d\*x))/d)^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rubi steps

$$\begin{aligned} \int \frac{(a+b \log(c\sqrt{x}))^p}{x^2} dx &= (2c^2) \text{Subst} \left( \int e^{-2x} (a+bx)^p dx, x, \log(c\sqrt{x}) \right) \\ &= -2^{-p} c^2 e^{\frac{2a}{b}} \Gamma \left( 1+p, \frac{2(a+b \log(c\sqrt{x}))}{b} \right) (a+b \log(c\sqrt{x}))^p \left( \frac{a+b \log(c\sqrt{x})}{b} \right)^{-p} \end{aligned}$$

**Mathematica [A]** time = 0.0481133, size = 73, normalized size = 1.

$$c^2 (-2^{-p}) e^{\frac{2a}{b}} (a+b \log(c\sqrt{x}))^p \left( \frac{a+b \log(c\sqrt{x})}{b} \right)^{-p} \text{Gamma} \left( p+1, \frac{2(a+b \log(c\sqrt{x}))}{b} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*Sqrt[x]])^p/x^2,x]

[Out]  $-\left(\frac{c^2 E^{\left(\frac{2a}{b}\right)} \Gamma\left[1 + p, \left(\frac{2(a + b \operatorname{Log}[c \sqrt{x}])}{b}\right)\right]}{b}\right) \cdot (a + b \operatorname{Log}[c \sqrt{x}])^p / \left(2^p \left(\frac{a + b \operatorname{Log}[c \sqrt{x}]}{b}\right)^p\right)$

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (a + b \ln(c\sqrt{x}))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^(1/2)))^p/x^2,x)

[Out] int((a+b\*ln(c\*x^(1/2)))^p/x^2,x)

**Maxima [A]** time = 1.35093, size = 65, normalized size = 0.89

$$\frac{2 \left( b \log(c\sqrt{x}) + a \right)^{p+1} c^2 e^{\left(\frac{2a}{b}\right)} E_{-p} \left( \frac{2 \left( b \log(c\sqrt{x}) + a \right)}{b} \right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^(1/2)))^p/x^2,x, algorithm="maxima")

[Out]  $-2 \cdot (b \cdot \log(c \cdot \sqrt{x}) + a)^{p+1} \cdot c^2 \cdot e^{\left(\frac{2a}{b}\right)} \cdot \exp\_integral\_e(-p, \frac{2 \cdot (b \cdot \log(c \cdot \sqrt{x}) + a)}{b}) / b$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{(b \log(c\sqrt{x}) + a)^p}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^(1/2)))^p/x^2,x, algorithm="fricas")

[Out] integral((b\*log(c\*sqrt(x)) + a)^p/x^2, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \log(c\sqrt{x}))^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*(1/2)))\*\*p/x\*\*2,x)

```
[Out] Integral((a + b*log(c*sqrt(x)))**p/x**2, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \log(c\sqrt{x}) + a)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(c*x^(1/2)))^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*sqrt(x)) + a)^p/x^2, x)
```

$$3.189 \quad \int \frac{(a+b \log(c\sqrt{x}))^p}{x^3} dx$$

**Optimal.** Leaf size=75

$$c^4 (-2^{-2p-1}) e^{\frac{4a}{b}} (a+b \log(c\sqrt{x}))^p \left( \frac{a+b \log(c\sqrt{x})}{b} \right)^{-p} \text{Gamma} \left( p+1, \frac{4(a+b \log(c\sqrt{x}))}{b} \right)$$

[Out] -((2^(-1 - 2\*p))\*c^4\*E^((4\*a)/b)\*Gamma[1 + p, (4\*(a + b\*Log[c\*Sqrt[x]]))/b]\*(a + b\*Log[c\*Sqrt[x]])^p)/((a + b\*Log[c\*Sqrt[x]])/b)^p

**Rubi [A]** time = 0.0583778, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2310, 2181}

$$c^4 (-2^{-2p-1}) e^{\frac{4a}{b}} (a+b \log(c\sqrt{x}))^p \left( \frac{a+b \log(c\sqrt{x})}{b} \right)^{-p} \text{Gamma} \left( p+1, \frac{4(a+b \log(c\sqrt{x}))}{b} \right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*Sqrt[x]])^p/x^3,x]

[Out] -((2^(-1 - 2\*p))\*c^4\*E^((4\*a)/b)\*Gamma[1 + p, (4\*(a + b\*Log[c\*Sqrt[x]]))/b]\*(a + b\*Log[c\*Sqrt[x]])^p)/((a + b\*Log[c\*Sqrt[x]])/b)^p

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x)/n]\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2181

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x)])/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])\*(c + d\*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rubi steps

$$\begin{aligned} \int \frac{(a+b \log(c\sqrt{x}))^p}{x^3} dx &= (2c^4) \text{Subst} \left( \int e^{-4x} (a+bx)^p dx, x, \log(c\sqrt{x}) \right) \\ &= -2^{-1-2p} c^4 e^{\frac{4a}{b}} \Gamma \left( 1+p, \frac{4(a+b \log(c\sqrt{x}))}{b} \right) (a+b \log(c\sqrt{x}))^p \left( \frac{a+b \log(c\sqrt{x})}{b} \right)^{-p} \end{aligned}$$

**Mathematica [A]** time = 0.0449017, size = 75, normalized size = 1.

$$c^4 (-2^{-2p-1}) e^{\frac{4a}{b}} (a+b \log(c\sqrt{x}))^p \left( \frac{a+b \log(c\sqrt{x})}{b} \right)^{-p} \text{Gamma} \left( p+1, \frac{4(a+b \log(c\sqrt{x}))}{b} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*Sqrt[x]])^p/x^3,x]

[Out] -((2^(-1 - 2\*p)\*c^4\*E^((4\*a)/b)\*Gamma[1 + p, (4\*(a + b\*Log[c\*Sqrt[x]])/b)\*(a + b\*Log[c\*Sqrt[x]])^p]/((a + b\*Log[c\*Sqrt[x]])/b)^p)

**Maple [F]** time = 0.041, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} (a + b \ln(c\sqrt{x}))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^(1/2)))^p/x^3,x)

[Out] int((a+b\*ln(c\*x^(1/2)))^p/x^3,x)

**Maxima [A]** time = 1.24732, size = 65, normalized size = 0.87

$$\frac{2(b \log(c\sqrt{x}) + a)^{p+1} c^4 e^{\left(\frac{4a}{b}\right)} E_{-p}\left(\frac{4(b \log(c\sqrt{x}) + a)}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^(1/2)))^p/x^3,x, algorithm="maxima")

[Out] -2\*(b\*log(c\*sqrt(x)) + a)^(p + 1)\*c^4\*e^(4\*a/b)\*exp\_integral\_e(-p, 4\*(b\*log(c\*sqrt(x)) + a)/b)/b

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \log(c\sqrt{x}) + a)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^(1/2)))^p/x^3,x, algorithm="fricas")

[Out] integral((b\*log(c\*sqrt(x)) + a)^p/x^3, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*(1/2)))\*\*p/x\*\*3,x)



[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \log(c\sqrt{x}) + a)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^(1/2)))^p/x^3,x, algorithm="giac")

[Out] integrate((b\*log(c\*sqrt(x)) + a)^p/x^3, x)

$$3.190 \quad \int \frac{(a+b \log(c\sqrt{x}))^p}{x^4} dx$$

**Optimal.** Leaf size=80

$$c^6 (-2^{-p}) 3^{-p-1} e^{\frac{6a}{b}} (a+b \log(c\sqrt{x}))^p \left( \frac{a+b \log(c\sqrt{x})}{b} \right)^{-p} \text{Gamma} \left( p+1, \frac{6(a+b \log(c\sqrt{x}))}{b} \right)$$

[Out]  $-\left(\left(3^{-1-p}\right) c^6 E^{\left(\frac{6a}{b}\right)} \text{Gamma}\left[1+p, \frac{6(a+b \text{Log}[c \text{Sqrt}[x]])}{b}\right]\right) / b * (a+b \text{Log}[c \text{Sqrt}[x]])^p / \left(2^p * \left(\frac{a+b \text{Log}[c \text{Sqrt}[x]]}{b}\right)^p\right)$

**Rubi [A]** time = 0.0583173, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2310, 2181}

$$c^6 (-2^{-p}) 3^{-p-1} e^{\frac{6a}{b}} (a+b \log(c\sqrt{x}))^p \left( \frac{a+b \log(c\sqrt{x})}{b} \right)^{-p} \text{Gamma} \left( p+1, \frac{6(a+b \log(c\sqrt{x}))}{b} \right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Log[c\*Sqrt[x]])^p/x^4, x]

[Out]  $-\left(\left(3^{-1-p}\right) c^6 E^{\left(\frac{6a}{b}\right)} \text{Gamma}\left[1+p, \frac{6(a+b \text{Log}[c \text{Sqrt}[x]])}{b}\right]\right) / b * (a+b \text{Log}[c \text{Sqrt}[x]])^p / \left(2^p * \left(\frac{a+b \text{Log}[c \text{Sqrt}[x]]}{b}\right)^p\right)$

#### Rule 2310

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[(d\*x)^(m+1)/(d\*n\*(c\*x^n)^((m+1)/n)), Subst[Int[E^(((m+1)\*x)/n)\*(a+b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2181

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))\*((c\_.) + (d\_.)\*(x\_)^(m\_)), x\_Symbol] :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m+1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m]+1)\*(-((f\*g\*Log[F])\*(c + d\*x))/d)^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rubi steps

$$\begin{aligned} \int \frac{(a+b \log(c\sqrt{x}))^p}{x^4} dx &= (2c^6) \text{Subst} \left( \int e^{-6x} (a+bx)^p dx, x, \log(c\sqrt{x}) \right) \\ &= -2^{-p} 3^{-1-p} c^6 e^{\frac{6a}{b}} \Gamma \left( 1+p, \frac{6(a+b \log(c\sqrt{x}))}{b} \right) (a+b \log(c\sqrt{x}))^p \left( \frac{a+b \log(c\sqrt{x})}{b} \right)^{-p} \end{aligned}$$

**Mathematica [A]** time = 0.0461448, size = 80, normalized size = 1.

$$c^6 (-2^{-p}) 3^{-p-1} e^{\frac{6a}{b}} (a+b \log(c\sqrt{x}))^p \left( \frac{a+b \log(c\sqrt{x})}{b} \right)^{-p} \text{Gamma} \left( p+1, \frac{6(a+b \log(c\sqrt{x}))}{b} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Log[c\*Sqrt[x]])^p/x^4,x]

[Out]  $-\left(\frac{3^{-1-p}c^6E^{\left(\frac{6a}{b}\right)}\Gamma\left[1+p,\left(\frac{6(a+b\text{Log}[c\text{Sqrt}[x]])}{b}\right)\right]}{2^p}\right)^p$

**Maple [F]** time = 0.04, size = 0, normalized size = 0.

$$\int \frac{1}{x^4} (a + b \ln(c\sqrt{x}))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*ln(c\*x^(1/2)))^p/x^4,x)

[Out] int((a+b\*ln(c\*x^(1/2)))^p/x^4,x)

**Maxima [A]** time = 1.34096, size = 65, normalized size = 0.81

$$\frac{2(b \log(c\sqrt{x}) + a)^{p+1} c^6 e^{\left(\frac{6a}{b}\right)} E_{-p}\left(\frac{6(b \log(c\sqrt{x}) + a)}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^(1/2)))^p/x^4,x, algorithm="maxima")

[Out]  $-2*(b*\log(c*\text{sqrt}(x)) + a)^{(p + 1)}*c^6*e^{(6*a/b)}*\text{exp\_integral\_e}(-p, 6*(b*\log(c*\text{sqrt}(x)) + a)/b)/b$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \log(c\sqrt{x}) + a)^p}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^(1/2)))^p/x^4,x, algorithm="fricas")

[Out] integral((b\*log(c\*sqrt(x)) + a)^p/x^4, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*ln(c\*x\*\*(1/2)))\*\*p/x\*\*4,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \log(c\sqrt{x}) + a)^p}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*log(c\*x^(1/2)))^p/x^4,x, algorithm="giac")

[Out] integrate((b\*log(c\*sqrt(x)) + a)^p/x^4, x)

### 3.191 $\int x^{-1+n} (a + b \log(cx^n))^p dx$

**Optimal.** Leaf size=65

$$\frac{e^{-\frac{a}{b}} (a + b \log(cx^n))^p \left(-\frac{a+b \log(cx^n)}{b}\right)^{-p} \Gamma\left(p+1, -\frac{a+b \log(cx^n)}{b}\right)}{cn}$$

[Out]  $(\Gamma[1 + p, -((a + b \cdot \text{Log}[c \cdot x^n])/b)]) \cdot (a + b \cdot \text{Log}[c \cdot x^n])^p / (c \cdot E^{(a/b)} \cdot n \cdot -((a + b \cdot \text{Log}[c \cdot x^n])/b))^p$

**Rubi [A]** time = 0.0512487, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2310, 2181}

$$\frac{e^{-\frac{a}{b}} (a + b \log(cx^n))^p \left(-\frac{a+b \log(cx^n)}{b}\right)^{-p} \Gamma\left(p+1, -\frac{a+b \log(cx^n)}{b}\right)}{cn}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{(-1 + n)} \cdot (a + b \cdot \text{Log}[c \cdot x^n])^p, x]$

[Out]  $(\Gamma[1 + p, -((a + b \cdot \text{Log}[c \cdot x^n])/b)]) \cdot (a + b \cdot \text{Log}[c \cdot x^n])^p / (c \cdot E^{(a/b)} \cdot n \cdot -((a + b \cdot \text{Log}[c \cdot x^n])/b))^p$

#### Rule 2310

$\text{Int}[(a \cdot) + \text{Log}[(c \cdot)(x \cdot)^{(n \cdot)}] \cdot (b \cdot)]^{(p \cdot)} \cdot ((d \cdot)(x \cdot))^{(m \cdot)}, x\_Symbol]$   
 $\rightarrow \text{Dist}[(d \cdot x)^{(m + 1)} / (d \cdot n \cdot (c \cdot x^n)^{(m + 1)/n}), \text{Subst}[\text{Int}[E^{((m + 1) \cdot x)/n} \cdot (a + b \cdot x)^p, x], x, \text{Log}[c \cdot x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p\}, x]$

#### Rule 2181

$\text{Int}[(F \cdot)^{((g \cdot) \cdot (e \cdot) + (f \cdot)(x \cdot))} \cdot ((c \cdot) + (d \cdot)(x \cdot))^{(m \cdot)}, x\_Symbol]$   
 $\rightarrow -\text{Simp}[(F \cdot)^{(g \cdot (e - (c \cdot f)/d))} \cdot (c + d \cdot x)^{\text{FracPart}[m]} \cdot \Gamma[m + 1, -((f \cdot g \cdot \text{Log}[F])/d) \cdot (c + d \cdot x)] / (d \cdot (-((f \cdot g \cdot \text{Log}[F])/d))^{\text{IntPart}[m] + 1} \cdot (-((f \cdot g \cdot \text{Log}[F]) \cdot (c + d \cdot x))/d))^{\text{FracPart}[m]}], x] /; \text{FreeQ}\{F, c, d, e, f, g, m\}, x] \&\& \text{IntegerQ}[m]$

#### Rubi steps

$$\int x^{-1+n} (a + b \log(cx^n))^p dx = \frac{\text{Subst}\left(\int e^x (a + bx)^p dx, x, \log(cx^n)\right)}{cn}$$

$$= \frac{e^{-\frac{a}{b}} \Gamma\left(1 + p, -\frac{a+b \log(cx^n)}{b}\right) (a + b \log(cx^n))^p \left(-\frac{a+b \log(cx^n)}{b}\right)^{-p}}{cn}$$

**Mathematica [A]** time = 0.0236361, size = 65, normalized size = 1.

$$\frac{e^{-\frac{a}{b}} (a + b \log(cx^n))^p \left(-\frac{a+b \log(cx^n)}{b}\right)^{-p} \Gamma\left(p+1, -\frac{a+b \log(cx^n)}{b}\right)}{cn}$$

Antiderivative was successfully verified.

[In] Integrate[x<sup>(-1 + n)</sup>\*(a + b\*Log[c\*x<sup>n</sup>])<sup>p</sup>,x]

[Out] (Gamma[1 + p, -((a + b\*Log[c\*x<sup>n</sup>])/b)]\*(a + b\*Log[c\*x<sup>n</sup>])<sup>p</sup>/(c\*E<sup>(a/b)</sup>\*n\*(-((a + b\*Log[c\*x<sup>n</sup>])/b))<sup>p</sup>)

**Maple [F]** time = 1.768, size = 0, normalized size = 0.

$$\int x^{-1+n} (a + b \ln(cx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x<sup>(-1+n)</sup>\*(a+b\*ln(c\*x<sup>n</sup>))<sup>p</sup>,x)

[Out] int(x<sup>(-1+n)</sup>\*(a+b\*ln(c\*x<sup>n</sup>))<sup>p</sup>,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-1+n)</sup>\*(a+b\*log(c\*x<sup>n</sup>))<sup>p</sup>,x, algorithm="maxima")

[Out] Exception raised: RuntimeError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((b \log(cx^n) + a)^p x^{n-1}, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-1+n)</sup>\*(a+b\*log(c\*x<sup>n</sup>))<sup>p</sup>,x, algorithm="fricas")

[Out] integral((b\*log(c\*x<sup>n</sup>) + a)<sup>p</sup>\*x<sup>(n - 1)</sup>, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-1+n)</sup>\*(a+b\*ln(c\*x<sup>n</sup>))<sup>p</sup>,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \log(cx^n) + a)^p x^{n-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(a+b*log(c*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((b*log(c*x^n) + a)^p*x^(n - 1), x)
```

### 3.192 $\int (dx^q)^m (a + b \log(cx^n))^p dx$

**Optimal.** Leaf size=114

$$\frac{x(dx^q)^m e^{-\frac{amq+a}{bn}} (cx^n)^{-\frac{mq+1}{n}} (a + b \log(cx^n))^p \left(-\frac{(mq+1)(a+b \log(cx^n))}{bn}\right)^{-p} \text{Gamma}\left(p + 1, -\frac{(mq+1)(a+b \log(cx^n))}{bn}\right)}{mq + 1}$$

[Out] (x\*(d\*x^q)^m\*Gamma[1 + p, -(((1 + m\*q)\*(a + b\*Log[c\*x^n]))/(b\*n))]\*(a + b\*Log[c\*x^n])^p)/(E^((a + a\*m\*q)/(b\*n))\*(1 + m\*q)\*(c\*x^n)^((1 + m\*q)/n)\*(-(((1 + m\*q)\*(a + b\*Log[c\*x^n]))/(b\*n))))^p)

**Rubi [A]** time = 0.107144, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$ , Rules used = {15, 2310, 2181}

$$\frac{x(dx^q)^m e^{-\frac{amq+a}{bn}} (cx^n)^{-\frac{mq+1}{n}} (a + b \log(cx^n))^p \left(-\frac{(mq+1)(a+b \log(cx^n))}{bn}\right)^{-p} \text{Gamma}\left(p + 1, -\frac{(mq+1)(a+b \log(cx^n))}{bn}\right)}{mq + 1}$$

Antiderivative was successfully verified.

[In] Int[(d\*x^q)^m\*(a + b\*Log[c\*x^n])^p,x]

[Out] (x\*(d\*x^q)^m\*Gamma[1 + p, -(((1 + m\*q)\*(a + b\*Log[c\*x^n]))/(b\*n))]\*(a + b\*Log[c\*x^n])^p)/(E^((a + a\*m\*q)/(b\*n))\*(1 + m\*q)\*(c\*x^n)^((1 + m\*q)/n)\*(-(((1 + m\*q)\*(a + b\*Log[c\*x^n]))/(b\*n))))^p)

#### Rule 15

Int[(u\_)\*((a\_)\*(x\_)^(n\_))^(m\_), x\_Symbol] := Dist[(a^IntPart[m]\*(a\*x^n)^FracPart[m])/x^(n\*FracPart[m]), Int[u\*x^(m\*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

#### Rule 2310

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_)^(m\_)), x\_Symbol] := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)\*x)/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

#### Rule 2181

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F]\*(c + d\*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rubi steps



$$\begin{aligned} \int (dx^q)^m (a + b \log(cx^n))^p dx &= (x^{-mq} (dx^q)^m) \int x^{mq} (a + b \log(cx^n))^p dx \\ &= \frac{\left(x (cx^n)^{-\frac{1+mq}{n}} (dx^q)^m\right) \text{Subst}\left(\int e^{\frac{(1+mq)x}{n}} (a + bx)^p dx, x, \log(cx^n)\right)}{n} \\ &= \frac{e^{-\frac{a+amq}{bn}} x (cx^n)^{-\frac{1+mq}{n}} (dx^q)^m \Gamma\left(1 + p, -\frac{(1+mq)(a+b \log(cx^n))}{bn}\right) (a + b \log(cx^n))^p \left(-\frac{(1+mq)(a+b \log(cx^n))}{bn}\right)^{-p}}{1 + mq} \end{aligned}$$

**Mathematica [A]** time = 0.151183, size = 118, normalized size = 1.04

$$\frac{x^{-mq} (dx^q)^m (a + b \log(cx^n))^p \exp\left(-\frac{(mq+1)(a+b \log(cx^n))-bn \log(x)}{bn}\right) \left(-\frac{(mq+1)(a+b \log(cx^n))}{bn}\right)^{-p} \text{Gamma}\left(p + 1, -\frac{(mq+1)(a+b \log(cx^n))}{bn}\right)}{mq + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x^q)^m\*(a + b\*Log[c\*x^n])^p,x]

[Out] ((d\*x^q)^m\*Gamma[1 + p, -(((1 + m\*q)\*(a + b\*Log[c\*x^n]))/(b\*n))]\*(a + b\*Log[c\*x^n])^p)/(E^(((1 + m\*q)\*(a - b\*n\*Log[x] + b\*Log[c\*x^n]))/(b\*n))\*(1 + m\*q)\*x^(m\*q)\*(-(((1 + m\*q)\*(a + b\*Log[c\*x^n]))/(b\*n))))^p)

**Maple [F]** time = 1.508, size = 0, normalized size = 0.

$$\int (dx^q)^m (a + b \ln(cx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^q)^m\*(a+b\*ln(c\*x^n))^p,x)

[Out] int((d\*x^q)^m\*(a+b\*ln(c\*x^n))^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^q)^m (b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^q)^m\*(a+b\*log(c\*x^n))^p,x, algorithm="maxima")

[Out] integrate((d\*x^q)^m\*(b\*log(c\*x^n) + a)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx^q)^m (b \log(cx^n) + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^q)^m*(a+b*log(c*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((d*x^q)^m*(b*log(c*x^n) + a)^p, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**q)**m*(a+b*ln(c*x**n))**p,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx^q)^m (b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^q)^m*(a+b*log(c*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((d*x^q)^m*(b*log(c*x^n) + a)^p, x)
```

$$3.193 \quad \int (d_1 x^{q_1})^{m_1} (d_2 x^{q_2})^{m_2} (a + b \log(cx^n))^p dx$$

**Optimal.** Leaf size=136

$$\frac{x (d_1 x^{q_1})^{m_1} (d_2 x^{q_2})^{m_2} (a + b \log(cx^n))^p e^{-\frac{a(m_1 q_1 + m_2 q_2 + 1)}{bn}} (cx^n)^{-\frac{m_1 q_1 + m_2 q_2 + 1}{n}} \left( -\frac{(m_1 q_1 + m_2 q_2 + 1)(a + b \log(cx^n))}{bn} \right)^{-p} \Gamma(p)}{m_1 q_1 + m_2 q_2 + 1}$$

[Out] (x\*(d1\*x^q1)^m1\*(d2\*x^q2)^m2\*Gamma[1 + p, -(((1 + m1\*q1 + m2\*q2)\*(a + b\*Log[c\*x^n]))/(b\*n))]\*(a + b\*Log[c\*x^n])^p)/(E^((a\*(1 + m1\*q1 + m2\*q2))/(b\*n))\*(1 + m1\*q1 + m2\*q2)\*(c\*x^n)^((1 + m1\*q1 + m2\*q2)/n)\*(-(((1 + m1\*q1 + m2\*q2)/(b\*n))\*(a + b\*Log[c\*x^n]))/(b\*n))))^p)

**Rubi [A]** time = 0.16834, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {15, 2310, 2181}

$$\frac{x (d_1 x^{q_1})^{m_1} (d_2 x^{q_2})^{m_2} (a + b \log(cx^n))^p e^{-\frac{a(m_1 q_1 + m_2 q_2 + 1)}{bn}} (cx^n)^{-\frac{m_1 q_1 + m_2 q_2 + 1}{n}} \left( -\frac{(m_1 q_1 + m_2 q_2 + 1)(a + b \log(cx^n))}{bn} \right)^{-p} \Gamma(p)}{m_1 q_1 + m_2 q_2 + 1}$$

Antiderivative was successfully verified.

[In] Int[(d1\*x^q1)^m1\*(d2\*x^q2)^m2\*(a + b\*Log[c\*x^n])^p,x]

[Out] (x\*(d1\*x^q1)^m1\*(d2\*x^q2)^m2\*Gamma[1 + p, -(((1 + m1\*q1 + m2\*q2)\*(a + b\*Log[c\*x^n]))/(b\*n))]\*(a + b\*Log[c\*x^n])^p)/(E^((a\*(1 + m1\*q1 + m2\*q2))/(b\*n))\*(1 + m1\*q1 + m2\*q2)\*(c\*x^n)^((1 + m1\*q1 + m2\*q2)/n)\*(-(((1 + m1\*q1 + m2\*q2)/(b\*n))\*(a + b\*Log[c\*x^n]))/(b\*n))))^p)

### Rule 15

Int[(u\_)\*((a\_)\*(x\_)^(n\_))^(m\_), x\_Symbol] := Dist[(a^IntPart[m]\*(a\*x^n)^FracPart[m])/x^(n\*FracPart[m]), Int[u\*x^(m\*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

### Rule 2310

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)^(p\_)\*((d\_)\*(x\_)^(m\_)), x\_Symbol] := Dist[(d\*x)^(m + 1)/(d\*n\*(c\*x^n)^((m + 1)/n)), Subst[Int[E^((m + 1)\*x/n)\*(a + b\*x)^p, x], x, Log[c\*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

### Rule 2181

Int[(F\_)^((g\_)\*((e\_) + (f\_)\*(x\_)))\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x)])/d\*(c + d\*x)]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])\*(c + d\*x)/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

### Rubi steps

$$\begin{aligned}
\int (d_1 x^{q_1})^{m_1} (d_2 x^{q_2})^{m_2} (a + b \log(cx^n))^p dx &= \left( x^{-m_1 q_1} (d_1 x^{q_1})^{m_1} \right) \int x^{m_1 q_1} (d_2 x^{q_2})^{m_2} (a + b \log(cx^n))^p dx \\
&= \left( x^{-m_1 q_1 - m_2 q_2} (d_1 x^{q_1})^{m_1} (d_2 x^{q_2})^{m_2} \right) \int x^{m_1 q_1 + m_2 q_2} (a + b \log(cx^n))^p dx \\
&= \frac{\left( x (cx^n)^{-\frac{1+m_1 q_1 + m_2 q_2}{n}} (d_1 x^{q_1})^{m_1} (d_2 x^{q_2})^{m_2} \right) \text{Subst} \left( \int e^{\frac{(1+m_1 q_1 + m_2 q_2)x}{n}} (a + b \log(e^{\frac{x}{n}}))^p dx \right)}{e^{-\frac{a(1+m_1 q_1 + m_2 q_2)}{bn}} x (cx^n)^{-\frac{1+m_1 q_1 + m_2 q_2}{n}} (d_1 x^{q_1})^{m_1} (d_2 x^{q_2})^{m_2} \Gamma \left( 1 + p, -\frac{(1+m_1 q_1 + m_2 q_2)x}{n} \right)} \\
&= \frac{n}{1 + m_1 q_1}
\end{aligned}$$

**Mathematica [A]** time = 0.226893, size = 142, normalized size = 1.04

$$\frac{(d_1 x^{q_1})^{m_1} (d_2 x^{q_2})^{m_2} x^{-m_1 q_1 - m_2 q_2} (a + b \log(cx^n))^p \exp\left(-\frac{(m_1 q_1 + m_2 q_2 + 1)(a + b \log(cx^n) - bn \log(x))}{bn}\right) \left(-\frac{(m_1 q_1 + m_2 q_2 + 1)(a + b \log(cx^n))}{bn}\right)}{m_1 q_1 + m_2 q_2 + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(d1\*x^q1)^m1\*(d2\*x^q2)^m2\*(a + b\*Log[c\*x^n])^p,x]

[Out] (x^(-(m1\*q1) - m2\*q2)\*(d1\*x^q1)^m1\*(d2\*x^q2)^m2\*Gamma[1 + p, -(((1 + m1\*q1 + m2\*q2)\*(a + b\*Log[c\*x^n]))/(b\*n))]\*(a + b\*Log[c\*x^n])^p)/(E^(((1 + m1\*q1 + m2\*q2)\*(a - b\*n\*Log[x] + b\*Log[c\*x^n]))/(b\*n))\*(1 + m1\*q1 + m2\*q2)\*(-(((1 + m1\*q1 + m2\*q2)\*(a + b\*Log[c\*x^n]))/(b\*n))))^p)

**Maple [F]** time = 19.344, size = 0, normalized size = 0.

$$\int (d_1 x^{q_1})^{m_1} (d_2 x^{q_2})^{m_2} (a + b \ln(cx^n))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d1\*x^q1)^m1\*(d2\*x^q2)^m2\*(a+b\*ln(c\*x^n))^p,x)

[Out] int((d1\*x^q1)^m1\*(d2\*x^q2)^m2\*(a+b\*ln(c\*x^n))^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (d_1 x^{q_1})^{m_1} (d_2 x^{q_2})^{m_2} (b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d1\*x^q1)^m1\*(d2\*x^q2)^m2\*(a+b\*log(c\*x^n))^p,x, algorithm="maxima")

[Out] integrate((d1\*x^q1)^m1\*(d2\*x^q2)^m2\*(b\*log(c\*x^n) + a)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((d_1 x^{q_1})^{m_1} (d_2 x^{q_2})^{m_2} (b \log(cx^n) + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d1\*x^q1)^m1\*(d2\*x^q2)^m2\*(a+b\*log(c\*x^n))^p,x, algorithm="fricas")

[Out] integral((d1\*x^q1)^m1\*(d2\*x^q2)^m2\*(b\*log(c\*x^n) + a)^p, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d1\*x\*\*q1)\*\*m1\*(d2\*x\*\*q2)\*\*m2\*(a+b\*ln(c\*x\*\*n))\*\*p,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (d_1 x^{q_1})^{m_1} (d_2 x^{q_2})^{m_2} (b \log(cx^n) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d1\*x^q1)^m1\*(d2\*x^q2)^m2\*(a+b\*log(c\*x^n))^p,x, algorithm="giac")

[Out] integrate((d1\*x^q1)^m1\*(d2\*x^q2)^m2\*(b\*log(c\*x^n) + a)^p, x)



# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*   is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*   antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
22       If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25       "C"],
26     If[FreeQ[result,Integrate] && FreeQ[result,Int],
27       "C",
28       "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
```

```

38 (*5 = hyperpergeometric function*)
39 (*6 = appell function*)
40 (*7 = rootsum function*)
41 (*8 = integrate function*)
42 (*9 = unknown function*)
43
44
45 ExpnType[expn_] :=
46   If[AtomQ[expn],
47     1,
48     If[ListQ[expn],
49       Max[Map[ExpnType,expn]],
50       If[Head[expn]===Power,
51         If[IntegerQ[expn[[2]]],
52           ExpnType[expn[[1]]],
53           If[Head[expn[[2]]]===Rational,
54             If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
55               1,
56               Max[ExpnType[expn[[1]],2]],
57             Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
58             If[Head[expn]===Plus || Head[expn]===Times,
59               Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
60             If[ElementaryFunctionQ[Head[expn]],
61               Max[3,ExpnType[expn[[1]]],
62             If[SpecialFunctionQ[Head[expn]],
63               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
64             If[HypergeometricFunctionQ[Head[expn]],
65               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
66             If[AppellFunctionQ[Head[expn]],
67               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
68             If[Head[expn]===RootSum,
69               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
70             If[Head[expn]===Integrate || Head[expn]===Int,
71               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
72             9]]]]]]]]]]
73
74
75 ElementaryFunctionQ[func_] :=
76   MemberQ[{
77     Exp,Log,
78     Sin,Cos,Tan,Cot,Sec,Csc,
79     ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
80     Sinh,Cosh,Tanh,Coth,Sech,Csch,
81     ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
82   },func]
83
84
85 SpecialFunctionQ[func_] :=
86   MemberQ[{
87     Erf, Erfc, Erfi,
88     FresnelS, FresnelC,
89     ExpIntegralE, ExpIntegralEi, LogIntegral,
90     SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
91     Gamma, LogGamma, PolyGamma,
92     Zeta, PolyLog, ProductLog,
93     EllipticF, EllipticE, EllipticPi
94   },func]
95
96
97 HypergeometricFunctionQ[func_] :=
98   MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
99
100

```



```

101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1},func]

```

## 4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 # if leaf size is "too large". Set at 500,000
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 # see problem 156, file Apostol_Problems
11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
14     debug:=false;
15
16     leaf_count_result:=leafcount(result);
17     #do NOT call ExpnType() if leaf size is too large. Recursion problem
18     if leaf_count_result > 500000 then
19         return "B";
20     fi;
21
22     leaf_count_optimal:=leafcount(optimal);
23
24     ExpnType_result:=ExpnType(result);
25     ExpnType_optimal:=ExpnType(optimal);
26
27     if debug then
28         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
29             ExpnType_optimal);
30     fi;
31
32 # If result and optimal are mathematical expressions,
33 # GradeAntiderivative[result,optimal] returns
34 # "F" if the result fails to integrate an expression that
35 # is integrable
36 # "C" if result involves higher level functions than necessary
37 # "B" if result is more than twice the size of the optimal
38 # antiderivative
39 # "A" if result can be considered optimal
40
41 #This check below actually is not needed, since I only
42 #call this grading only for passed integrals. i.e. I check
43 #for "F" before calling this. But no harm of keeping it here.
44 #just in case.
45
46 if not type(result,freeof('int')) then
47     return "F";
48 end if;
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then
56             if debug then

```

```

57         print("both result and optimal complex");
58         fi;
59         #both result and optimal complex
60         if leaf_count_result<=2*leaf_count_optimal then
61             return "A";
62         else
63             return "B";
64         end if
65     else #result contains complex but optimal is not
66         if debug then
67             print("result contains complex but optimal is not");
68         fi;
69         return "C";
70     end if
71 else # result do not contain complex
72     # this assumes optimal do not as well
73     if debug then
74         print("result do not contain complex, this assumes optimal do
not as well");
75     fi;
76     if leaf_count_result<=2*leaf_count_optimal then
77         if debug then
78             print("leaf_count_result<=2*leaf_count_optimal");
79         fi;
80         return "A";
81     else
82         if debug then
83             print("leaf_count_result>2*leaf_count_optimal");
84         fi;
85         return "B";
86     end if
87 end if
88 else #ExpnType(result) > ExpnType(optimal)
89     if debug then
90         print("ExpnType(result) > ExpnType(optimal)");
91     fi;
92     return "C";
93 end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417
102 is_contains_complex:= proc(expression)
103     return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)

```

```

119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'`^`') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'`+`') or type(expn,'`*`') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))
149   elif AppellFunctionQ(op(0,expn)) then
150     max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152     max(8,apply(max,map(ExpnType,[op(expn)]))) else
153     9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159   member(func,[
160     exp,log,ln,
161     sin,cos,tan,cot,sec,csc,
162     arcsin,arccos,arctan,arccot,arcsec,arccsc,
163     sinh,cosh,tanh,coth,sech,csch,
164     arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168   member(func,[
169     erf,erfc,erfi,
170     FresnelS,FresnelC,
171     Ei,Ei,Li,Si,Ci,Shi,Chi,
172     GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173     EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177   member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181   member(func,[AppellF1])

```

```

182 end proc:
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

### 4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #     Port of original Maple grading function by
3 #     Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #     added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:
42         if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43             return True

```

```

44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,``^`)
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn
72 )))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,``+`) or
77 type(expn,``*`)
78     m1 = expnType(expn.args[0])
79     m2 = expnType(list(expn.args[1:]))
80     return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82     return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84     m1 = max(map(expnType, list(expn.args)))
85     return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88     m1 = max(map(expnType, list(expn.args)))
89     return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
90     elif is_appell_function(expn.func):
91     m1 = max(map(expnType, list(expn.args)))
92     return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
93     elif isinstance(expn,RootSum):
94     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
95 ,Apply[List,expn]],7]],
96     return max(7,m1)
97     elif str(expn).find("Integral") != -1:
98     m1 = max(map(expnType, list(expn.args)))
99     return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
100     else:
101     return 9
102
103 #main function
104 def grade_antiderivative(result,optimal):
105
106     leaf_count_result = leaf_count(result)

```

```

102 leaf_count_optimal = leaf_count(optimal)
103
104 expnType_result = expnType(result)
105 expnType_optimal = expnType(optimal)
106
107 if str(result).find("Integral") != -1:
108     return "F"
109
110 if expnType_result <= expnType_optimal:
111     if result.has(I):
112         if optimal.has(I): #both result and optimal complex
113             if leaf_count_result <= 2*leaf_count_optimal:
114                 return "A"
115             else:
116                 return "B"
117         else: #result contains complex but optimal is not
118             return "C"
119     else: # result do not contain complex, this assumes optimal do not as
well
120         if leaf_count_result <= 2*leaf_count_optimal:
121             return "A"
122         else:
123             return "B"
124 else:
125     return "C"

```

#### 4.0.4 SageMath grading function

```

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by
2 #     Albert Rich to use with Sagemath. This is used to
3 #     grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #     'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
(flatten(tree(anti))))
33         return round(1.35*len(flatten(tree(anti)))) #fudge factor
34             #since this estimate of leaf count is bit lower than

```

```

35         #what it should be compared to Mathematica's
36
37 def is_sqrt(expr):
38     debug=False;
39     if expr.operator() == operator.pow: #isinstance(expr,Pow):
40         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
41             if debug: print ("expr is sqrt")
42             return True
43         else:
44             return False
45     else:
46         return False
47
48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func , " is special_function")
83         else:
84             print ("func ", func , " is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','
91     hypergeometric_U']
92
93 def is_appell_function(func):
94     return func.name() in ['hypergeometric'] #[appellf1] can't find this in
95     sagemath

```

```

95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
104             return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list: #isinstance(expn,list):
121         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
127     elif expn.operator() == operator.pow: #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
129             return expnType(expn.operands()[0]) #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
132                 return 1
133             else:
134                 return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137     elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138         m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139         m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
141     elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
142         return max(3,expnType(expn.operands()[0]))
143     elif is_special_function(expn.operator()): #is_special_function(expn.func)
144         m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))

```



```

145     return max(4,m1)    #max(4,m1)
146     elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
148         return max(5,m1)    #max(5,m1)
149     elif is_appell_function(expn.operator()):
150         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
151         return max(6,m1)    #max(6,m1)
152     elif str(expn).find("Integral") != -1: #this will never happen, since it
153         #is checked before calling the grading function that is passed.
154         #but kept it here.
155         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
156         return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
157     else:
158         return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)
176
177     if expnType_result <= expnType_optimal:
178         if result.has(I):
179             if optimal.has(I): #both result and optimal complex
180                 if leaf_count_result <= 2*leaf_count_optimal:
181                     return "A"
182                 else:
183                     return "B"
184             else: #result contains complex but optimal is not
185                 return "C"
186         else: # result do not contain complex, this assumes optimal do not as
well
187             if leaf_count_result <= 2*leaf_count_optimal:
188                 return "A"
189             else:
190                 return "B"
191     else:
192         return "C"

```